Classical Mechanics
Phy 235, Lecture 17.

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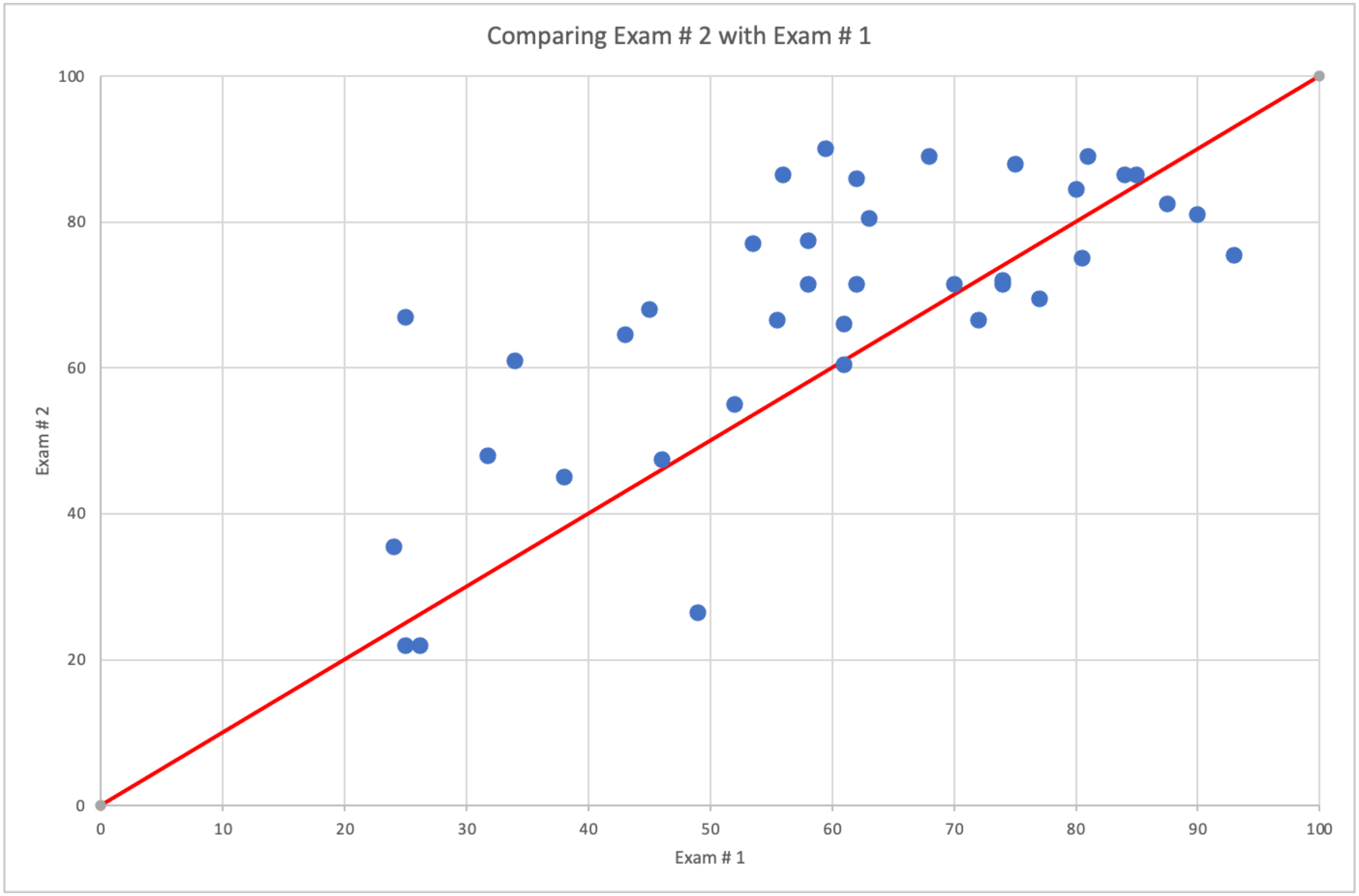
Amazing …… classical mechanics at work.

