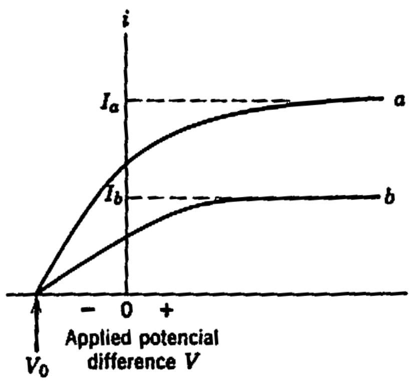
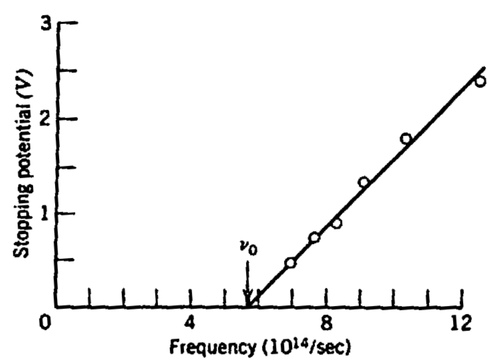
Studies of many properties of light, such as diffraction, required the use of the wave-model of light (see Table below). One effect, **the photoelectric effect (which is the ejection of electrons from a surface when it is exposed to light)**, could not be explained using the wave-model of light.

	Explained by Wave Model	Explained by Particle Model
Reflection	Yes	Yes
Refraction	Yes	Yes
Interference	Yes	No
Diffraction	Yes	No
Polarization	Yes	No
Photoelectric Effect	No	Yes





Detailed studies of the maximum electron energy and the number of emitted electrons, obtained by measuring the voltage required to decelerate/stop them and the electron current, show that:

- The number of emitted electrons, obtained from the maximum current at positive potential differences, is proportional to the intensity of the light.
- The maximum electron energy, obtained from the stopping potential V_0, is independent of the intensity of the light.
- The maximum electron energy is dependent on the frequency of the light.

Robert Millikan made major contributions to the study of the photoelectric effect and in 1923 he was awarded the Nobel Prize for this work.

Albert Einstein proposed the following explanation of the photoelectric effect:

- Light is a collection of photons.
- Each photon has an energy $h\nu$.

- When the intensity of a beam of light increases, the number of photons increases, but the energy of each individual photon remains the same.
- The incident photon is absorbed by a single electron and the energy of the emitted electron is

$$K_{e^-} = h\nu - W$$

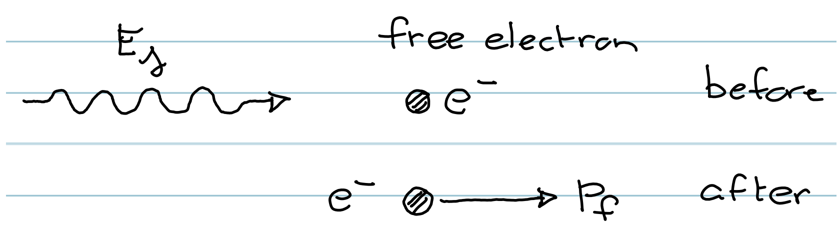
  where W is the work function of the material (the energy required to break the bond between the electron and the atom).
- The potential difference required to stop the electron is

$$V_0 = \frac{K_{e^-}}{e} = \frac{h}{e}\nu - \frac{W}{e}$$

  The slope of the stopping potential as function of the frequency is equal to h/e and can be used to determine Planck's constant. The measured slope is in excellent agreement with the accepted value of Planck's constant.

In 1921, Albert Einstein received the Nobel Prize for his explanation of the photoelectric effect.

Note: Only electrons bound to atoms can absorb photons. Consider what is required for a free electron to absorb a photon by examining the constraints imposed by conservation of linear momentum and conservation of energy (note: the interaction has to be elastic since the electron does not have internal structure).



