Classical Mechanics
Phy 235, Lecture 22.

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KLM.
Not only for transportation of people.

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Announcements

- No recitations and office hours this week.
- The due date of the Phy 235 term paper is Wednesday November 24 at noon:
  - You need to submit the draft and the final version in pdf format to the dropbox link that is posted on the PHY 235 web. Use the following naming convention: **FirstDraftPhy235XXYYYYYYYY.pdf** and **FinalPaperPhy235XXYYYYYYYY.pdf** where XX is your last name and YYYYYYYY is your student id number.
  - You need to discuss the draft with the writing center fellows, I need to receive a confirmation from them that they have discussed the draft with you, and I need to be able to see that you addressed their comments on your draft.
- Homework set # 10 is due on Friday December 3 at noon.
- Optional homework assignments # 3 is due on Friday December 10 at noon.

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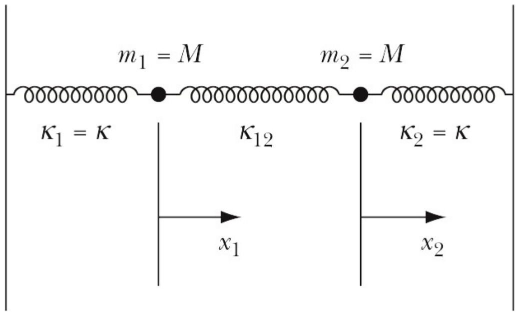
Two Coupled Harmonic Oscillators.



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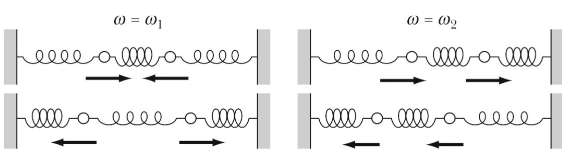
Two Coupled Harmonic Oscillators. Two modes.



Antisymmetrical mode (out of phase)   Symmetrical mode (in phase)

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Two Coupled Harmonic Oscillators.

- Two approaches:
- Approach 1:
  - Write down the coupled equations of motion.
  - Try trial functions for x_1 and x_2 with the same frequency.
  - The two frequency will have different amplitudes.
- Approach 2:
  - Cary out a coordinate transformation to decouple the coupled equations.
  - Solve each decoupled equation.
  - Each solution may have a different frequency.
  - Use the solutions of the decoupled equations and the "inverse" coordinate transformation to find the solution.

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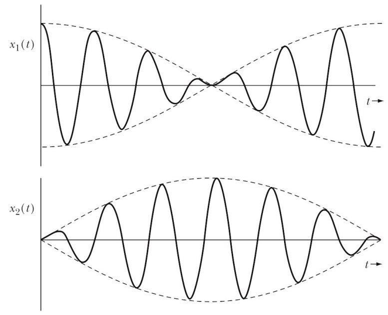
Weak Coupling



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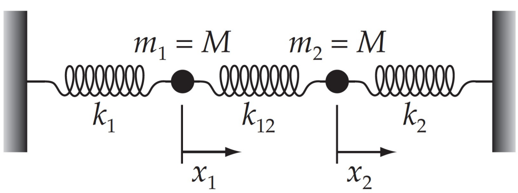
Problem 12.1.

Reconsider the problem of two coupled oscillators discussion in Section 12.2 in the event that the three springs all have different force constants. Find the two characteristic frequencies, and compare the magnitudes with the natural frequencies of the two oscillators in the absence of coupling.



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Problem 12.3.

Two identical harmonic oscillators (with masses M and natural frequencies ω_0) are coupled such that by adding to the system a mass m, common to both oscillators, the equations of motion become

$$\left.\begin{aligned} &\ddot{x}_1 + \frac{m}{M}\ddot{x}_2 + \omega_0^2\, x_1 = 0 \\ &\ddot{x}_2 + \frac{m}{M}\ddot{x}_1 + \omega_0^2\, x_2 = 0 \end{aligned}\right]$$

Solve this pair of coupled equations, and obtain the frequencies of the normal modes of the system.

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2 Minute 41 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 2 minute 41 second intermission.
- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.

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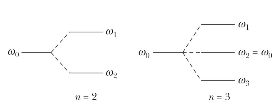
The General Problem of Coupled Oscillations.

Observations from our studies of 2 coupled oscillators:

- The coupling in a system with two degrees of freedom results in two characteristic frequencies.
- The two characteristic frequencies in a system with two degree of freedom are pushed towards lower and higher energies compared to the non-coupled frequency.

Now we will continue the discussion of n coupled oscillators.



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N Coupled Oscillators

- We will have n coupled equations (A and m are the amplitude and mass tensors):

$$\sum_k \left(A_{kj} - \omega^2 m_{kj}\right) a_k = 0$$

- This set of equation will have non-trivial solutions if

$$\begin{vmatrix} A_{11}-\omega^2 m_{11} & A_{12}-\omega^2 m_{12} & A_{13}-\omega^2 m_{13} & \ldots\ldots \\ A_{12}-\omega^2 m_{12} & A_{22}-\omega^2 m_{22} & A_{23}-\omega^2 m_{23} & \ldots\ldots \\ A_{13}-\omega^2 m_{13} & A_{32}-\omega^2 m_{32} & A_{33}-\omega^2 m_{33} & \ldots\ldots \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \end{vmatrix} = 0$$

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Steps

- Follow these steps in order to solve most coupled oscillator problems:
  - Choose generalized coordinates.
  - Determine the A and m tensors.
  - Determine the eigen frequency and the eigen vectors.
  - Determine the scale factors required to match the initial conditions.
  - Determine the normal coordinates.

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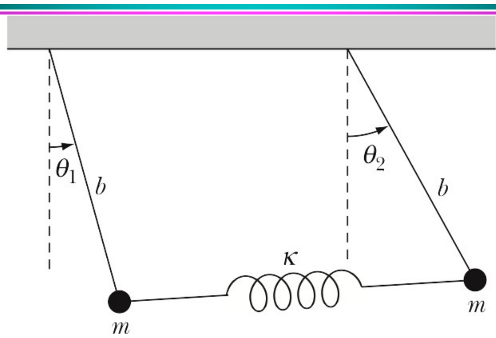
Example 12.4.



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ENOUGH FOR TODAY?

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