Exam # 2.

- Grades were distributed via email on Tuesday.
- Exams are or were returned during recitations on Tuesday and Thursday.
- If you feel your exam was not graded properly, you need to tell me. Do not complain to your TAs.
- Any requests for regrades for specific problems should be made by Wednesday November 10 (end of lecture). I will need the following:
  - Your blue book(s).
  - A written explanation why you feel you deserve more points.

Give a physicist some numbers, he will analyze them!



A quick summary

- From Mondays lecture:

$$\left(d\bar{r}\right)_{fixed} = d\bar{\theta} \times \bar{r}$$

$$v_f = \left(\frac{d\bar{r}\,'}{dt}\right)_{fixed} = \left(\frac{d\bar{R}}{dt}\right)_{fixed} + \left(\frac{d\bar{r}}{dt}\right)_{rotating} + \bar{\omega} \times \bar{r} = V + v_r + \bar{\omega} \times \bar{r}$$

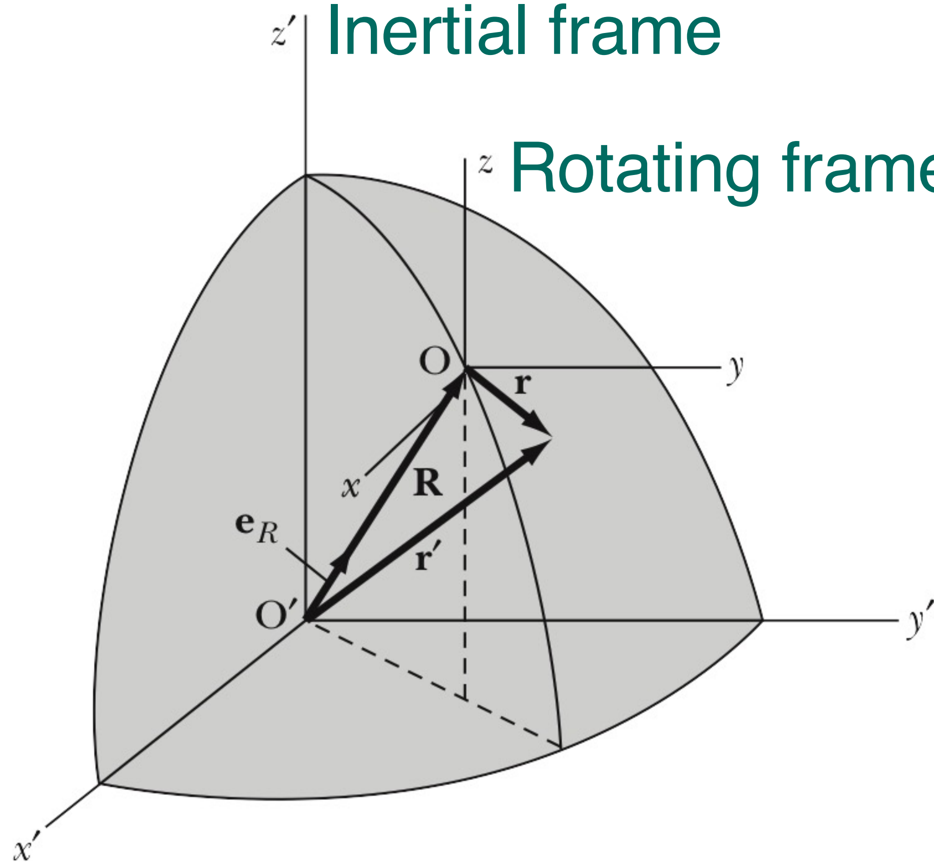
$$\bar{a}_f = \left(\frac{d\bar{v}_f}{dt}\right)_{fixed} = \left(\frac{d\bar{V}}{dt}\right)_{fixed} + \left(\frac{d\bar{v}_r}{dt}\right)_{fixed} + \left(\frac{d\bar{\omega}}{dt}\right)_{fixed} \times \bar{r} + \bar{\omega} \times \left(\frac{d\bar{r}}{dt}\right)_{fixed} =$$