The initial linear momentum of the system is just the linear momentum of the photon since we assume the electron is at rest.

$$p_i = p_\gamma = \frac{E_\gamma}{c}$$

The total initial energy is the sum of the energy of the photon and the rest energy of the electron.

$$E_i = E_\gamma + mc^2$$

After the absorption, the final linear momentum and energy will be equal to the final linear momentum and energy of the electron.

$$p_f = p_e$$

$$E_f = \sqrt{\left(p_e c\right)^2 + \left(mc^2\right)^2}$$

Since linear momentum and energy must be conserved, we rewrite these relations in the following way:

$$p_e = \frac{E_\gamma}{c}$$

$$E_f^2 = \left(p_e c\right)^2 + \left(mc^2\right)^2 = E_\gamma^2 + \left(mc^2\right)^2 = E_i^2 = \left(E_\gamma + mc^2\right)^2 = E_\gamma^2 + 2E_\gamma\left(mc^2\right) + \left(mc^2\right)^2$$

The last equation can be used to show that energy conservation is only satisfied if

$$2E_\gamma\left(mc^2\right) = 0 \quad \Rightarrow \quad E_\gamma = 0$$

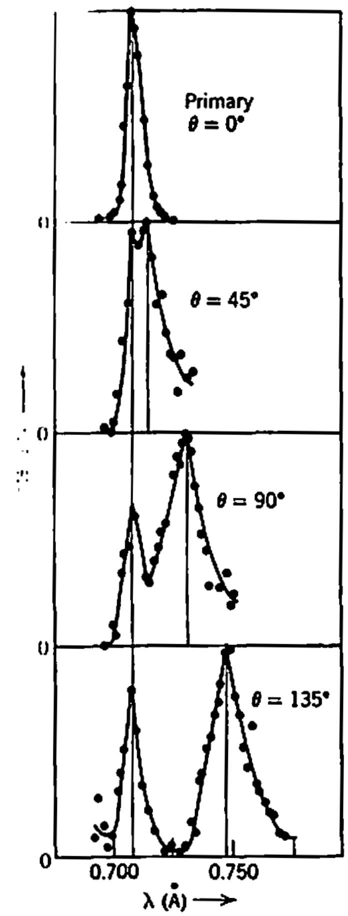
We thus conclude that a free electron cannot absorb a photon.

The particle nature of light was confirmed by a series of experiments carried out by Arthur Compton, who studied the scattering of X-rays from matter. In 1927, Compton received the Nobel Prize for his work.

In the wave-model, X-rays with a well-defined wavelength would be considered electromagnetic waves with a well-defined frequency. When the electromagnetic waves are incident on a target containing free electrons, the free electrons will start to oscillate with the frequency of the incident electromagnetic waves. The electromagnetic waves generated by the oscillating electrons will have the same frequency as the frequency of the incident waves, and the wavelength of the scattered waves would thus be the same as the wavelength of the incident waves. Compton observed, that the wavelength contributions of the scattered waves contains two components:

- One component has a wavelength identical to the wavelength of the incident wave.
- One component has a wavelength that is larger than the wavelength of the incident wave.

The component with the longer wavelengths, clearly visible in the distributions shown in the Figure on the right, can only be explained if it is assumed that the X-rays behave like particles and the scattering of the X-rays from the free electrons can be described in terms of two-dimensional elastic scattering.



Consider the kinematics of the scattering process, shown in the Figure at the bottom of the page. Conservation of linear momentum in the x and the y direction requires:

