# Classical Mechanics
# Phy 235, Lecture 25.

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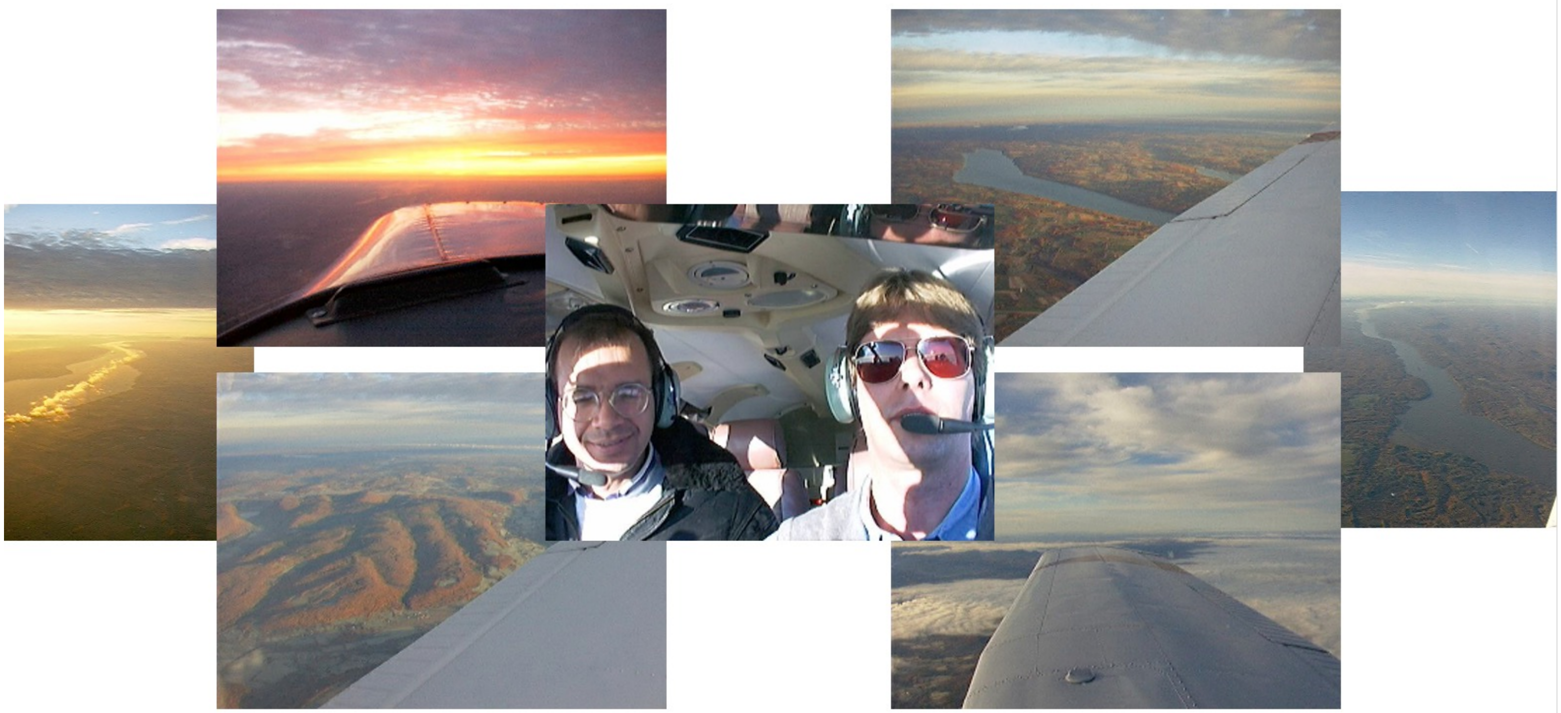
# Yesterday: Sunday December 5.
# An important evening: pakjes avond.