$$= \left(\frac{d\bar{V}}{dt}\right)_{fixed} + \left\{\left(\frac{d\bar{v}_r}{dt}\right)_{rotating} + \bar{\omega} \times \bar{v}_r\right\} + \left(\frac{d\bar{\omega}}{dt}\right)_{fixed} \times \bar{r} + \bar{\omega} \times \left\{\left(\frac{d\bar{r}}{dt}\right)_{rotating} + \bar{\omega} \times \bar{r}\right\} =$$

$$= \left(\frac{d\bar{V}}{dt}\right)_{fixed} + \left(\frac{d\bar{v}_r}{dt}\right)_{rotating} + 2\bar{\omega} \times \bar{v}_r + \dot{\bar{\omega}} \times \bar{r} + \bar{\omega} \times \left\{\bar{\omega} \times \bar{r}\right\}$$

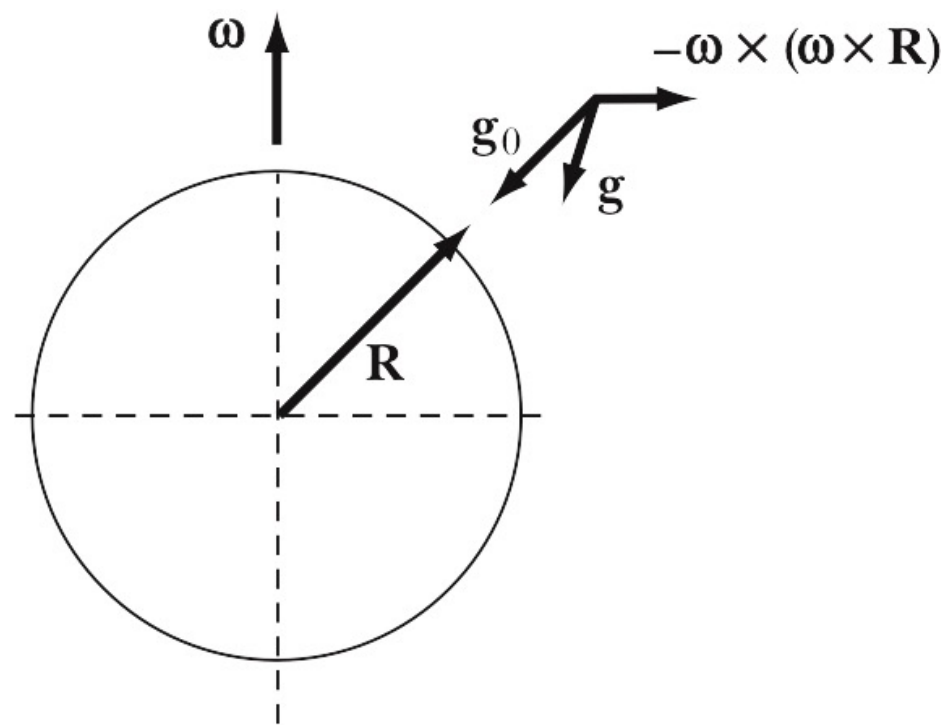
The centripetal force.

- The Earth is not a good inertial reference frame.
- The biggest "non-inertial" effect is due to the daily rotation around its axis.
- We use a rotating *xyz* frame, fixed on the surface of the Earth, and a fixed inertial reference frame *x'y'z'* whose origin is located at the center of the Earth.



The centripetal force.

- Consider a pendulum at rest in our reference frame on the surface of the Earth.
- In the inertial reference frame we would measure a gravitational acceleration g_0.
- In the rotating reference frame (our laboratory on Earth) we measure an acceleration g.



Force on a pendulum.

- The effective force on our pendulum is

$$\bar{F}_{eff} = m\bar{a}_f - m\bar{\omega}\times\{\bar{\omega}\times\bar{r}\} = m\{\bar{g}_0 - \bar{\omega}\times\{\bar{\omega}\times\bar{r}\}\}$$

- Since the pendulum is at rest in our rotation reference frame, the velocity $v_{\mathrm{r}} = 0$ m/s.

