Classical Mechanics
Phy 235, Lecture 14.

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The KLM became The Flying Dutchman (minus the doom and gloom).

Course Comments

- Exam # 2 tomorrow: 10/26 between 8 am and 9.30 am.
- Extra office hours today:
  - Frank Wolfs: 12 pm - 2 pm, B&L 203A
  - Elizabeth Champion: 2 pm - 3 pm, B&L 304
  - Margaret Porcelli: 6 pm - 7 pm POA.

- There will be no recitations this week.
- There will be no office hours on Wednesday 10/27 and Thursday 10/28.

- The next regular homework set is due on 11/5.
- Optional homework set 2 is due on 10/29.

Newton's third law.

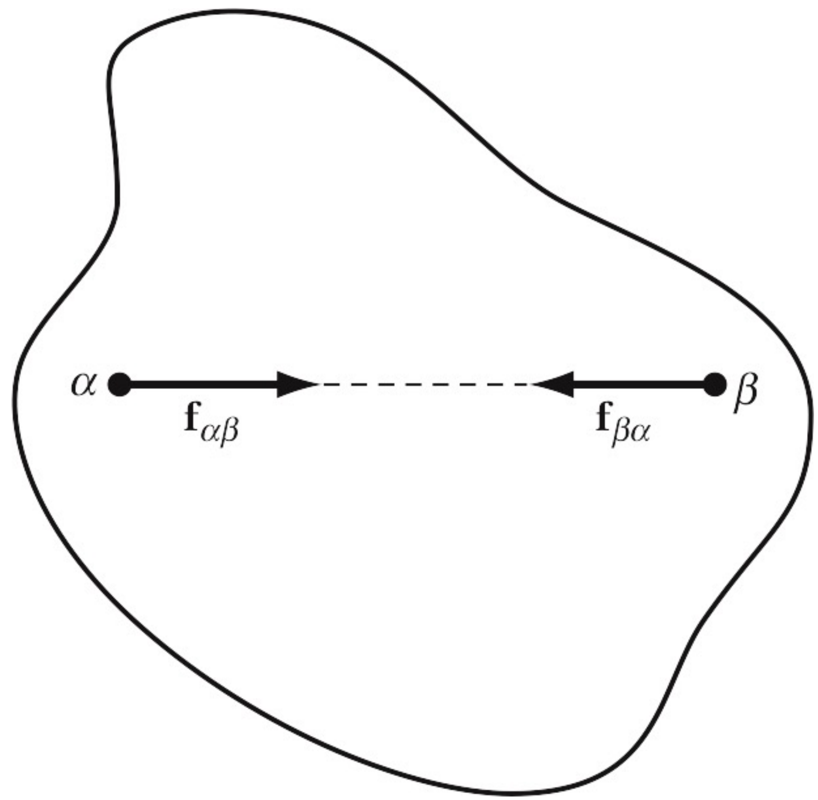
- Weak form of Newton's law:

$$\vec{F}_{\alpha\beta} = -\vec{F}_{\beta\alpha}$$

- Strong form of Newton's law:

$$\vec{F}_{\alpha\beta} = -\vec{F}_{\beta\alpha}$$

and the forces must lie on the straight line connecting the two particles.



