Classical Mechanics
Phy 235, Lecture 16.

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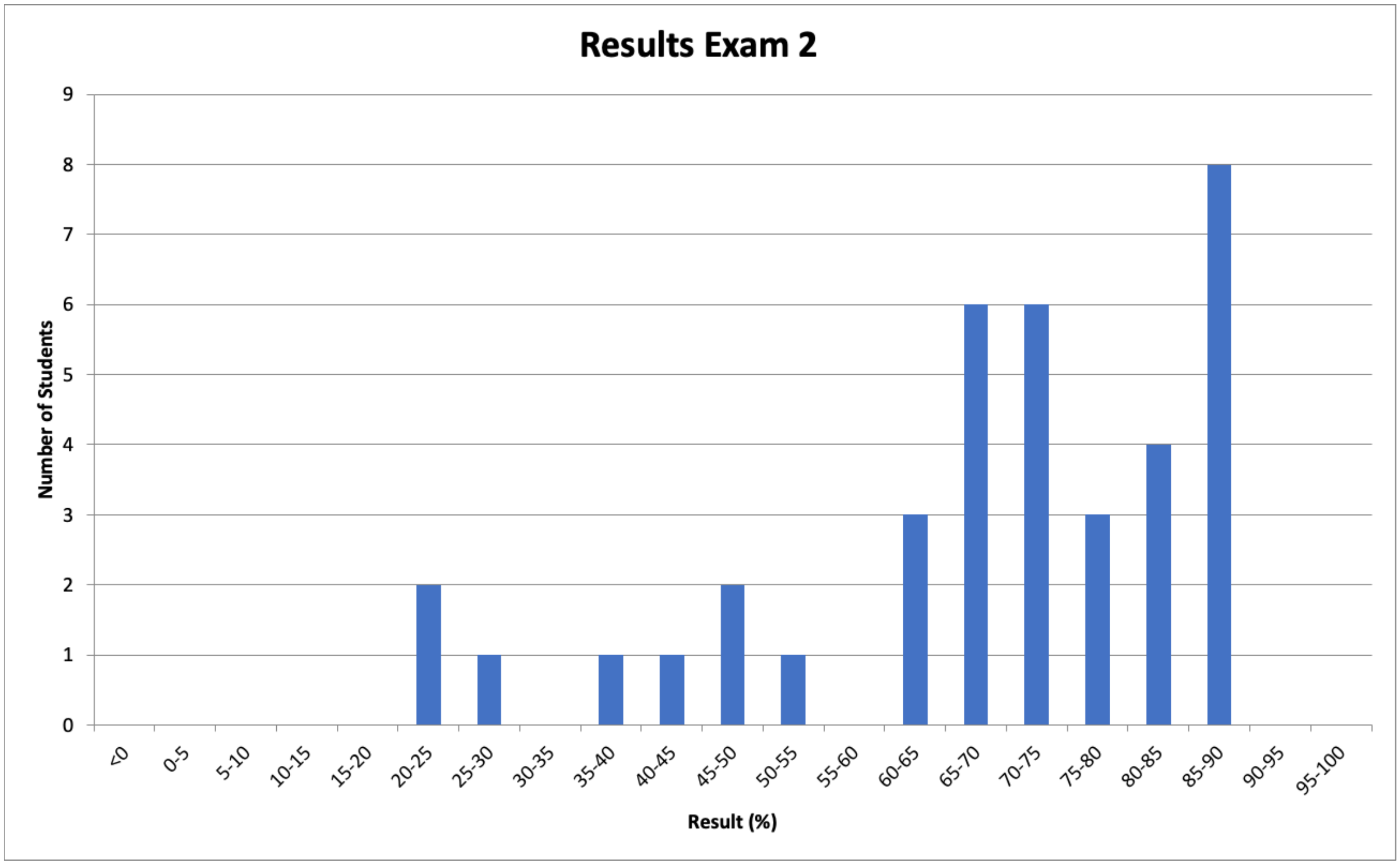
Comments on Exam 2

- Problem 1:
  - Similar, but simpler than Example problem 5.1.
  - Average score: 16.1/25
- Problem 2:
  - Homework problem (HW # 5, Problem # 3)
  - Average score: 20.1/25
- Problem 3:
  - Example problem 7.10.
  - Average score: 14.4/25
- Problem 4:
  - The KLM is 102 years old. See lecture 9, slide 2.
  - Average score: 14.5/25

Results Exam # 2



Motion in Non-Inertial Reference Frames.