# Course Announcements.

- There will be office hours today to help you with questions about Exam # 3. Details were distributed via email. Come prepared to ask questions about the exam.
- Exam # 3 can be picked up on Monday December 13 during my office hours.
- Any questions related to the grading of exam # 3 need to be submitted to Prof. Wolfs by Friday December 17. Drop your exam and a note describing why you feel you deserve more points in the PHY 235 homework locker.
- The final exam will cover Chapter 1 – 13. The exam will take place on Tuesday December 14, 12.30 pm – 3.30 pm in B&L 109.
- Extra office hours will be scheduled for Monday December 13.

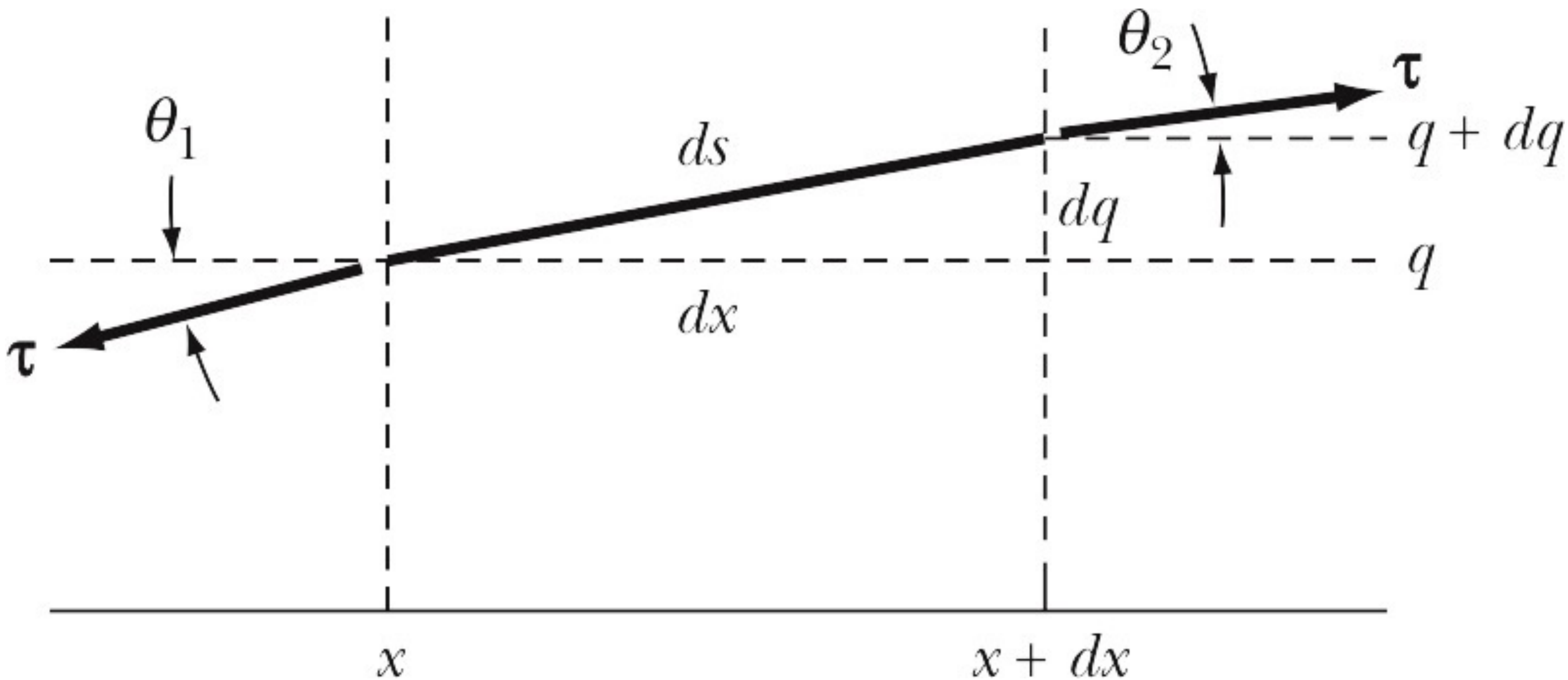
# On Wednesday ……
# the truth about Bernoulli.