- The gravitational acceleration we measure in our Earth frame is equal to

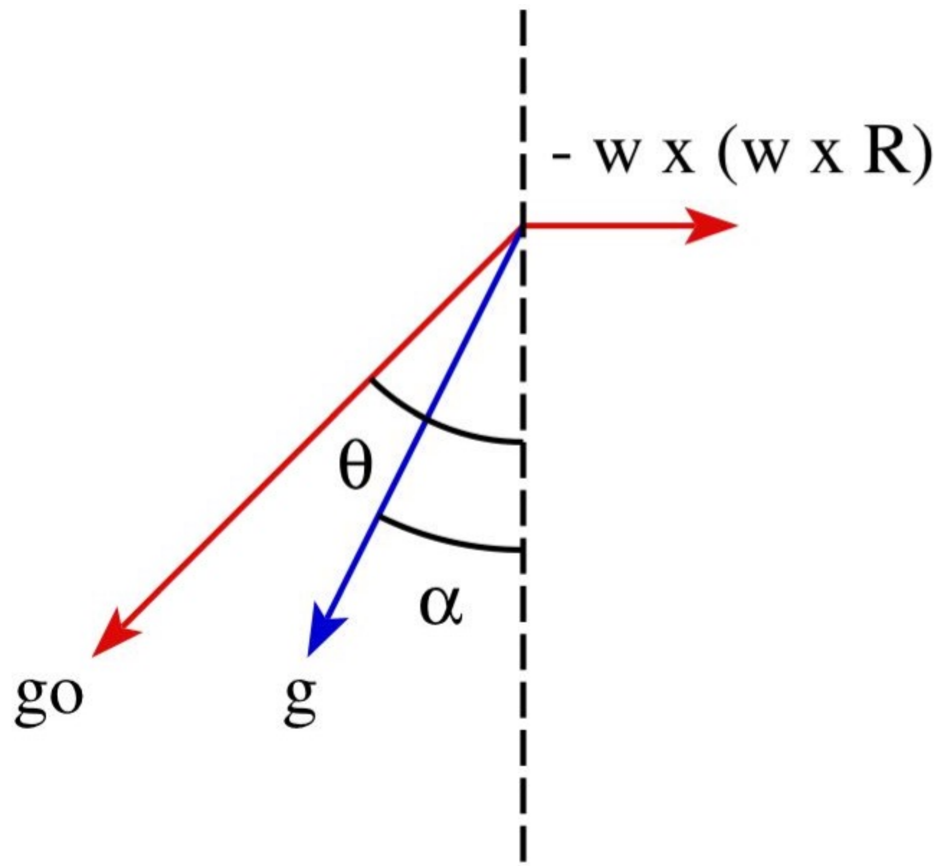
$$\bar{g} = \bar{g}_0 - \bar{\omega}\times\{\bar{\omega}\times\bar{r}\}$$

Correction due to rotation.
Note: correction is position dependent.

Force on a pendulum.

- The effect of the rotation of the Earth is a change in the equilibrium angle of the pendulum.
- The change in angle is equal to