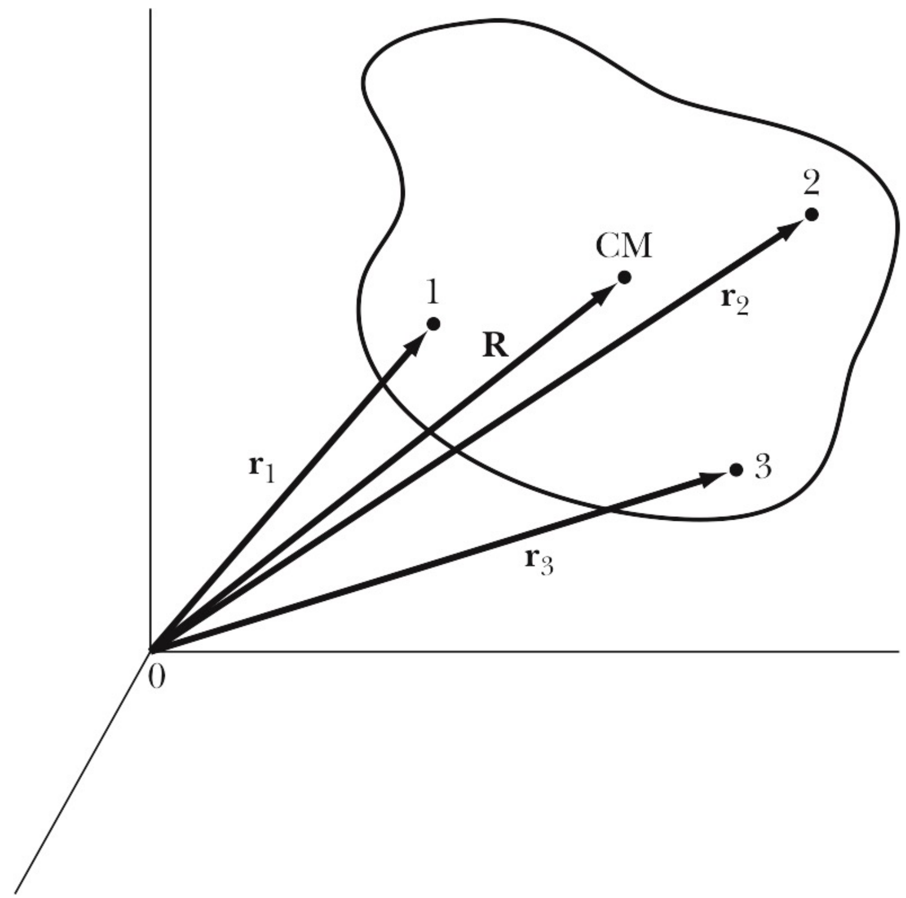
Center of Mass

- Definitions of center of mass:
  - Discrete mass distribution:

$$\bar{R}_{cm} = \frac{\sum_i m_i \bar{r}_i}{\sum_i m_i} = \frac{1}{M}\sum_i m_i \bar{r}_i$$

  - Continuous mass distribution:

$$\bar{R}_{cm} = \frac{1}{M}\int \bar{r}\, dm$$



Problem 9.1.

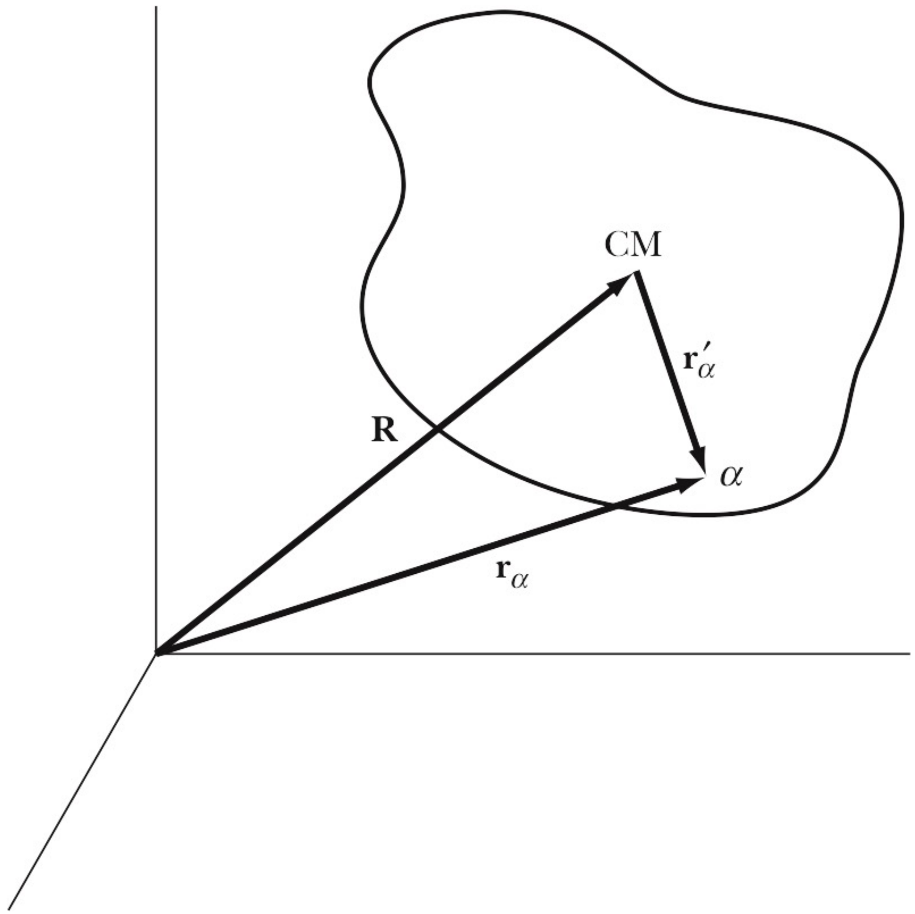
Find the center of mass of a hemispherical shell of constant density and inner radius r_1 and outer radius r_2.

