Do not turn the pages of the exam until you are instructed to do so.

Exam rules: You may use **only** a writing instrument while taking this test. You may **not** consult any calculators, computers, books, nor each other.

Problems 1 and 2 must be answered in exam booklet 1. Problems 3 and 4 must be answered in exam booklet 2. The answers need to be well motivated and expressed in terms of the variables used in the problem. You will receive partial credit where appropriate, but only when we can read your solution. Answers that are not motivated will not receive any credit, even if correct.

At the end of the exam, you must hand in your exam, the blue exam booklets, and the equation sheet. All items must be clearly labeled with your name, your student ID number, and the day/time of your recitation. **If any of these items are missing, we will not grade your exam, and you will receive a score of 0 points.**

You are required to complete the following *Honor Pledge for Exams*. Copy and sign the pledge before starting your exam.

"I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own."

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Name: ____________________________________

Signature: ________________________________

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Good Luck !

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Problem 1 (25 points) **Answer in booklet 1**

Consider the first few low-lying states of the helium atom. When the helium atom is in its ground state, both electrons are in $n = 1$ states. When the Helium atom is in one of the first excited states, one electron is in an $n = 1$ state and the other electron is in an $n = 2$ state. The spatial wavefunction of electron i can be written as

$$(x_i; y_i; z_i) \text{ or } \quad (i) \tag{1}$$

where contains information about the spatial quantum numbers. The spin wavefunction of the two-electron system, $(s_{z;1}; s_{z;2})$, can be written as combinations of the following four functions

$$(+\frac{1}{2}; +\frac{1}{2}); (+\frac{1}{2}; -\frac{1}{2}); (-\frac{1}{2}; +\frac{1}{2}); \text{ and } (-\frac{1}{2}; -\frac{1}{2}) \tag{2}$$

(a) **(5 points)** Write down all possible total wavefunctions for the ground state of helium ($n_1 = 1; n_2 = 1$).

(b) **(5 points)** Write down all possible total wavefunctions for the first low-lying excited states of helium ($n_1 = 1; n_2 = 2$).

(c) **(5 points)** Make an energy diagram showing the location of the low-lying energy levels of helium ($n_1 = 1$ and $n_2 = 1; 2$), assuming there is no Coulomb interaction between the two electrons. Label each level with the n and ` values of each of the two electrons and their total spin. **Note: the actual location of these energy levels is not important, but their relative position is.**

(d) **(5 points)** Now include the effect of the Coulomb interaction between the electrons, but ignore the exchange force. What is the effect of the Coulomb interaction on the energy of the states shown in the diagram constructed in (c)? In the energy diagram, indicate the shifts and/or splitting of the energy levels of the low-lying states of helium due to the Coulomb interaction. Label each level with the n and ` values of each of the two electrons and their total spin. **Note: the relative shifts of the levels are important and need to be correctly motivated and drawn.**

(e) **(5 points)** Finally, include the effect of the exchange force. What is the effect of the exchange force on the energy of the states shown in the diagram constructed in (d)? In the energy diagram, indicate the shifts and/or splitting of the energy levels of the low-lying states of helium due to the exchange force. Label each level with the n and ` values of each of the two electrons and their total spin. **Note: the relative shifts of the levels are important and need to be correctly motivated and drawn.**

Your answers need to be well motivated and expressed in terms of the variables provided.

Problem 2 (25 points) **Answer in booklet 1**

In a one-dimensional system the number of energy states per unit energy is

$$N(\text{"}) = \frac{L}{h}\sqrt{\frac{2m}{\text{"}}} \tag{3}$$

where L is the length of the system and m is the mass of the electron. There are N electrons in this sample and each state can be occupied by at most two electrons.

(a) **(15 points)** Determine the Fermi energy at 0 K.

(b) **(10 points)** Determine the average energy per electron at 0 K.

Your answers need to be well motivated and expressed in terms of the variables provided.