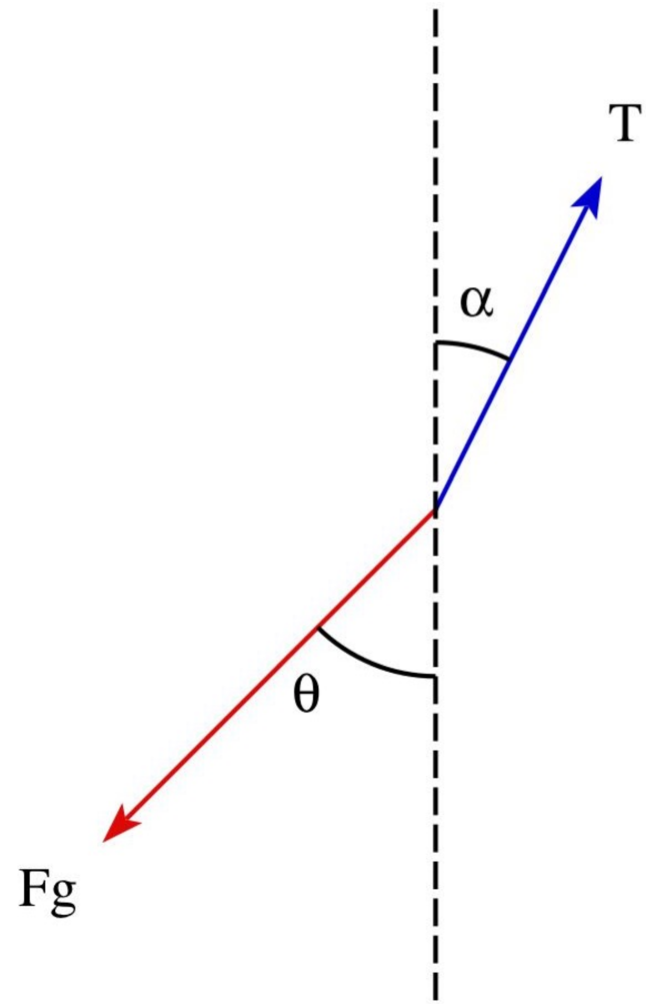
$$\Delta\theta = \theta - \alpha =$$
$$= \theta - \operatorname{atan}\left(\left(1 - \frac{\omega^2 R}{g_0}\right)\tan\theta\right)$$



Force on a pendulum.
As viewed in the fixed reference frame.

- We could have viewed this problem also from our fixed reference frame.
- In this reference frame, the mass is rotating, and there must thus be a net force with magnitude mv^2/r acting on it:

$$F_r = m\frac{v^2}{R\sin\theta} =$$
$$= m\omega^2 R\sin\theta$$



Force on a pendulum.
As viewed in the fixed reference frame.

- The net force is provided by the horizontal components of the tension T and the gravitational force:

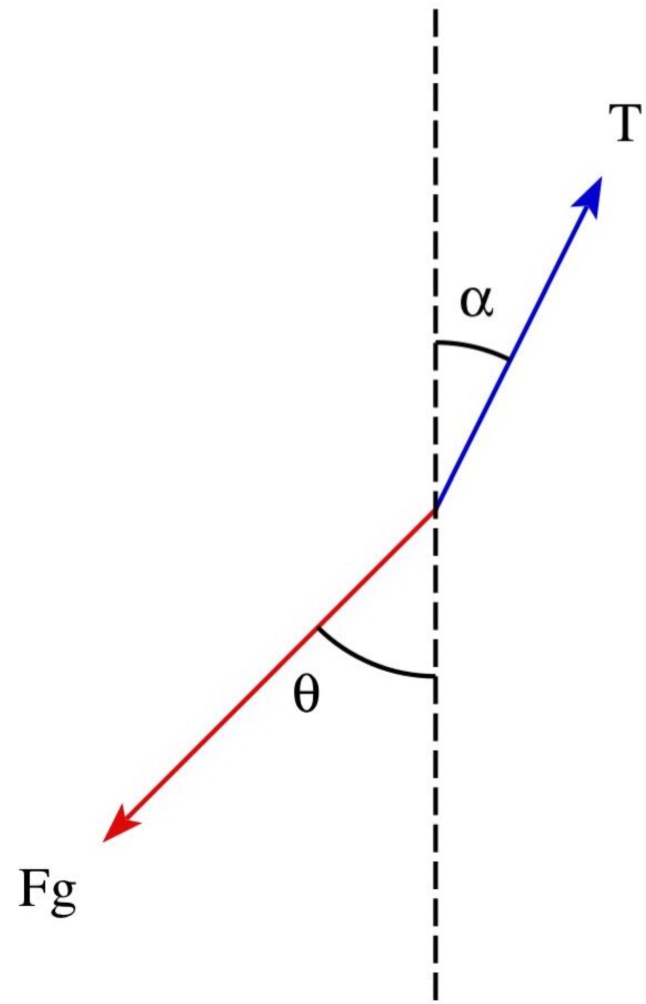
$$mg_0 \sin\theta - T\sin\alpha = m\omega^2 R\sin\theta$$

