Do not turn the pages of the exam until you are instructed to do so.

Exam rules: You may use **only** a writing instrument while taking this test. You may **not** consult any calculators, computers, books, nor each other.

Problems 1 and 2 must be answered in exam booklet 1. Problems 3 and 4 must be answered in exam booklet 2. The answers need to be well motivated and expressed in terms of the variables used in the problem. You will receive partial credit where appropriate, but only when we can read your solution. Answers that are not motivated will not receive any credit, even if correct.

At the end of the exam, you must hand in your exam, the blue exam booklets, and the equation sheet. All items must be clearly labeled with your name, your student ID number, and the day/time of your recitation. **If any of these items are missing, we will not grade your exam, and you will receive a score of 0 points.**

You are required to complete the following *Honor Pledge for Exams*. Copy and sign the pledge before starting your exam.

"I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own."

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Name: ____________________________________

Signature: ________________________________

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Good Luck !

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Problem 1 (25 points) **Answer in booklet 1**

Consider the lowest energy eigenfunctions of a one-electron atom.

(a) **(10 points)** What is the parity of the various $\psi_{3,2,m}$ wavefunctions?
Note: Make sure you provide an answer for each possible value of m.

(b) **(15 points)** Evaluate the electric dipole matrix element to determine whether transitions between the $\psi_{2,0,0}$ state and the $\psi_{1,0,0}$ state are possible.
Note: You **cannot** use the selection rules to justify your answer. You will need to evaluate the expectation value of the electric dipole matrix element.

Your answers need to be well motivated and expressed in terms of the variables provided.

Problem 2 (25 points) **Answer in booklet 1**

A particle of mass m and energy $E = \hbar^2 k^2/(2m)$ is incident from the left on a potential

$$V(x) = \begin{cases} \frac{\hbar^2 k^2}{2m} & 0 \le x \le L \\ 0 & x < 0 \text{ and } x > L \end{cases} \tag{1}$$

(a) **(5 points)** What is the wave function in the region $x < 0$?

Note: you do not yet have to determine the value of the constants that appear in the wavefunction.

Note 2: you can set the amplitude of the incident wave to 1.

(b) **(5 points)** What is the wave function in the region $0 \le x \le L$? **Note:** you do not yet have to determine the value of the constants that appear in the wavefunction.

(c) **(5 points)** What is the wave function in the region $x > L$?

Note: you do not yet have to determine the value of the constants that appear in the wavefunction.

(d) **(10 points)** Calculate the transmission coefficient.

Your answers need to be well motivated and expressed in terms of the variables provided. The wavenumber k should be treated as a known variable.

Problem 3 (25 points) **Answer in booklet 2**

Consider the ground-state wavefunction of the hydrogen atom.

(a) **(15 points)** What is the expectation value $< V >$ of the potential energy of the hydrogen atom when it is in its ground state?

(b) **(5 points)** Express the energy of the ground state of the hydrogen atom in terms of the expectation value $< V >$ of the potential energy.

(c) **(5 points)** What is the expectation value of the kinetic energy of the electron when the hydrogen atom is in its ground state?

Your answers need to be well motivated and expressed in terms of the variables provided.

Problem 4 (25 points) **Answer in booklet 2**

Short answer questions. No derivations are needed

(a) **(5 points)** The apparatus shown in Fig. 1 was used by Stern and Gerlach in 1922 to measure the magnetic dipole moment of silver atoms.

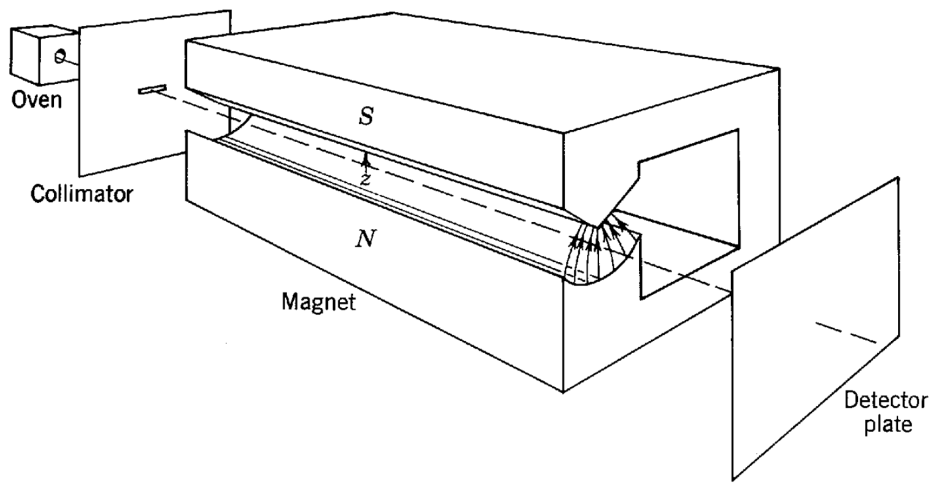

Figure 1: Schematic of the Stern and Gerlach experiment.