Linear Momentum.

- Linear momentum:

$$\bar{P} = \sum_i m_i \dot{\bar{r}}_i = \frac{d}{dt}\sum_i m_i \bar{r}_i =$$

$$= \frac{d}{dt}\left(M\bar{R}\right) = M\dot{\bar{R}}$$



Properties of Linear Momentum.

- The center of mass of a system moves as if it were a single particle with a mass equal to the total mass of the system, M, acted on by the total external force, and independent of the nature of the internal forces.

- The linear momentum of a system of particles is the same as that of a single particle of mass M, located at the position of the center of mass, and moving in the manner the center of mass is moving.

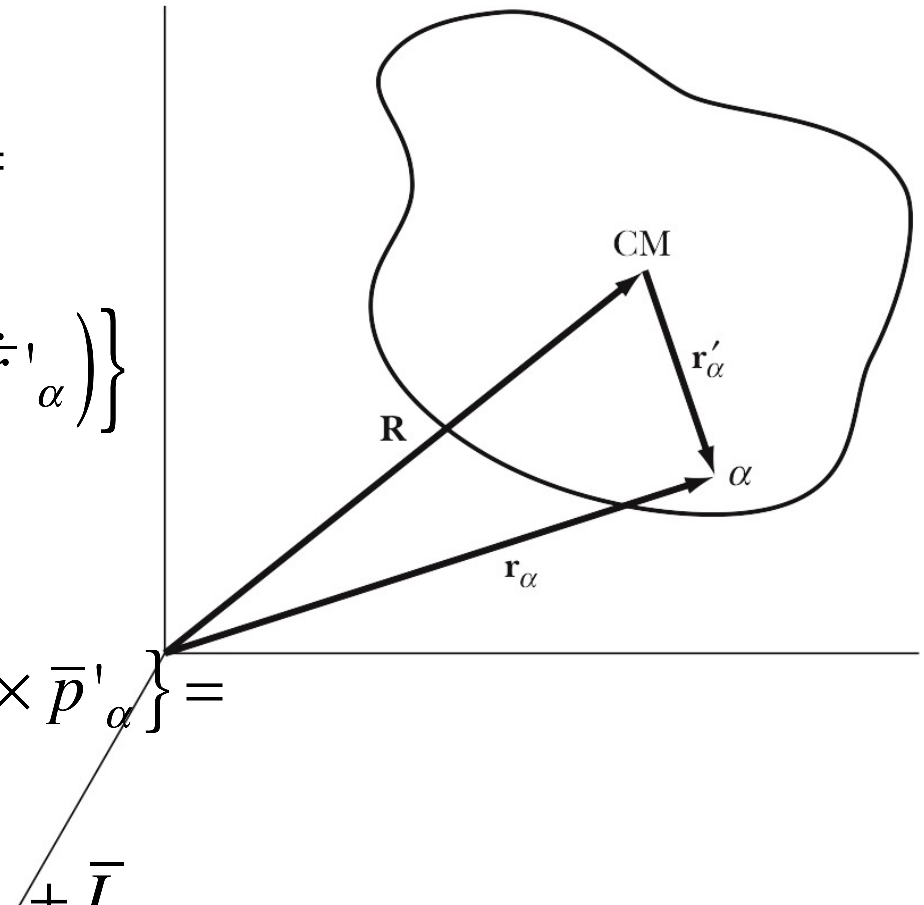
- The total linear momentum for a system free of external forces is constant and equal to the linear momentum of the center of mass.

Angular Momentum.