Problem 3 (25 points) **Answer in booklet 2**

Consider the the ^{12}C atom. The ground state configuration of this atom has a $1s^2 2s^2 2p^2$ configuration. The two electrons in the $2p$ state are optically active. Some of the energy levels of ^{12}C are shown in Fig. 1.

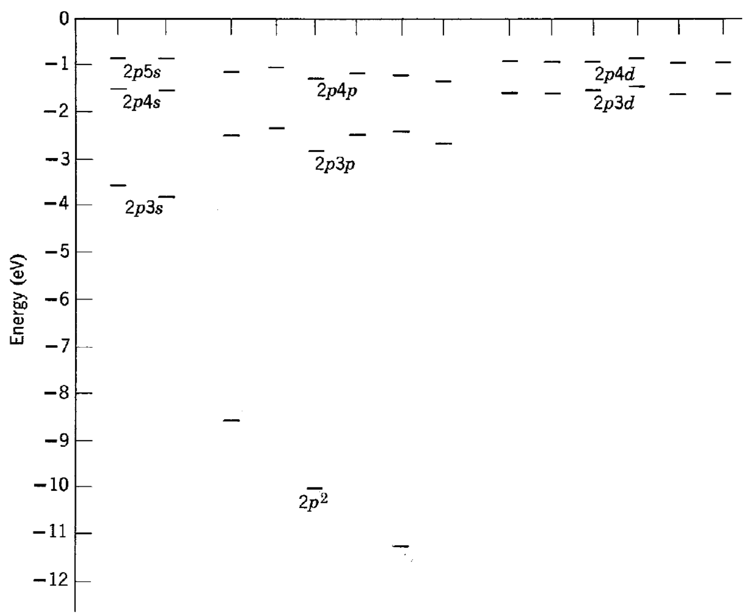

Figure 1: Some energy levels in ^{12}C.

Three different levels at energies around -10 eV correspond to both electrons in the $2p$ state. The higher lying states (between -4 eV and -1 eV) correspond to configurations where one electron is promoted to the $3s$, $4s$, or $5s$ state (on the left), or to the $3p$ or $4p$ state (in the middle), or to the $3d$ or $4d$ state (on the right).

(a) **(10 points)** What is the spectroscopic notation of the three states you expect to see when both electrons are in the $2p$ state?

Make sure you justify your answer. No points will be awarded for just writing down the spectroscopic notation of the three states.

(b) **(5 points)** Rank the three states identified in part (a) according to their energy.

Make sure you justify your answer.

(c) **(10 points)** What is the spectroscopic notation of the six states that can be populated when one $2p$ electron is promoted to a $3d$ state?

Make sure you justify your answer. No points will be awarded for just writing down the spectroscopic notation of the six states.

Your answers need to be well motivated.

Problem 4 (25 points) **Answer in booklet 2**

Short answer questions. No derivations are needed

(a) **(5 points)** Figure 2 shows measured transitions between atomic levels when there is no external field (top) and when there is a weak external field.

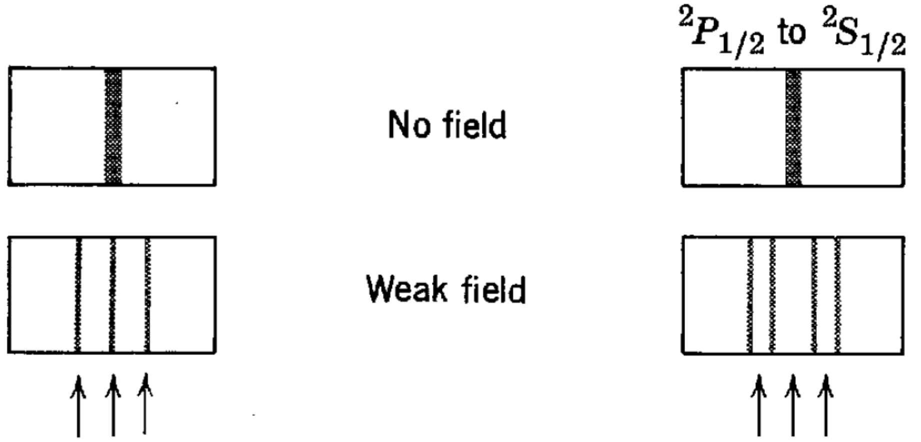

Figure 2: Transitions between atomic levels with and without an external magnetic field.

i. What is the name of the effect shown on the left-hand side in Fig. 2?
ii. What is the name of the effect shown on the right-hand side in Fig. 2?