$$p_0 = p_1 \cos\theta + p\cos\varphi$$
$$0 = p_1 \sin\theta - p\sin\varphi$$

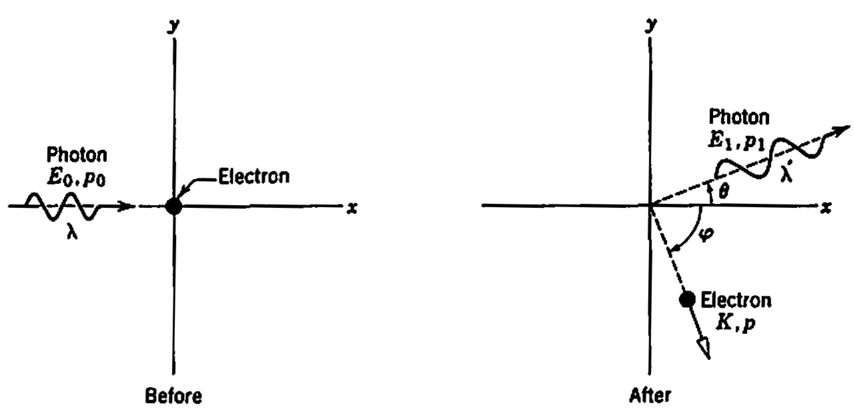
Conservation of energy requires:

$$E_0 + mc^2 = E_1 + K + mc^2$$

The conditions for conservation of linear momentum can be rewritten as

$$\left(p_0 - p_1\cos\theta\right)^2 = \left(p\cos\varphi\right)^2$$
$$\left(p_1\sin\theta\right)^2 = \left(p\sin\varphi\right)^2$$
$$\overline{p_0^2 + p_1^2 - 2p_0p_1\cos\theta = p^2}$$

The condition for conservation of energy can be rewritten in terms of the linear momenta of the X-rays before and after scattering, and can be used to determine the final kinetic energy of the electron:



$$K = E_0 - E_1 = cp_0 - cp_1 = \sqrt{(pc)^2 + (mc^2)^2} - mc^2$$

This equation can be rewritten as

$$c(p_0 - p_1) + mc^2 = \sqrt{(pc)^2 + (mc^2)^2} \quad \Rightarrow \quad \left(c(p_0 - p_1) + mc^2\right)^2 = (pc)^2 + (mc^2)^2 \quad \Rightarrow$$

$$(p_0 - p_1)^2 + 2(p_0 - p_1)(mc) + (mc)^2 = p^2 + (mc)^2 \quad \Rightarrow$$

$$p^2 = (p_0 - p_1)^2 + 2(p_0 - p_1)(mc)$$

We now have two different expressions for the linear momentum of the electron and can use these two expressions to derive a relation between the linear momenta of the X-rays before and after the scattering process:

$$p_0^2 + p_1^2 - 2p_0 p_1 \cos\theta = (p_0 - p_1)^2 + 2(p_0 - p_1)(mc) \quad \Rightarrow$$

$$2p_0 p_1 (1 - \cos\theta) = 2(p_0 - p_1)(mc) \quad \Rightarrow \quad (1 - \cos\theta) = \left(\frac{1}{p_1} - \frac{1}{p_0}\right)(mc) = \left(\frac{\lambda_1}{h} - \frac{\lambda_0}{h}\right)(mc) \quad \Rightarrow$$

$$\lambda_1 - \lambda_0 = \frac{h}{mc}(1 - \cos\theta) = \lambda_C (1 - \cos\theta)$$

The constant λ_C in the last equation is called the **Compton Wavelength** that is equal to 0.0243Å for scattering of electrons. The Compton wavelength decreases with increasing mass and scattering of the entire atom would produce a negligible Compton wavelength. The two peaks observed in the wavelength distributions of the scattered X rays correspond to scattering from electrons and scattering from atoms.

Note: The shift in wavelength is independent of the wavelength. The effect can therefore only be observed for small wavelengths due to the experimental resolution.