- Angular momentum:

$$\bar{L} = \sum_\alpha \bar{L}_\alpha = \sum_\alpha \left\{ \bar{r}_\alpha \times m_\alpha \dot{\bar{r}}_\alpha \right\} =$$

$$= \sum_\alpha \left\{ \left(\bar{R} + \bar{r}'_\alpha \right) \times m_\alpha \left(\dot{\bar{R}} + \dot{\bar{r}}'_\alpha \right) \right\}$$



$$\bar{L} = \left(\bar{R} \times \dot{\bar{R}} \right) \sum_\alpha m_\alpha + \sum_\alpha \left\{ \bar{r}'_\alpha \times \bar{p}'_\alpha \right\} =$$

$$= \bar{R} \times \bar{P} + \sum_\alpha \bar{L}_{\alpha, wrt, cm} = \bar{L}_{cm} + \bar{L}_{wrt, cm}$$

Properties of Angular Momentum.

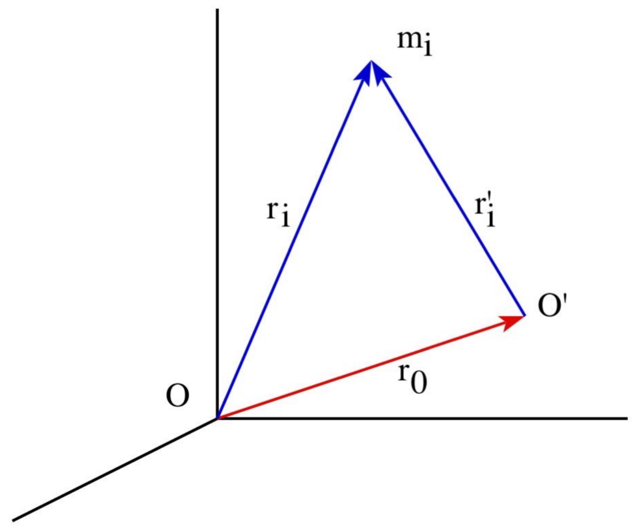
- The total angular momentum about an origin is the sum of the angular momentum of the center of mass about that origin and the angular momentum of the system about the position of the center of mass.

- If the net resultant torques about a given axis vanish, then the total angular momentum of the system about that axis remained constant in time.

- The total internal torque must vanish if the internal forces are central, and the angular momentum of an isolated system can not be altered without the application of external forces.

Problem 9.13.

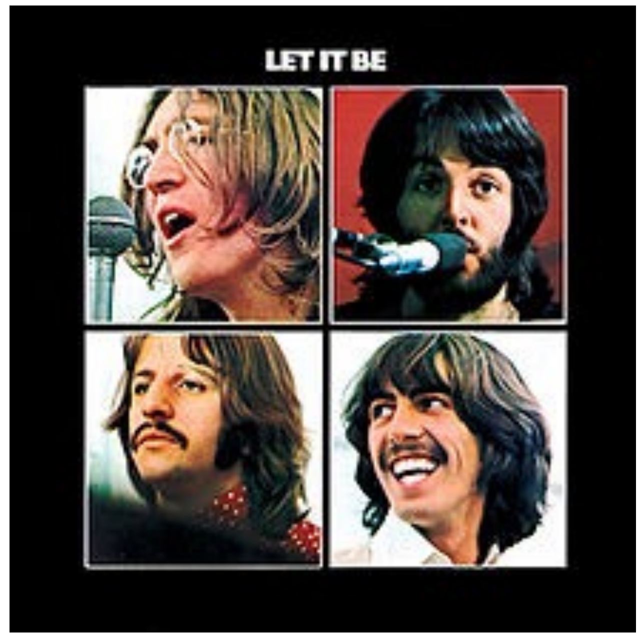
Even though the total force on a system of particles is zero, the net torque may not be zero. Show that the net torque has the same value in any coordinate system.



3 Minute 52 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 3 minute 52 second intermission.

- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.



