Classical Mechanics
Phy 235, Lecture 23.

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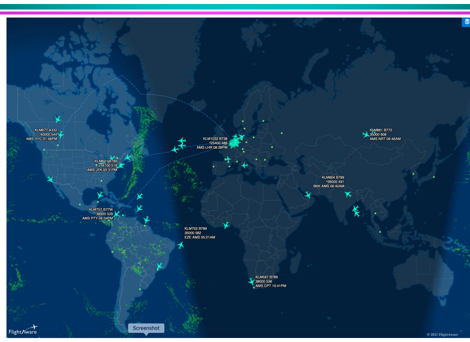
A beautiful landing at SFO.

2

A small country; a huge reach.
KLM planes flying at 3.04 pm on 11/23.



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N Coupled Oscillators

- We will have n coupled equations (A and m are the amplitude and mass tensors):

$$\sum_k \left(A_{kj} - \omega^2 m_{kj} \right) a_k = 0$$

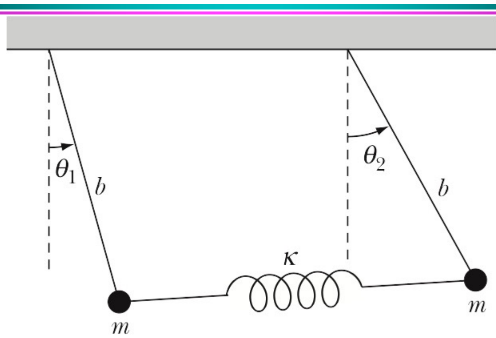
- This set of equation will have non-trivial solutions if

$$\begin{vmatrix} A_{11} - \omega^2 m_{11} & A_{12} - \omega^2 m_{12} & A_{13} - \omega^2 m_{13} & \ldots\ldots \\ A_{12} - \omega^2 m_{12} & A_{22} - \omega^2 m_{22} & A_{23} - \omega^2 m_{23} & \ldots\ldots \\ A_{13} - \omega^2 m_{13} & A_{32} - \omega^2 m_{32} & A_{33} - \omega^2 m_{33} & \ldots\ldots \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \end{vmatrix} = 0$$

Steps

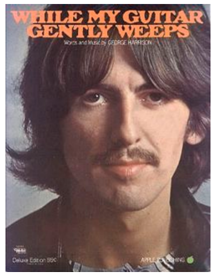
- Follow these steps in order to solve most coupled oscillator problems:
  - Choose generalized coordinates.
  - Determine the A and m tensors.
  - Determine the eigen frequency and the eigen vectors.
  - Determine the scale factors required to match the initial conditions.
  - Determine the normal coordinates.

Example 12.4.

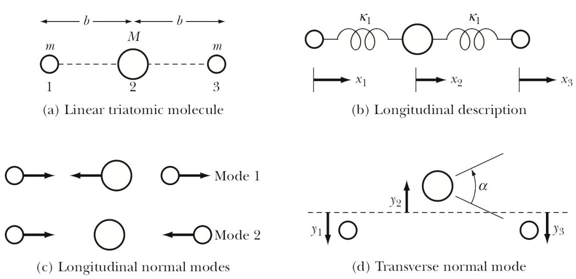


4 Minute 47 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 4 minute 47 second intermission.
- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.



Molecular Vibrations



Problem 12.21.

Three oscillators of equal mass *m* are coupled such that the potential energy of the system is given by

$$U = \frac{1}{2}\left[\kappa_1\left(x_1^2 + x_3^2\right) + \kappa_2 x_2^2 + \kappa_3\left(x_1 x_2 + x_2 x_3\right)\right]$$

where

$$\kappa_3 = \sqrt{2\kappa_1\kappa_2}$$

Find the eigen frequencies by solving the secular equation. What is the physical interpretation of the zero-frequency mode?

General steps to solve this type of problems.

- Find {A}.

$$\{\mathbf{A}\} = \begin{bmatrix} \kappa_1 & \frac{1}{2}\kappa_3 & 0 \\ \frac{1}{2}\kappa_3 & \kappa_2 & \frac{1}{2}\kappa_3 \\ 0 & \frac{1}{2}\kappa_3 & \kappa_1 \end{bmatrix}$$

- Solve secular determinant.

$$\begin{vmatrix} \kappa_1 - m\omega^2 & \frac{1}{2}\kappa_3 & 0 \\ \frac{1}{2}\kappa_3 & \kappa_2 - m\omega^2 & \frac{1}{2}\kappa_3 \\ 0 & \frac{1}{2}\kappa_3 & \kappa_1 - m\omega^2 \end{vmatrix} = 0$$

- Find {m}.

$$\{\mathbf{m}\} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}$$

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I wish you call a happy and safe thanksgiving holiday.

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ENOUGH FOR TODAY?

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