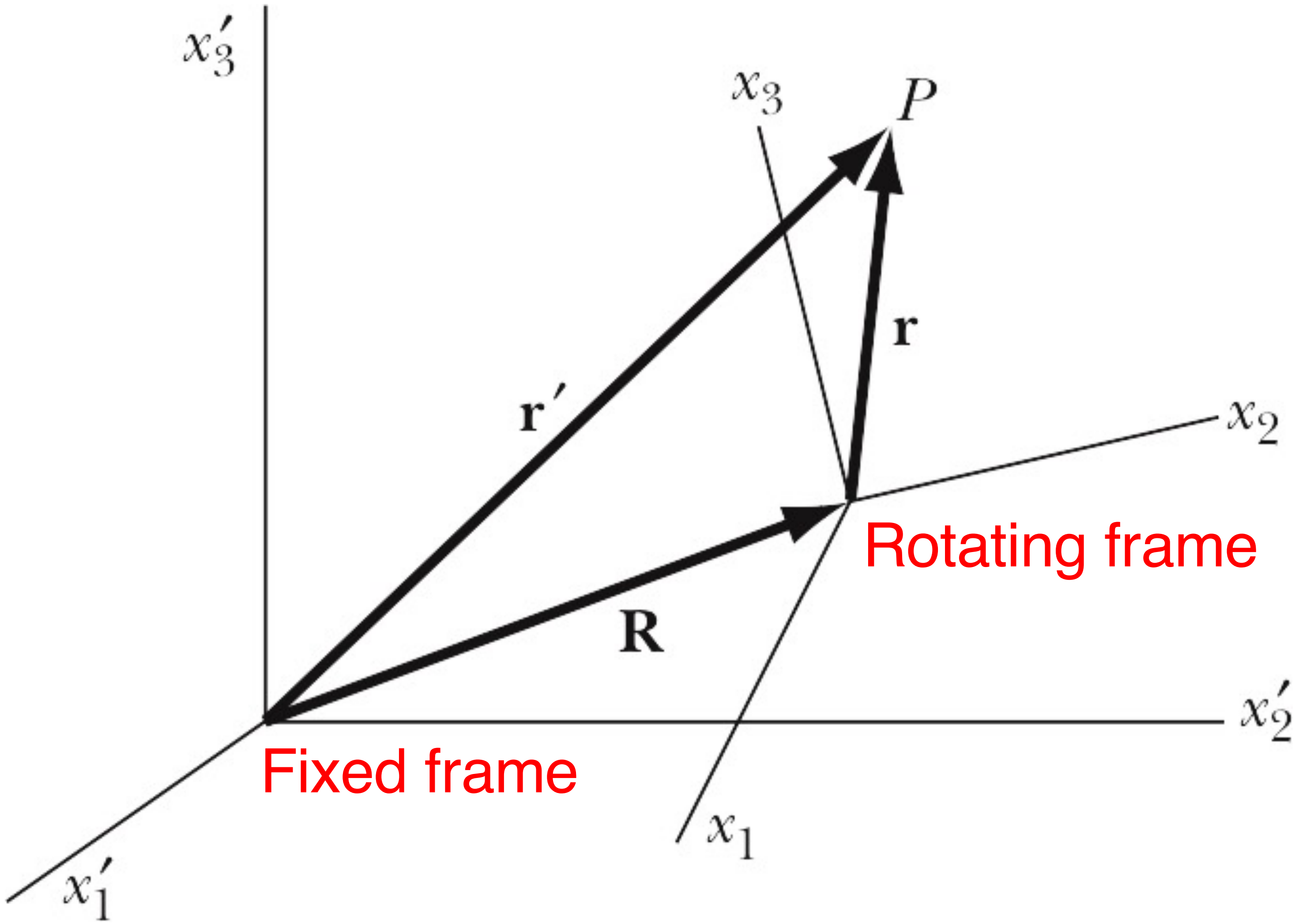
- The Earth is not an inertial reference frame:
  - The Earth is rotating around its axis.
  - The Earth is moving around the sun.
  - The sun is moving around the center of the Milky-Way galaxy.
- What is the effect of this non-inertial motion?
- All will be revealed in Chapter 10.

Problem 10.1.

Calculate the centrifugal acceleration due to the Earth's rotation on a particle on the surface of the Earth at the equator. Compare this result with the gravitational acceleration.

Compute also the centrifugal acceleration due to the motion of the Earth about the sun, and the motion of the sun around the center of the Milky Way.

Rotating Coordinate system.



The angular acceleration is the same in both reference frames.

- Relation between position vectors:

$$\left(\frac{d\bar{r}}{dt}\right)_{fixed} = \left(\frac{d\bar{r}}{dt}\right)_{rotating} + \bar{\omega} \times \bar{r}$$

- Relation between angular velocity vectors:

$$\left(\frac{d\bar{\omega}}{dt}\right)_{fixed} = \left(\frac{d\bar{\omega}}{dt}\right)_{rotating} + \bar{\omega} \times \bar{\omega} = \left(\frac{d\bar{\omega}}{dt}\right)_{rotating}$$

- Conclusion: the angular acceleration is the same in both reference frames.

Velocity in fixed (inertial) frame.