## The Physics of Flying!
## Lecture 26.



Views of New York State from 9000'.

# The wave equation.



# The wave equation.

- The ideal wave equation:

$$\frac{\partial^2 q}{\partial x^2} = \frac{\rho}{\tau}\frac{\partial^2 q}{\partial t^2}$$

- The "real" wave equation:

$$\frac{\partial^2 q}{\partial x^2} - \frac{D}{\tau}\frac{\partial q}{\partial t} + \frac{F(x,t)}{\tau} = \frac{\rho}{\tau}\frac{\partial^2 q}{\partial t^2}$$

# Problem 13.11.

- When a particular driving force is applied to a string, it is observed that the string's vibration is purely in the n$^{th}$ harmonic. Find the driving force.

$$f_s(t) = \int_0^L F(x,t) \sin \frac{s\pi x}{b} \, dx = 0 \quad \text{for } s \neq n$$

$$= \int_0^L F(x,t) \sin \frac{s\pi x}{b} \, dx \neq 0 \quad \text{for } s = n$$

# Solving the ideal wave equation.

- The ideal wave equation:

$$\frac{\partial^2 q}{\partial x^2} = \frac{\rho}{\tau}\frac{\partial^2 q}{\partial t^2}$$

- No dissipation: energy is conserved.
- Use **separation of variables** to solve the wave equation:

$$q(x,t) = \psi(x)\chi(t)$$

- This results on two differential equations:

$$\frac{v^2}{\psi}\frac{\partial^2 \psi}{\partial x^2} = \omega^2 \quad \Leftrightarrow \quad \frac{\partial^2 \psi}{\partial x^2} - \frac{\omega^2}{v^2}\psi = 0$$

$$\frac{1}{\chi}\frac{\partial^2 \chi}{\partial t^2} = \omega^2 \quad \Leftrightarrow \quad \frac{\partial^2 \chi}{\partial t^2} - \omega^2 \chi = 0$$