(b) **(5 points)** Consider the three processes shown in Fig. 3 for a two-level atom.

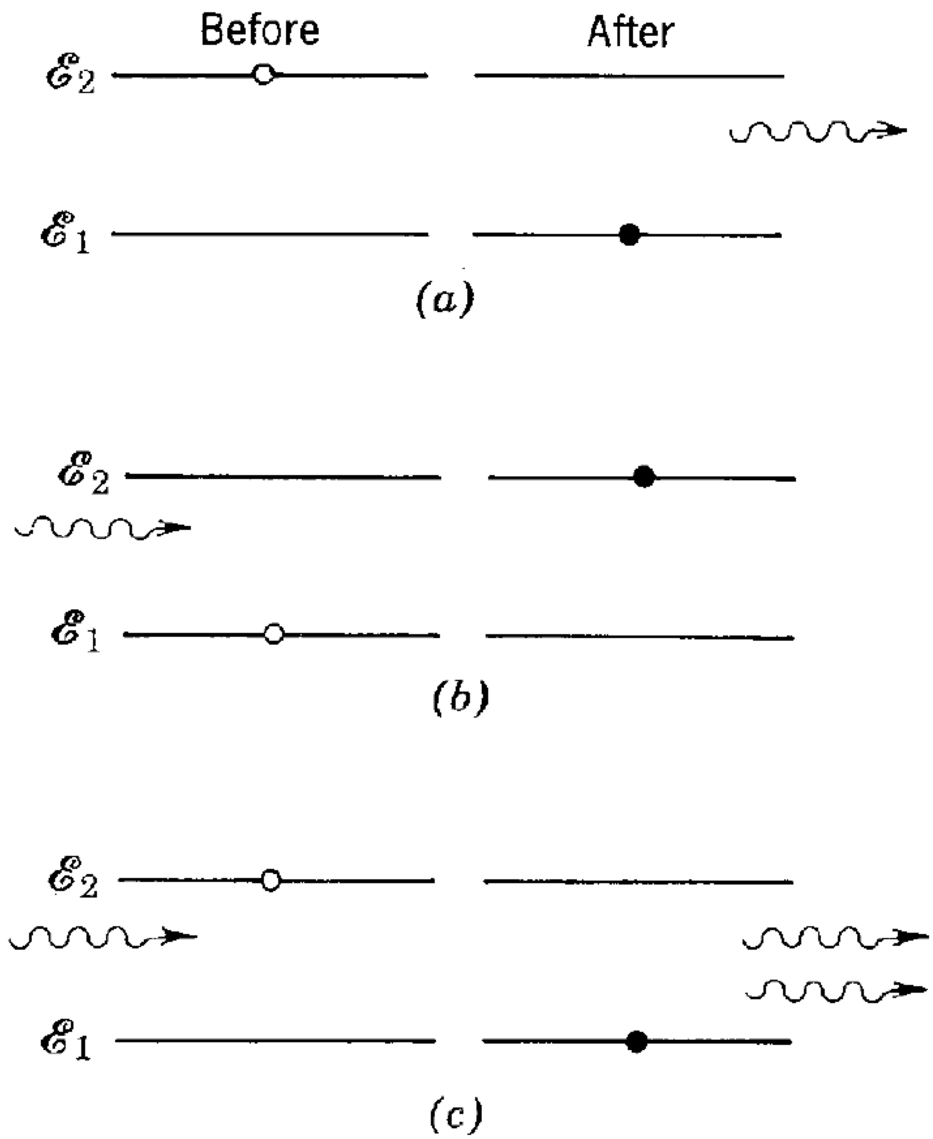

Figure 3: Atomic transitions

What are the names of the processes shown in Fig. 3a, Fig. 3b, and Fig. 3c?