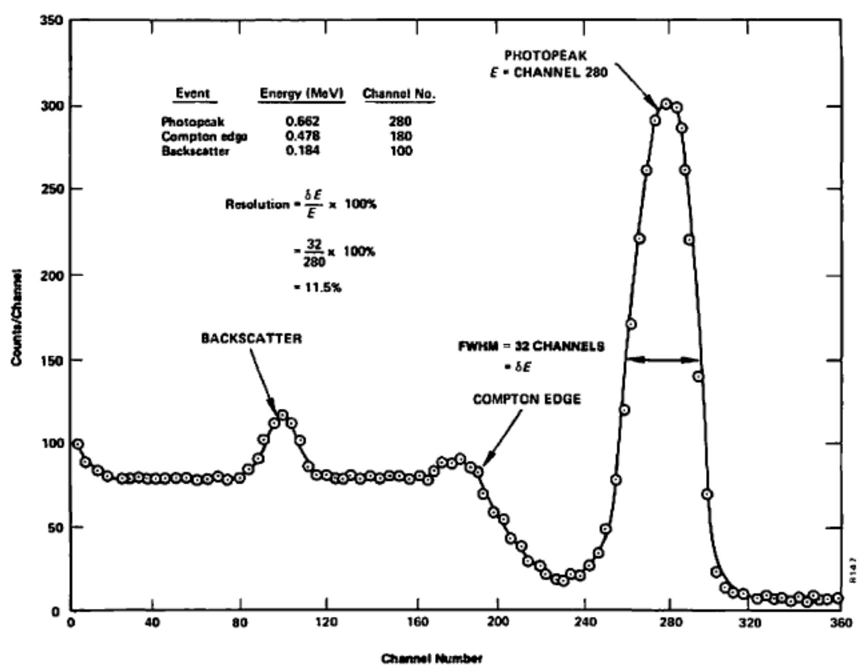
We can rewrite the relation between the initial and the final wavelengths in terms of the photon energies:

$$\lambda_1 - \lambda_0 = \frac{hc}{E_1} - \frac{hc}{E_0} = \frac{h}{mc}(1-\cos\theta) \quad \Rightarrow \quad \frac{1}{E_1} - \frac{1}{E_0} = \frac{1}{mc^2}(1-\cos\theta) \quad \Rightarrow$$

$$E_1 = \frac{1}{\dfrac{1}{E_0} + \dfrac{1}{mc^2}(1-\cos\theta)} = \frac{E_0}{1 + \dfrac{E_0}{mc^2}(1-\cos\theta)} \approx \frac{E_0}{1+2E_0(1-\cos\theta)}$$

In the last step we have assumed that all energies are specified in MeV and approximated 1/0.511 by 2.

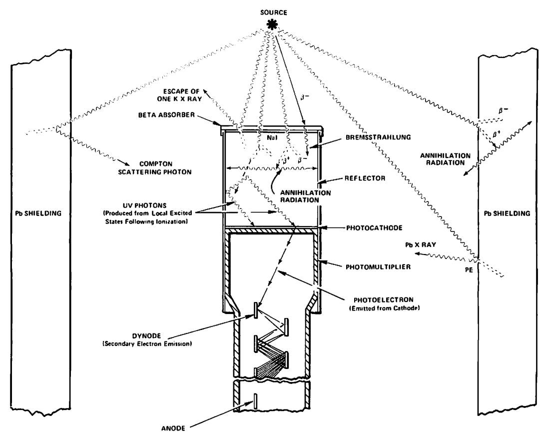
Consider the detection of the gamma rays emitted by a ^{137}Cs source. This source emits gamma rays with energies of 0.662 MeV. These gamma rays can be detected using a NaI detector (see energy spectrum at the bottom of this page). Gamma rays loose energy in matter as a result of their interactions with the electrons of the detector material. After several interactions, the photon will have transferred all of its energy to the electrons. The transfer of energy causes the electrons to emit ultra-violet photons that are converted into an electric signal using a photomultiplier tube. The integral of the electric signal is proportional to the number of ultra-violet photons and thus to the energy deposited in the detector. The Figure at the bottom of this page shows the measured energy deposited in a 2" NaI detector. In addition to a peak at channel 280, which corresponds to a deposition of 0.662 MeV, a significant number of interactions deposit a much smaller amount of energy. The largest energy loss of a photon in a single interaction happens when it back scatters (θ = 180°). The energy of a 0.662 MeV photon after