# 4 Minute 07 Second Intermission.

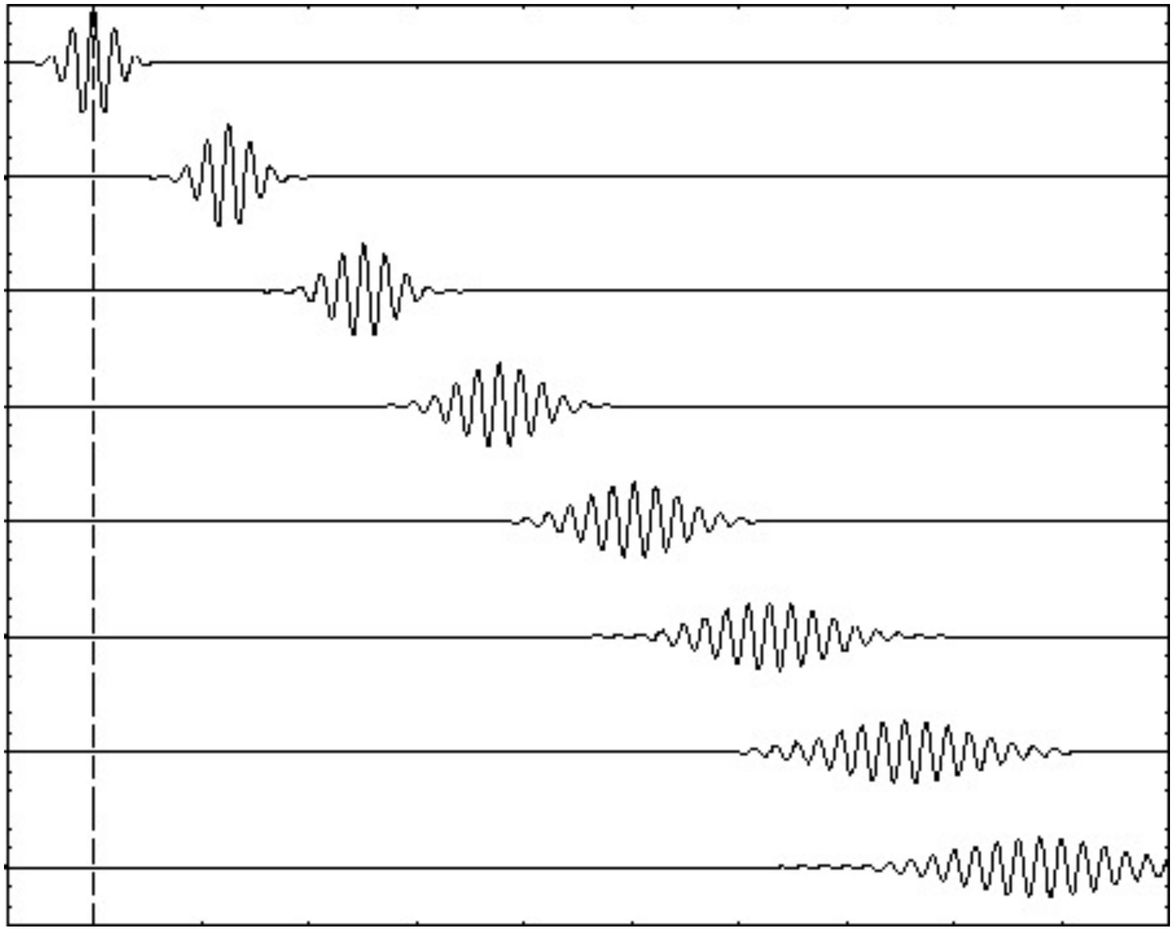
- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 4 minute 07 second intermission.

- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.

# Wave velocities

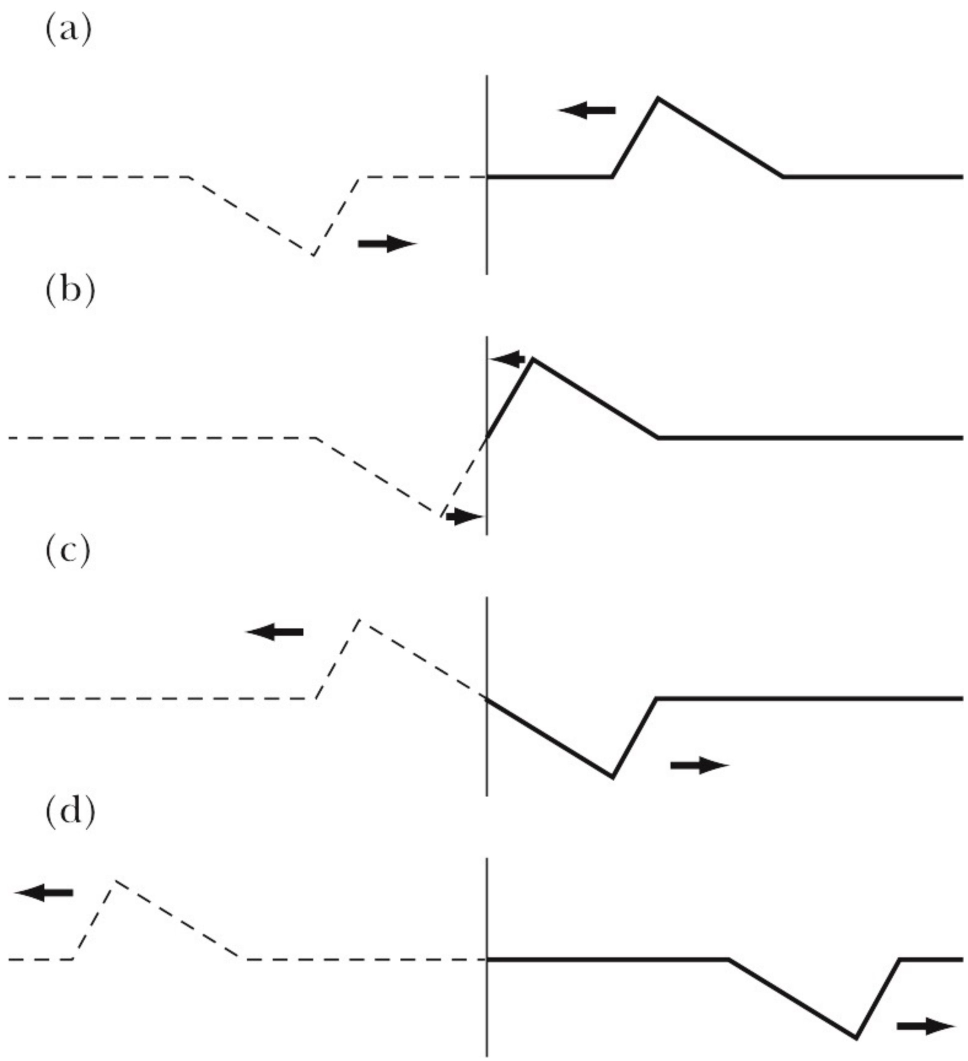
- Wave velocity:
  - Velocity that keeps the amplitude constant: $v$.
- Phase velocity:
  - Velocity that keeps the phase constant: $V$.
- The velocities depend on the wave number. When this is the case, the medium is called a **dispersive medium**.