What fundamental property of the electron was discovered as a result of these measurements?

(b) **(5 points)** Consider the three wavefunctions shown in Fig. 2.

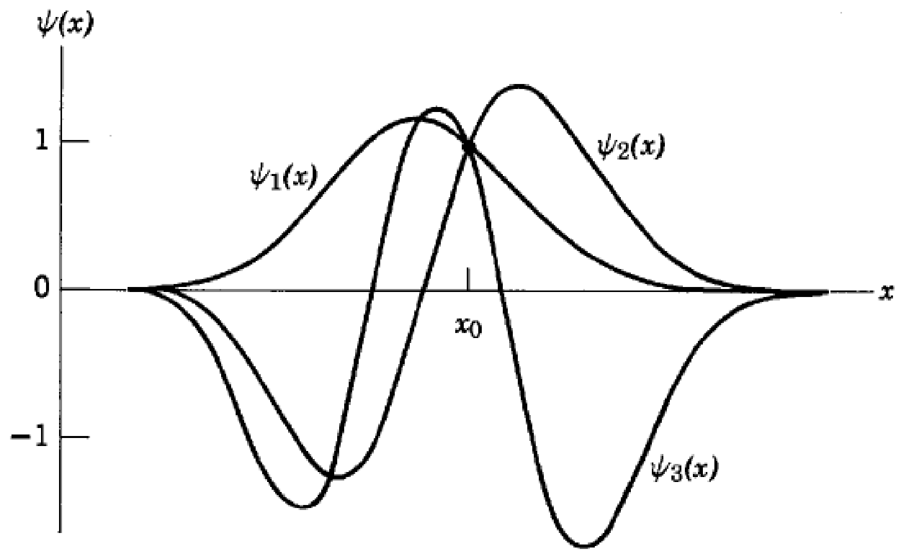

Figure 2: Eigenfunctions of a potential well.

Rank the three eigenfunctions (ψ_1, ψ_2, and ψ_3) based on their energy, starting with the eigenfunction with the highest energy and ending with the eigenfunction with the lowest energy.

(c) **(5 points)** Consider the four potential distributions shown on the left-hand side of Fig. 3. For each potential distribution shown in Fig. 3 the energy of the system is also indicated. Assume a particle of energy E is approaching the barrier from the left.