scattering by 180° is equal to

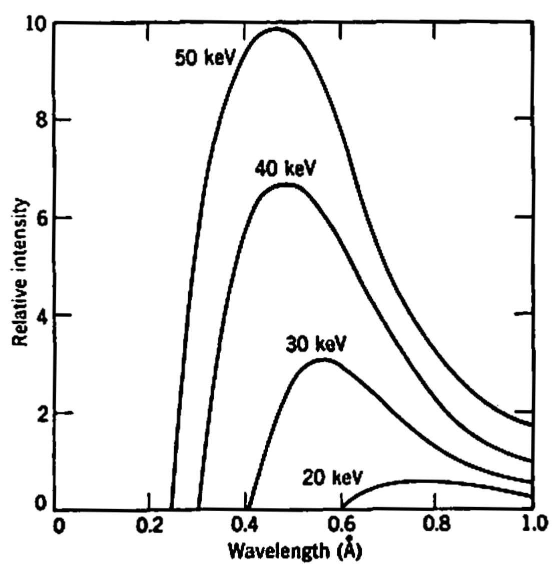
$$E_1 \approx \frac{E_0}{1+2E_0\left(1-\cos\theta\right)} = \frac{0.662}{1+1.324\left(1+1\right)} = 0.181 \text{ MeV}$$

The energy transferred to the electron is thus 0.662 MeV − 0.181 MeV = 0.481 MeV. If the scattered photon leaves the detector, the measured energy is thus 0.481 MeV. If the scattering angle is less than 180°, the energy transferred to the electron will be less than 0.481 MeV. This process is responsible for the Compton edge in the energy spectrum measured with the NaI detector. Another notable feature in the spectrum is a peak labeled "backscatter". This peak is due to gamma rays from the source, scattering from materials around the detector. Using the previous calculation of the maximum energy loss of photons in Compton scattering, we expect that a backscattered photon enters the detector with an energy of 0.181 MeV. If it deposits all of its energy in the detector, it will produce a peak at 0.181 MeV, which is where the backscattered peak is observed.



X-rays can be produced when electrons slow down. The initial studies of X-ray production were carried out with stopping electrons. The following observations were made in these experiments (see the Figure on the right):

- The energy (or wavelength) distributions of the X-rays are continuous.
- There is a minimum wavelength. No X-rays are observed with a wavelength below this minimum wavelength.
- The minimum wavelength depends on the energy loss of the electrons.
- The minimum wavelength does not depend on the stopping material.



These observations can only be explained using the particle picture of light.

Consider the scattering process shown in the Figure at the bottom of this page. Since energy is conserved in the interaction, the energy of the emitted X-ray is equal to

$$h\nu = K - K' = \frac{hc}{\lambda}$$

The maximum X-ray energy occurs when the electron stops (*K'* = 0). The corresponding minimum wavelength is equal to

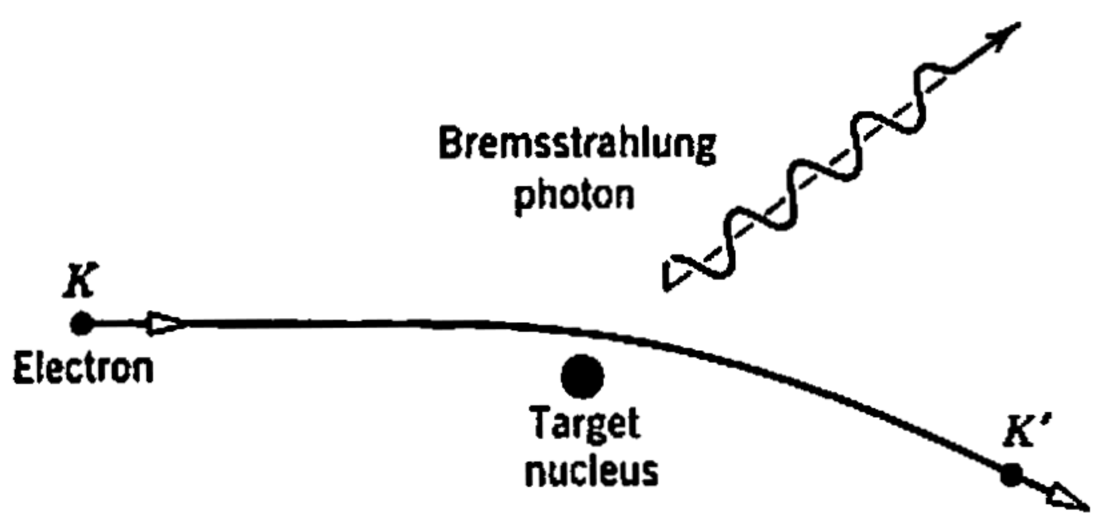
$$\lambda_{\min} = \frac{hc}{K} = \frac{\left(6.626\times10^{-34}\right)\left(2.998\times10^{8}\right)}{K} = \frac{1.986\times10^{-25}}{K}$$