(c) **(5 points)** Consider particle occupancy distributions shown in Fig. 4 as function of "=kT for two different values of .

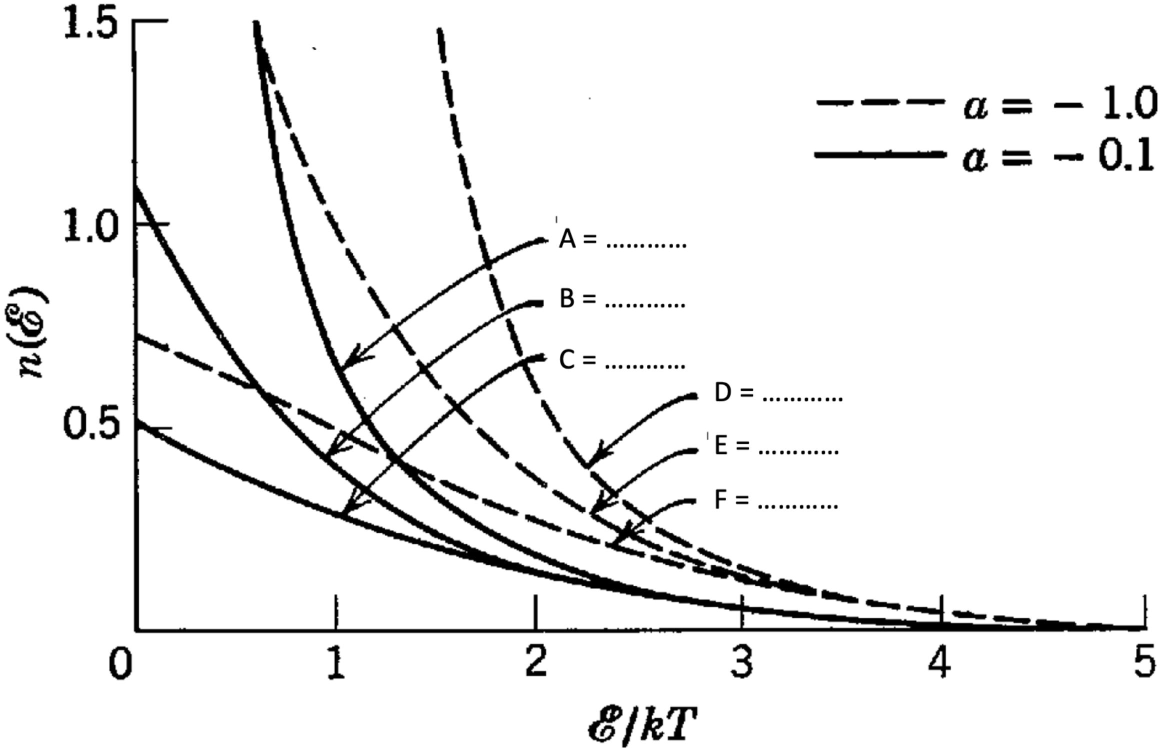

Figure 4: Variou particle occupancy distributions.

What is the name of each of the six occupancy distributions? In your blue booklet, label each distributions by the letter used in Fig. 4:

i. A = ...
ii. B = ...
iii. C = ...
iv. D = ...
v. E = ...
vi. F = ...

(d) **(5 points)** Figure 5 shows a poorly labelled figure.