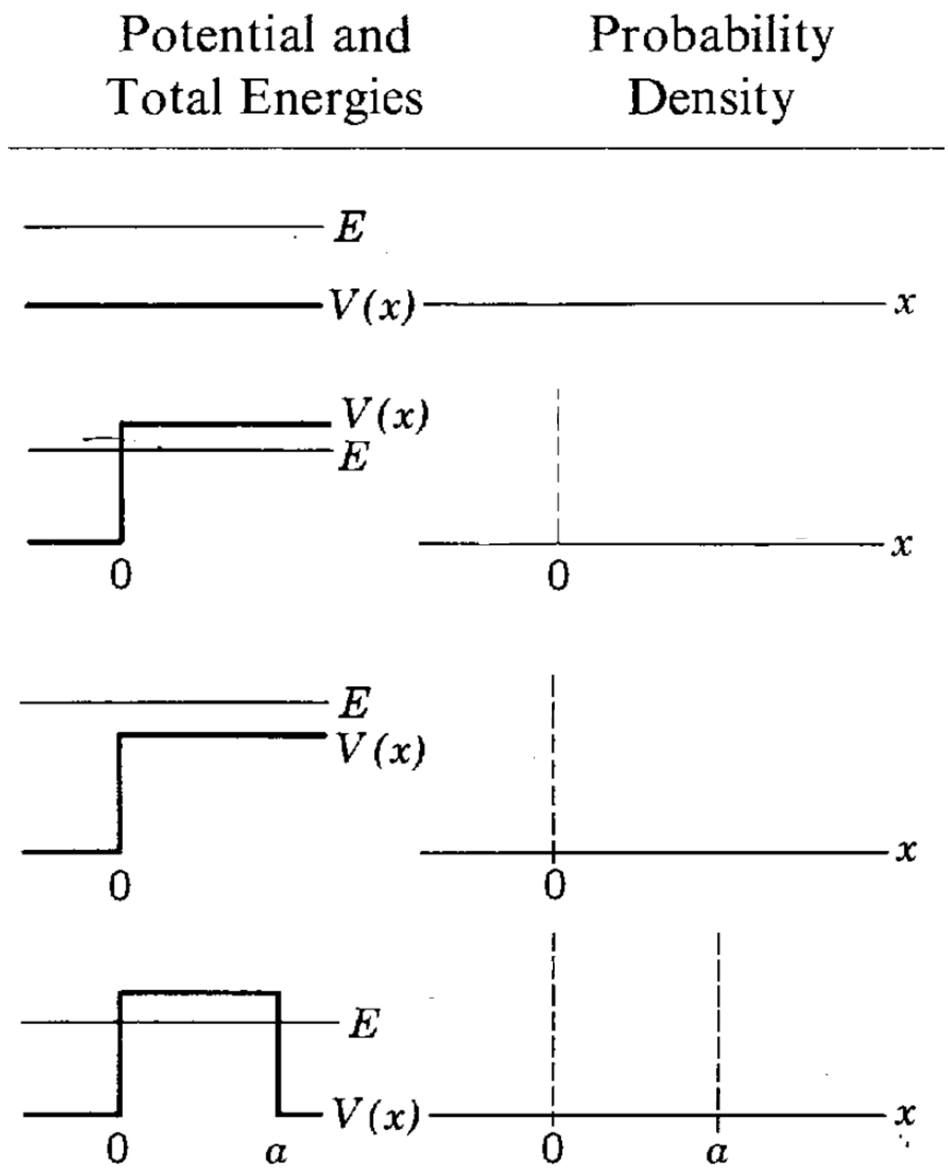

Figure 3: Various potentials and total energies.

For each potential distribution shown in Fig. 3, sketch the corresponding probability density distribution in your blue booklet. Make sure that your probability density distribution covers the same region in x that is covered by the potential distributions shown on the left-hand side in Fig. 3.

(d) **(5 points)** Consider the polar diagrams shown in Fig. 4. Note that the polar diagrams for $\pm m$ are the same and only the positive values of m are shown in this Figure.

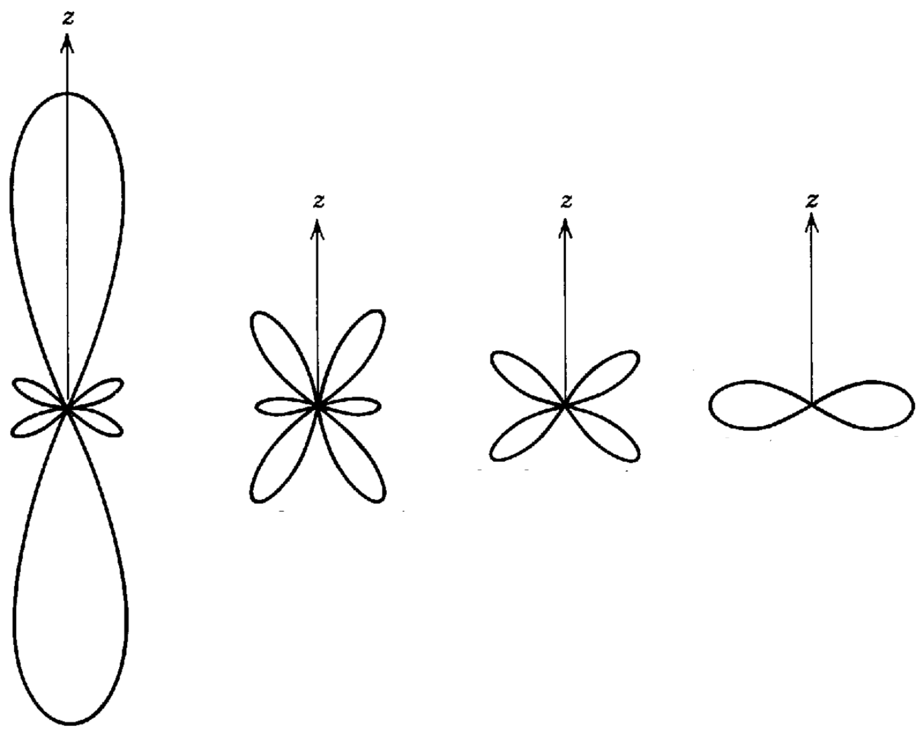

Figure 4: Polar diagrams for a one-electron atom.

The diagrams cover all possible m values for one specific value of ℓ. For each diagram, identify the value of ℓ and m. In your blue booklet, identify the four polar diagrams shown in Fig. 4 as left, middle left, middle right, and right.

(e) **(5 points)** What happens when the wind speed exceeds 30 miles per hour?

Figure 5: Video shown during lecture on February 24, 2022.

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