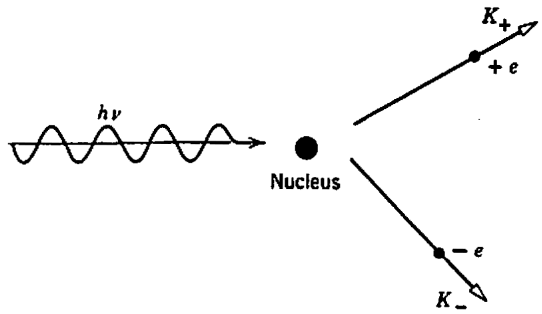
The kinetic energy of the electron is determined by the potential difference used to accelerate them. An electron with an energy of 20 keV is expected to produce a minimum wavelength of

$$\lambda_{\min} = \frac{1.986\times10^{-25}}{\left(20\times10^{3}\right)\left(1.602\times10^{-19}\right)} = 0.62\times10^{-10}\ \text{m}$$

which is in good agreement with the observed minimum wavelength. When the energy of the electron is doubled, the minimum wavelength should go down by a factor of two. This is also in good agreement with the observations.

The photoelectric effect and the Compton effect are two effects by which photons loose energy when they interact with matter. Another process by which photons loose energy is **pair production**.





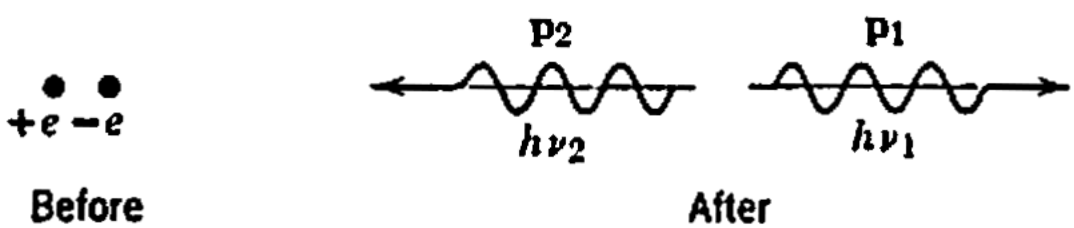
In pair production, the photon is converted into an electron-positron pair. Energy conservation in this process requires that

$$E_\gamma = E_{e^-} + E_{e^+} = K_{e^-} + K_{e^+} + 2m_e c^2$$

This process cannot happen in empty space. Why not? We can always find a reference frame in which the electron and the positron are back-to-back and have linear momenta of equal magnitude. In this frame, the total linear momentum of the pair is 0. Conservation of linear momentum requires that the linear momentum of the photon is also 0 but this can never happen. In the presence of a nucleus, the nucleus will ensure that linear momentum is conserved. However, the nucleus will increase the energy of the emitted positron and decrease the energy of the emitted electron due to the Coulomb interaction between the electron/positron and the positively charged nucleus.