- In the vertical direction, the net force must be 0:

$$T\cos\alpha = mg\cos\theta$$

- The angle α can now be determined:

$$\tan\alpha = \frac{T\sin\alpha}{T\cos\alpha} = \left(1 - \frac{\omega^2 R}{g_0}\right)\tan\theta$$

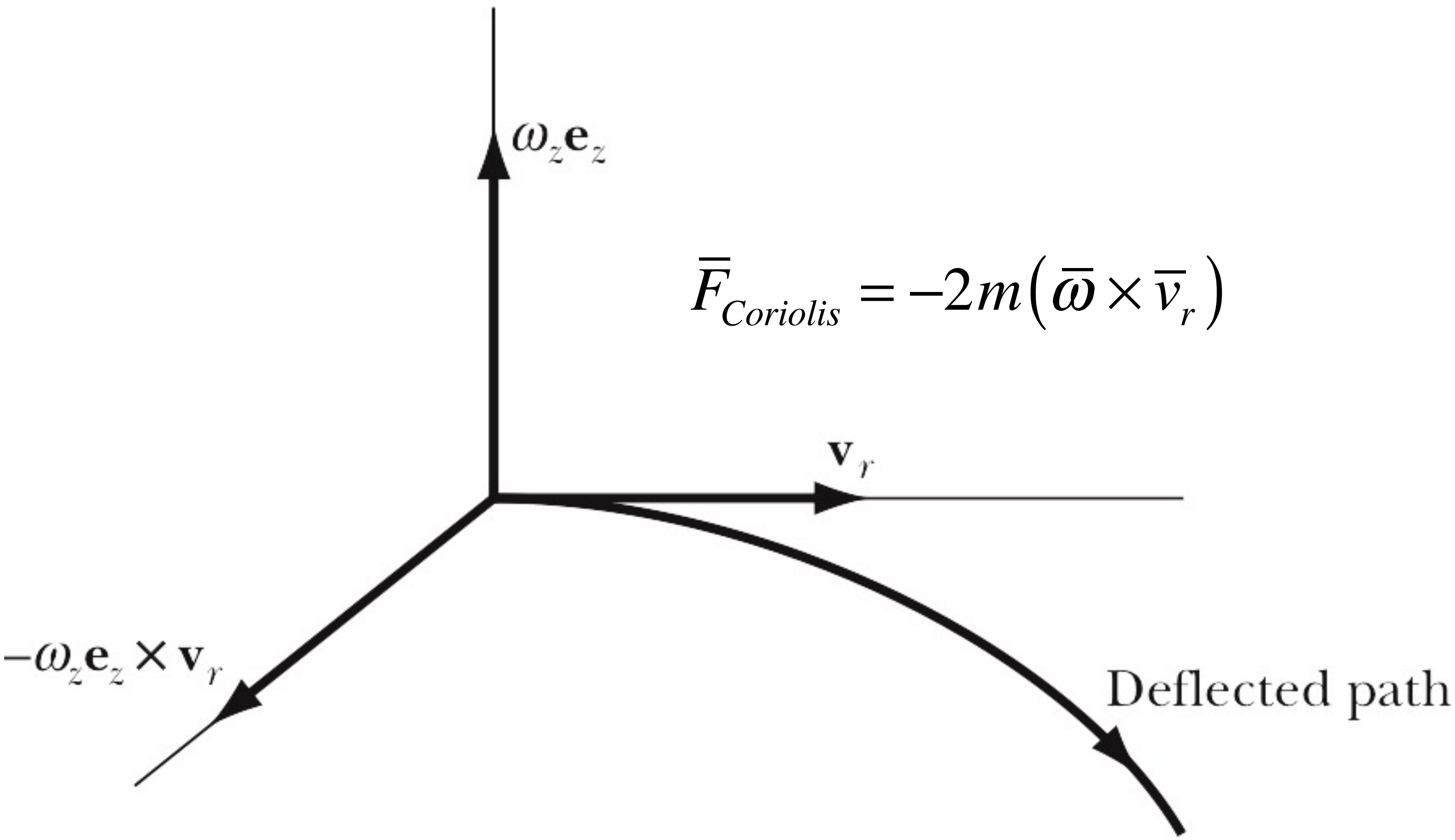


3 Minute 23 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 3 minute 23 second intermission.

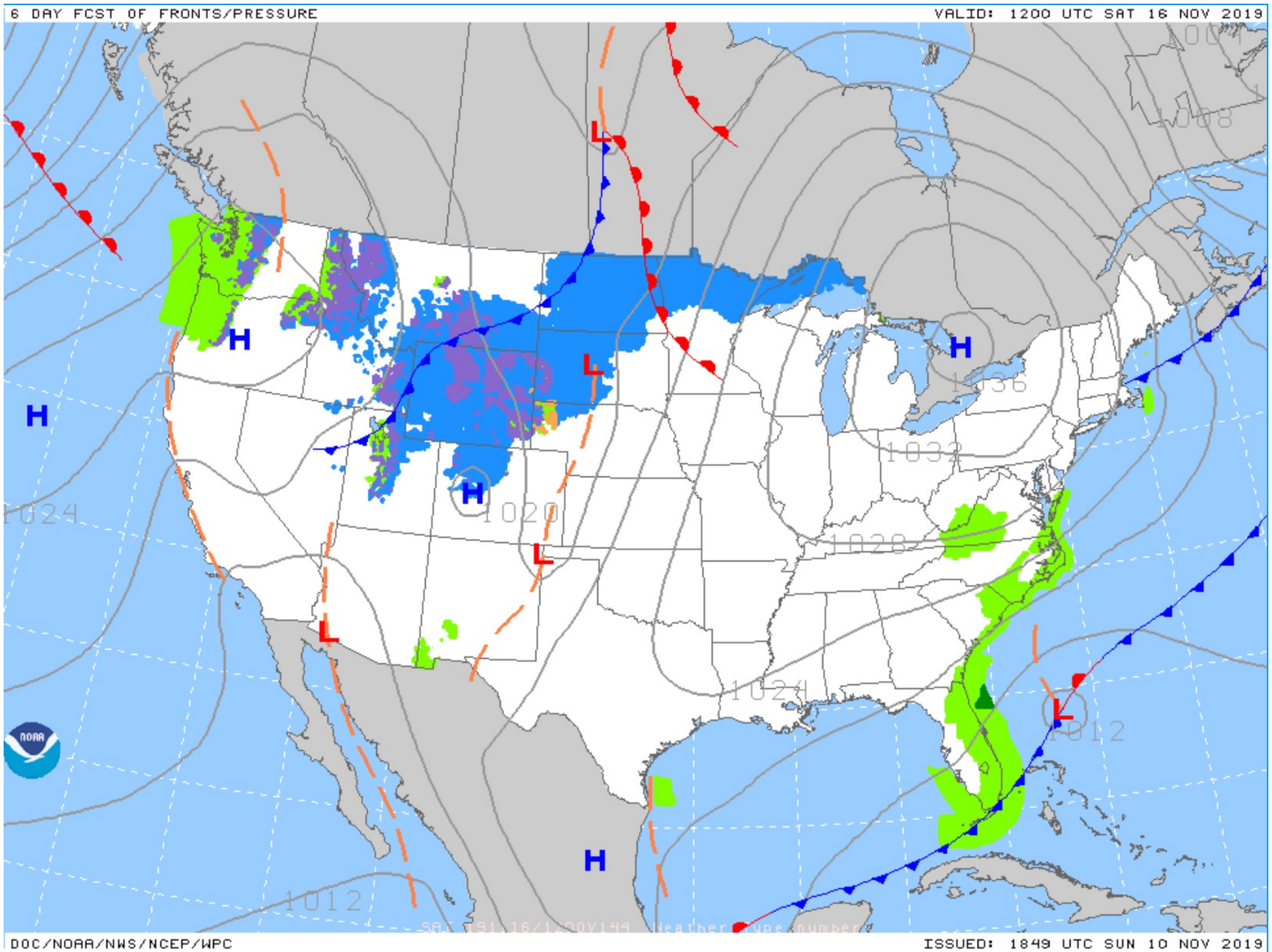
- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.