# Dispersion.
# Limits information transmission.



http://jick.net/skept/GWP/

# Wave Propagation: reflection at fixed end.



# Other wave features.

- Two wave travelling in opposite directions can create a **standing wave**:

$$q(x,t) = A\left\{e^{-ik(x+vt)} + e^{-ik(x-vt)}\right\} = 2Ae^{-ikx}\cos(\omega t)$$

- For the loaded string:
  - There is a minimum wave length. $$\lambda_n = \frac{2L}{n}$$
  - There is a maximum wave number. $$k_n = \frac{2\pi}{\lambda_n} = \frac{2\pi}{\left(\frac{2L}{n}\right)} \simeq \frac{\pi}{d}$$
  - There is a maximum frequency. $$\omega_n = 2\sqrt{\frac{\tau}{md}}\sin\left\{\frac{k_n d}{2}\right\} = 2\sqrt{\frac{\tau}{md}}$$

# Complex wavenumbers.

- Higher frequencies can be supported if the wavenumber becomes complex:

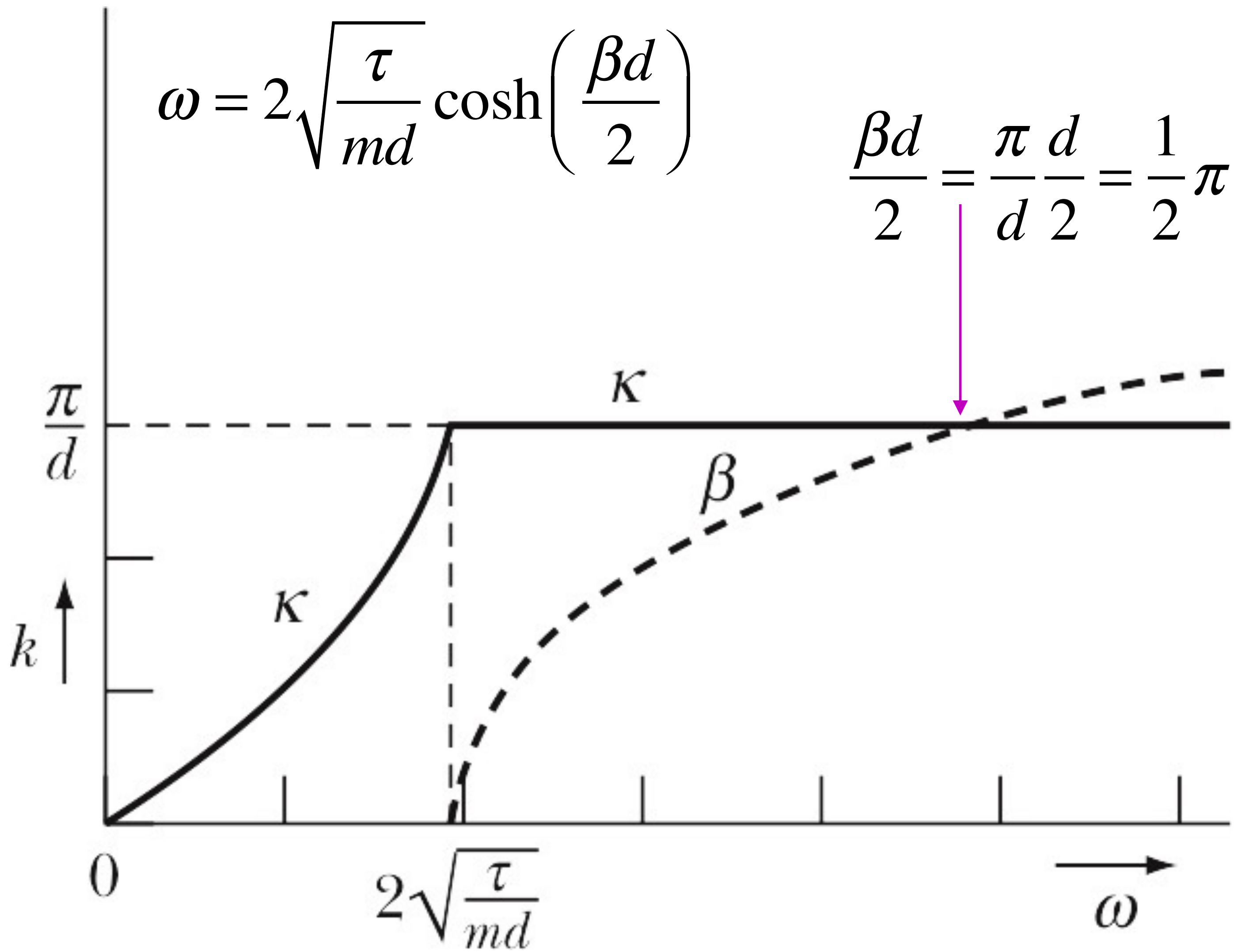
$$
\begin{aligned}
\omega &= 2\sqrt{\frac{\tau}{md}}\sin\left\{\frac{d}{2}(\kappa - i\beta)\right\} = \\
&= 2\sqrt{\frac{\tau}{md}}\left\{\sin\left(\frac{d}{2}\kappa\right)\cos\left(\frac{i\beta d}{2}\right) - \cos\left(\frac{d}{2}\kappa\right)\sin\left(\frac{i\beta d}{2}\right)\right\} = \\
&= 2\sqrt{\frac{\tau}{md}}\left\{\sin\left(\frac{d}{2}\kappa\right)\cosh\left(\frac{\beta d}{2}\right) - i\cos\left(\frac{d}{2}\kappa\right)\sinh\left(\frac{\beta d}{2}\right)\right\}
\end{aligned}
$$

Note: the equation in my notes is missing $\kappa$ in the last two steps.