The opposite of pair creation is **annihilation**. This process is indicated schematically in the Figure at the bottom of this page. Consider an electron and a positron that are initially at rest with respect to each other. The initial linear momentum of the system is thus 0. The electron and positron pair cannot be converted into a single photon since any photon has a non-zero linear momentum. However, conversion into a photon pair is possible, as shown on the right in the Figure. In order for the linear momentum of the system to be zero, the photons are emitted back-to-back and have the same energy and thus the same linear momentum. Conservation of energy requires that

$$E_i = 2m_e c^2 = E_f = E_{\gamma 1} + E_{\gamma 2} = 2E_{\gamma 1}$$



The energy of each photon is thus equal to

$$E_{\gamma 1} = \frac{1}{2}\left(2m_e c^2\right) = m_e c^2 = 0.511 \text{ MeV}$$

The back-to-back nature of the photons can be used to do ray tracing to determine the location of the source.

Consider an electron with linear momentum p. The energy of the electron can be calculated using the following relation:

$$E^2 = (pc)^2 + \left(mc^2\right)^2 \quad \Rightarrow \quad E = \pm\sqrt{(pc)^2 + \left(mc^2\right)^2}$$

The energy can thus take on positive and negative values and are schematically shown in the Figure at the bottom of this page. Dirac assumed that all negative energy levels are normally filled everywhere. In order for a free electron to be created, it needs to be moved from a negative energy level to a positive energy level. As can be seen in the energy diagram below, the minimum energy required to achieve this is $2mc^2$. When the electron is move to the positive energy levels, a vacancy is left behind in a negative energy level. This vacancy behaves like a positron!

Due to the various effects described in this Chapter, photons loose energy when they travel through matter. The main mechanisms are:

- The photoelectric effect. This effect dominates at low energies (< 0.5 MeV).
- Rayleigh and Compton scattering. The effect dominates at intermediate energies.
- Pair production. This effect dominates at energies above 5 MeV.

The probability of an interaction is expressed in terms of a **cross section**. When a photon flux I is incident on a slab of material with n atoms per unit area, the number of photons absorbed per unit time is equal to

$$N = \sigma I n$$

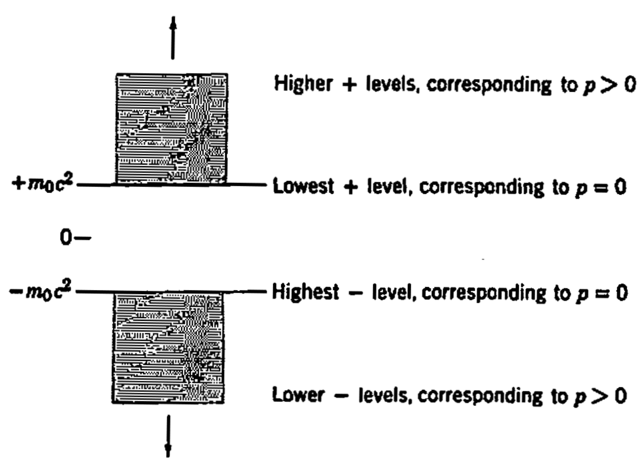


Figure 2-15 The energy levels of a free electron according to Dirac.

where σ is the cross section. The unit of cross section is area and the cross section depends on the atom and on the photon energy. For the strong interaction, the cross section is approximately equal to $\pi R_{\text{nucleus}}{}^2$. For the electromagnetic interaction, the cross section is approximately equal to $\pi R_{\text{atom}}{}^2$.

The intensity of the beam of photons after it passes though material depends on the thickness of the material. The intensity after a distance t is equal to

$$I(t) = I(0)e^{-\sigma\rho t}$$

where ρ is the density of the material. The product of the cross section and the density, $\sigma\rho$, is called the **attenuation coefficient**. The **attenuation length** is defined as $1/(\sigma\rho)$ and the thickness of material that attenuates the incident beam by a factor e.