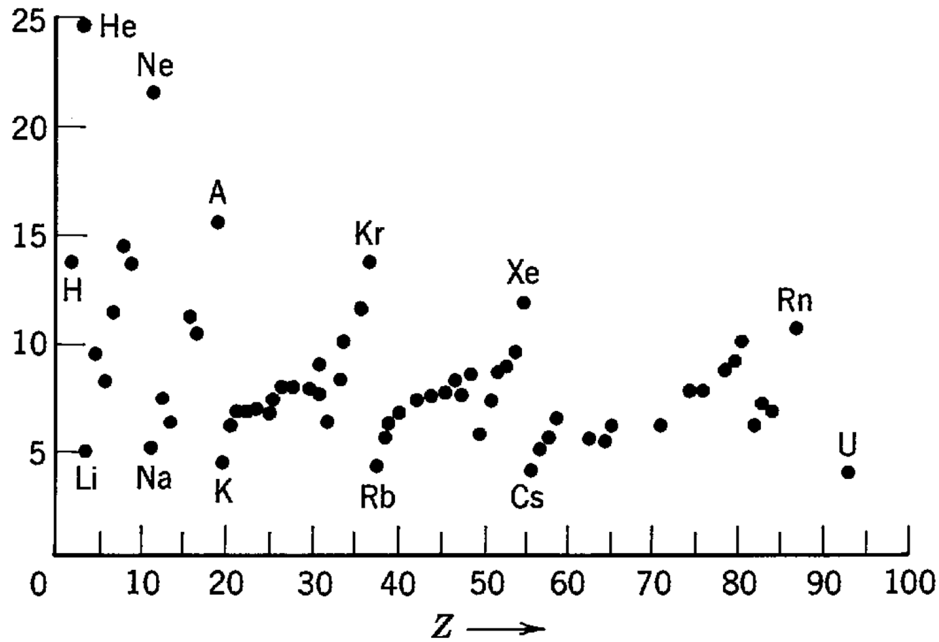

Figure 5: Poorly labelled figure.

What is the quantity that is shown on the vertical axis of Fig. 5?

(e) **(5 points)** In which country did Pieter Zeeman do the work for which he was awarded the 1902 Nobel Prize in Physics?

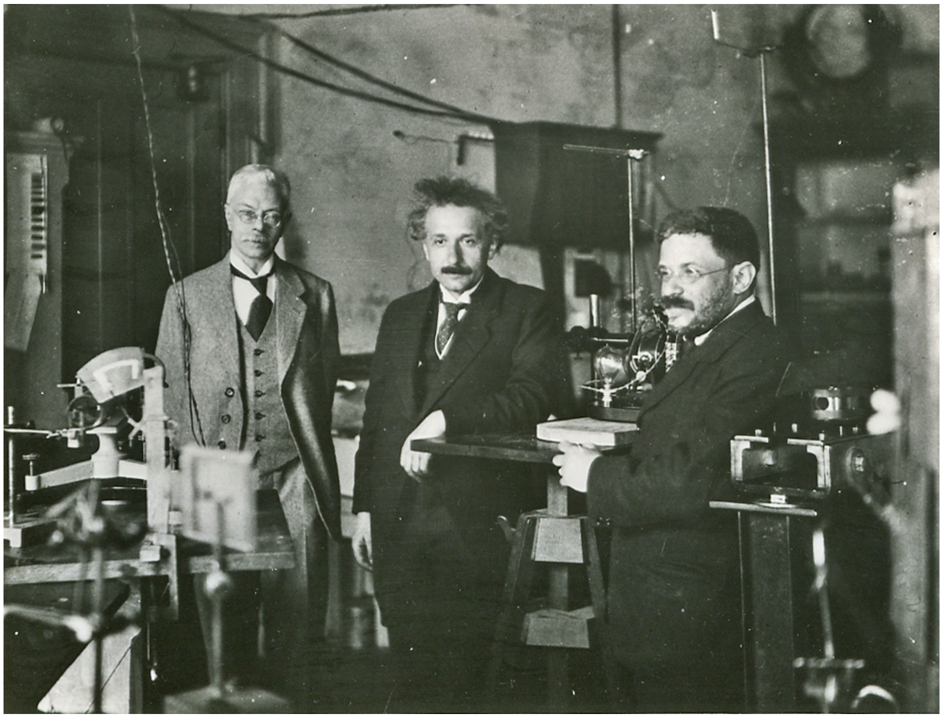

Figure 6: Zeeman, Einstein, and Ehrenfest.

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