Quantum Mechanics I - Module 2

1. Consider the following potential:

$$V(x) = \begin{cases} 0 & x < 0 \\ V_0 & x > 0 \end{cases}$$

where $E > V_0$. In class we solved (or will solve) the problem for this potential for a particle approaching the step from the left-hand side (see Section 6.4 in the textbook). In workshop, develop a solution for a particle that approaches the step from the right-hand side. Compare the reflection and transmission coefficients for this case with those obtained in the textbook.

2. Consider a system with potential energy given by:

$$V(x) = \frac{1}{2} m\omega^2 x^2$$

a) What kind of potential is this?
b) What does it mean for a function to be an eigenfunction of a system?
c) Show that: $\phi_0 = Ae^{-\alpha x^2}$ is an eigenfunction of this system for a particular value of α.
d) What is the value of α?
e) What is the value of the energy E for this eigenfunction?

3. What is an operator? What are the position, momentum, and energy operators in one dimension?

4. What is the expectation value? What is the general expression for the expectation value of an operator?

5. A large part of quantum mechanics is devoted to the study of the solutions of the Schrödinger equation for various potentials. What are the forms of the potential for the following systems:

(a) A free particle.
(b) A particle in a box of length a.
(c) A harmonic oscillator.

6. What are the conditions that the wavefunction must satisfy for the particle in a box problem?