Effects of the Coriolis Force.

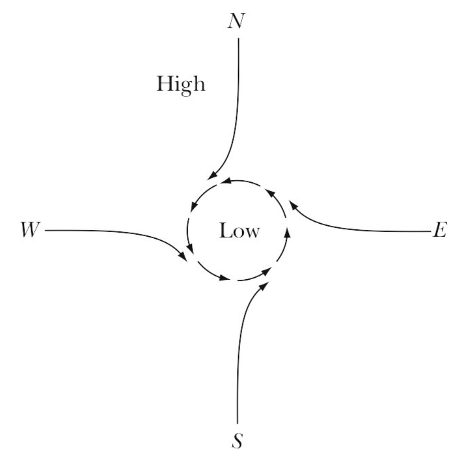


$$\bar{F}_{Coriolis} = -2m\left(\bar{\omega} \times \bar{v}_r\right)$$

Air flowing West to East: deflected South.
Air flowing East to West: deflect North.



Note: only applies to the Northern hemisphere.

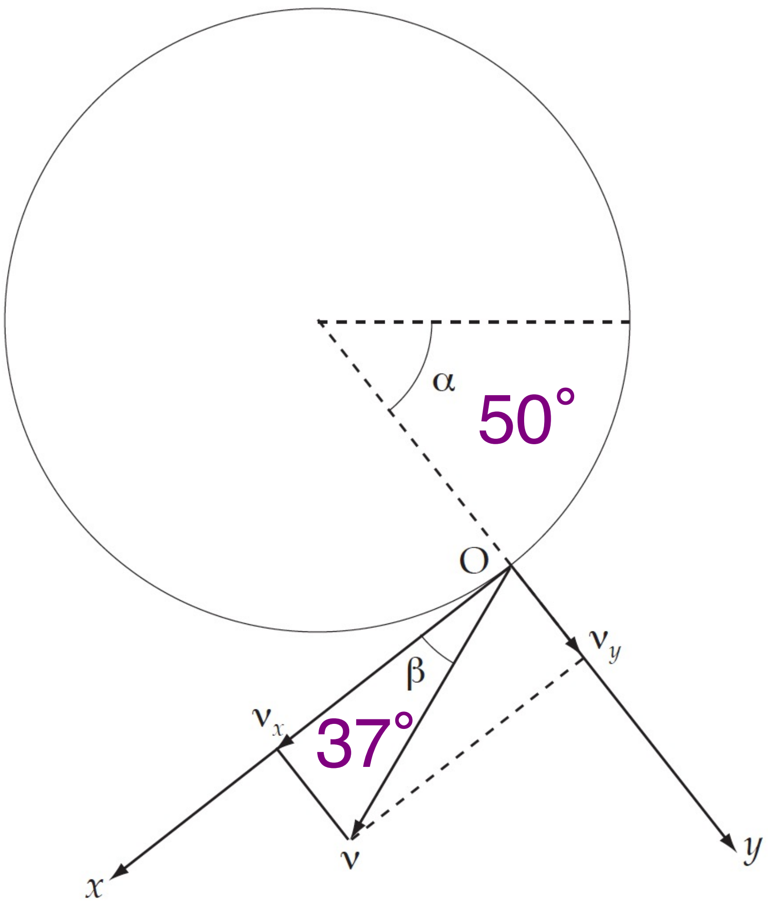


Problem 10.20.

- Calculate the effective gravitational acceleration g at the Earth's surface at the pole and at the equator. Take into account the difference in the equatorial (6378 km) and polar (6357) radius as well as the centrifugal force.

Problem 10.18.

A British warship fires a projectile due south near the Falkland Islands during World War I at latitude 50° South. If the shells are fired at 37° elevation with a speed of 800 m/s, by how much do the shells miss their target and in what direction?



ENOUGH FOR TODAY?