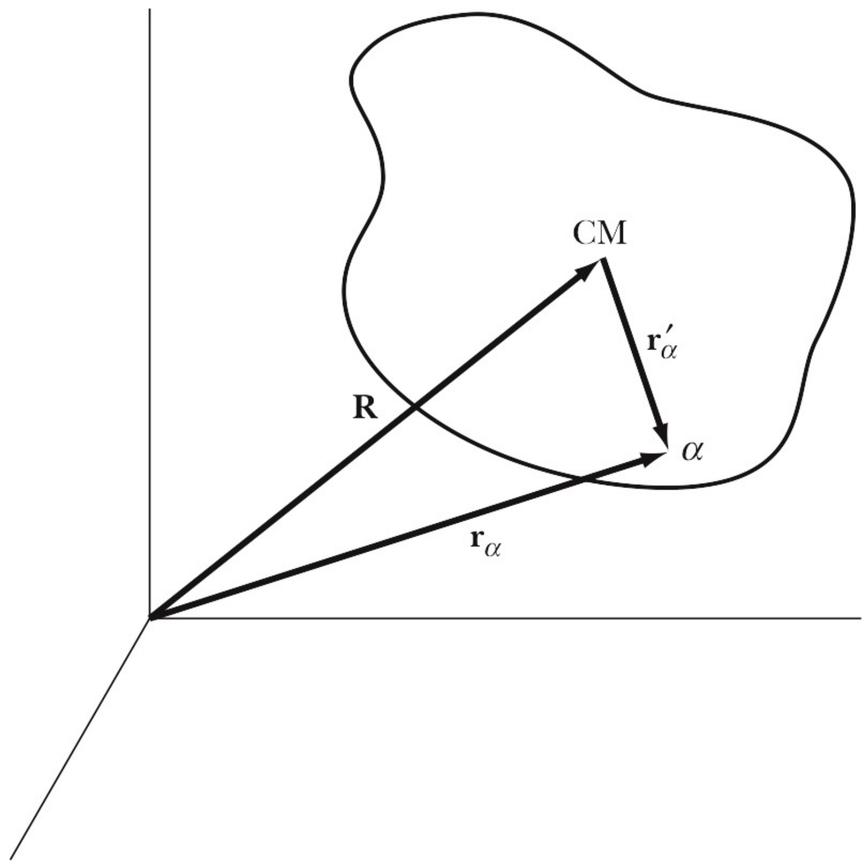
Energy

- Total energy = kinetic energy + potential energy.
- Kinetic energy:

$$T = \frac{1}{2}MV^2 + \sum_i \frac{1}{2} m_i v_i'^2$$

- Potential energy:

$$U = \sum_i U_{i,ext} + \sum_i \sum_{j<i} U_{ij,\mathrm{int}}$$



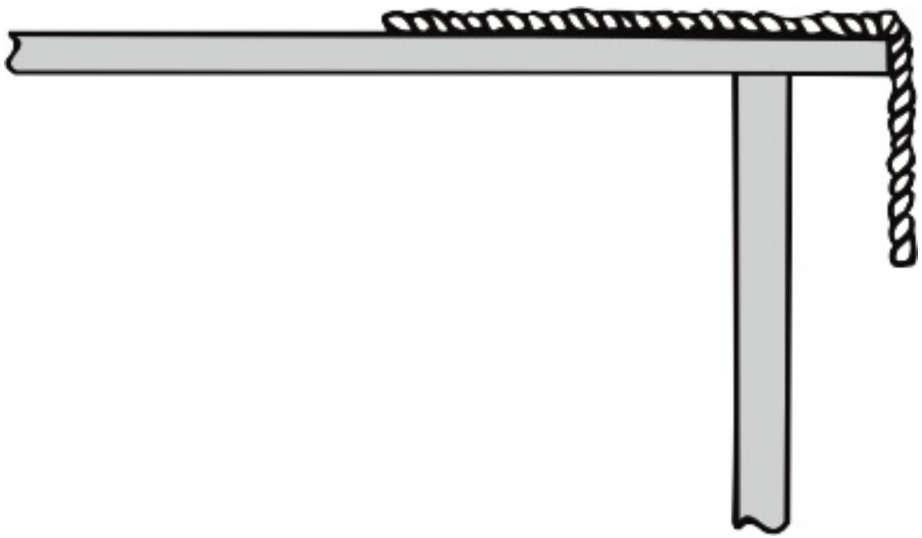
Properties of Energy.

- The total kinetic energy of the system is equal to the sum of the kinetic energy of a particle of mass M moving with the velocity of the center of mass and the kinetic energy of the motion of the individual particles relative to the center of mass.
- The total energy for a conservative system is constant.

Problem 9.21.

A flexible rope of length 1.0 m slides from a frictionless table top as shown in the Figure on the right.
The rope is initially released from rest with 30 cm hanging over the edge of the table.

Find the time at which the left end of the rope reaches the edge of the table.



ENOUGH FOR TODAY?