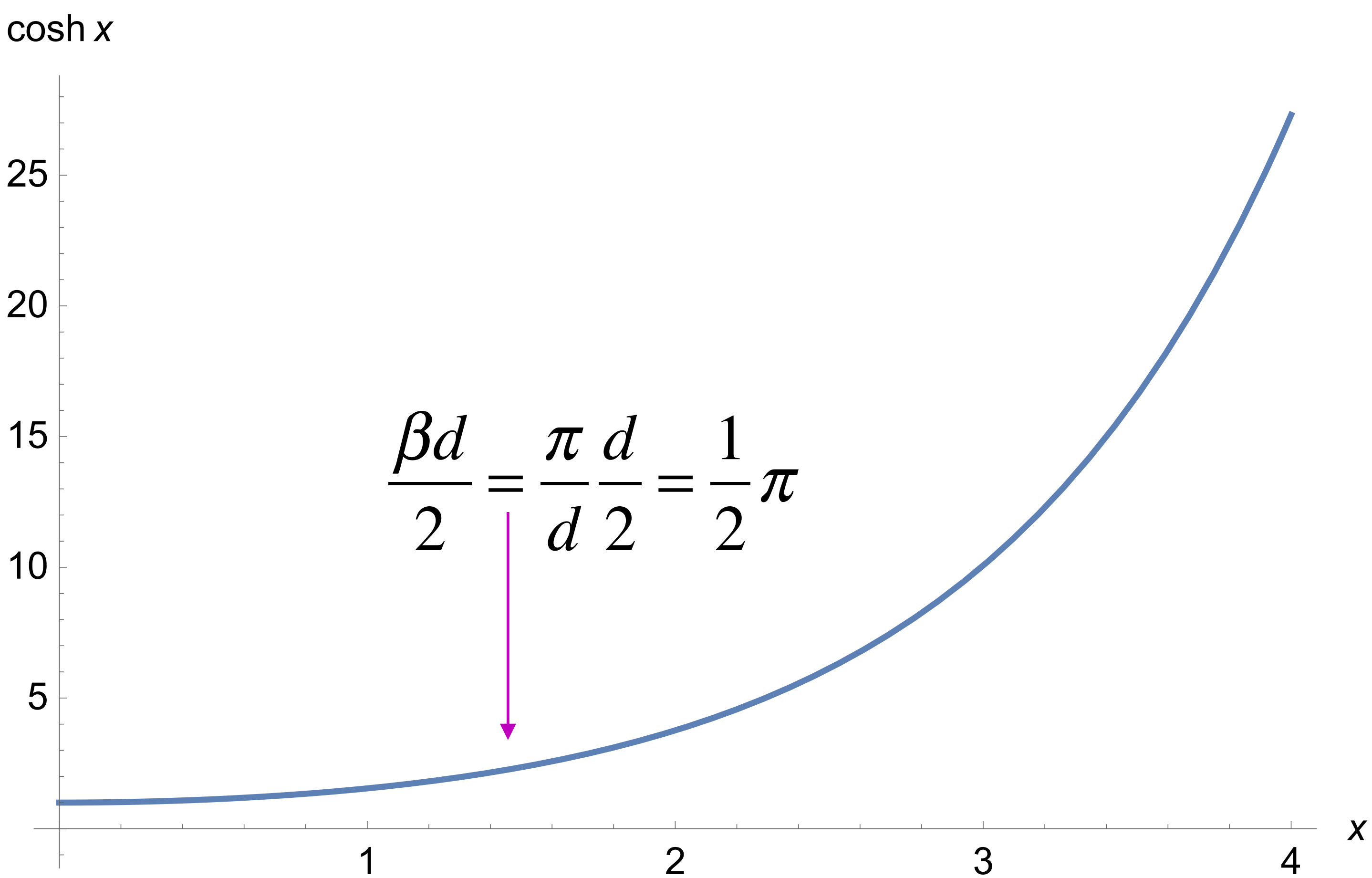
# Solutions at high frequencies.

$$\omega = 2\sqrt{\frac{\tau}{md}}\cosh\left(\frac{\beta d}{2}\right)$$

$$\frac{\beta d}{2} = \frac{\pi}{d}\frac{d}{2} = \frac{1}{2}\pi$$



# $\cosh(x)$ function.



# ENOUGH FOR TODAY?