Velocity of the origin of the rotating frame.

$$v_f = \left(\frac{d\bar{r}\,'}{dt}\right)_{fixed} = \left(\frac{d\bar{R}}{dt}\right)_{fixed} + \left(\frac{d\bar{r}}{dt}\right)_{rotating} + \bar{\omega}\times\bar{r} = V + v_r + \bar{\omega}\times\bar{r}$$

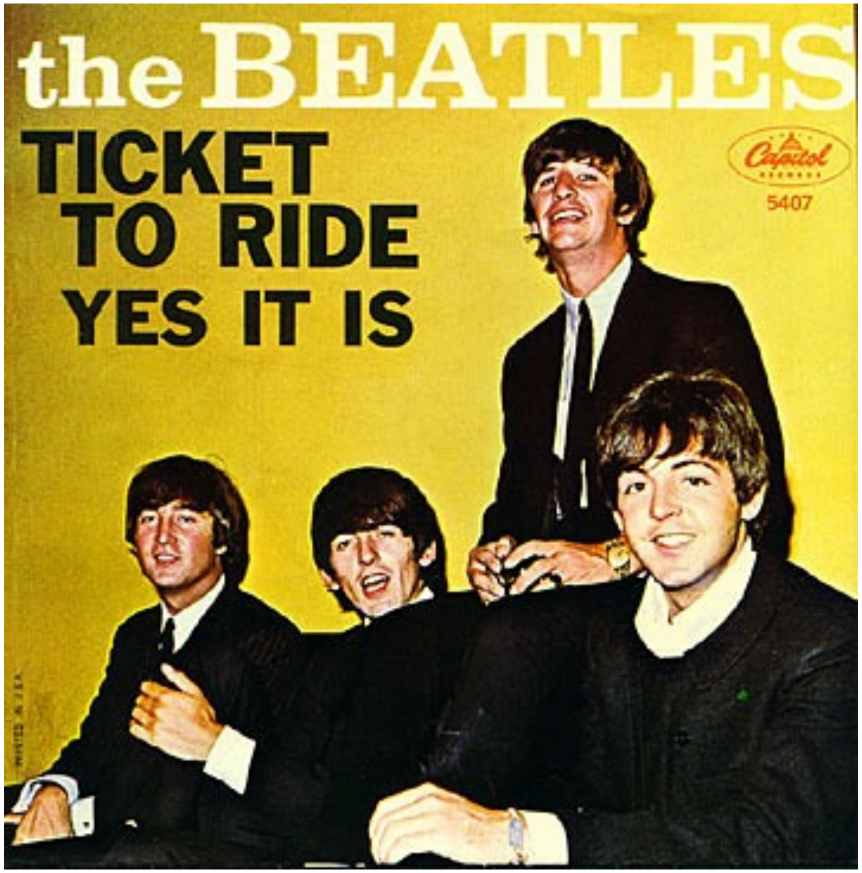
Velocity in fixed frame.

Velocity in rotating frame.

3 Minute 10 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 3 minute 10 second intermission.

- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.



Newton's laws in rotating reference frames.

- Only in the fixed reference frame can we use Newton's second law:

$$\bar{a}_f = \left(\frac{d\bar{v}_f}{dt}\right)_{fixed} = \frac{\bar{F}}{m}$$

- The acceleration in the fixed reference frame can also be expressed as:

$$\bar{a}_f = \left(\frac{d\bar{V}}{dt}\right)_{fixed} + \underbrace{\left(\frac{d\bar{v}_r}{dt}\right)_{rotating}}_{\text{Acceleration observed in rotating frame.}} + 2\bar{\omega}\times\bar{v}_r + \dot{\bar{\omega}}\times\bar{r} + \bar{\omega}\times\{\bar{\omega}\times\bar{r}\}$$

Acceleration observed in rotating frame.

Observed acceleration in rotating reference frame.

- An observer in the rotating reference frame will observe an acceleration:

$$\bar{a}_r = \left(\frac{d\bar{v}}{dt}\right)_{rotating}$$

- This acceleration can be expressed in terms of the acceleration in the inertial frame:

$$\bar{a}_r = \bar{a}_f - \left(\frac{d\bar{V}}{dt}\right)_{fixed} - 2\bar{\omega}\times\bar{v}_r - \dot{\bar{\omega}}\times\bar{r} - \bar{\omega}\times\{\bar{\omega}\times\bar{r}\}$$

The effective force.

Using the **effective force**

Coriolis force.

$$\bar{F}_{eff} = m\bar{a}_f - m\dot{\bar{\omega}} \times \bar{r} - 2m\bar{\omega} \times \bar{v}_r - m\bar{\omega} \times \{\bar{\omega} \times \bar{r}\}$$

Centripetal force.

Not Dutch!
So forget it.

an observer in the rotating frame will be able to determine the acceleration in the rotating frame by dividing the effective force by the mass of the object.

Problem 10-6.

A bucket of water is set spinning about its symmetry axis. Determine the shape of the water in the bucket.

ENOUGH FOR TODAY?