Classical Mechanics
Phy 235, Lecture 13.

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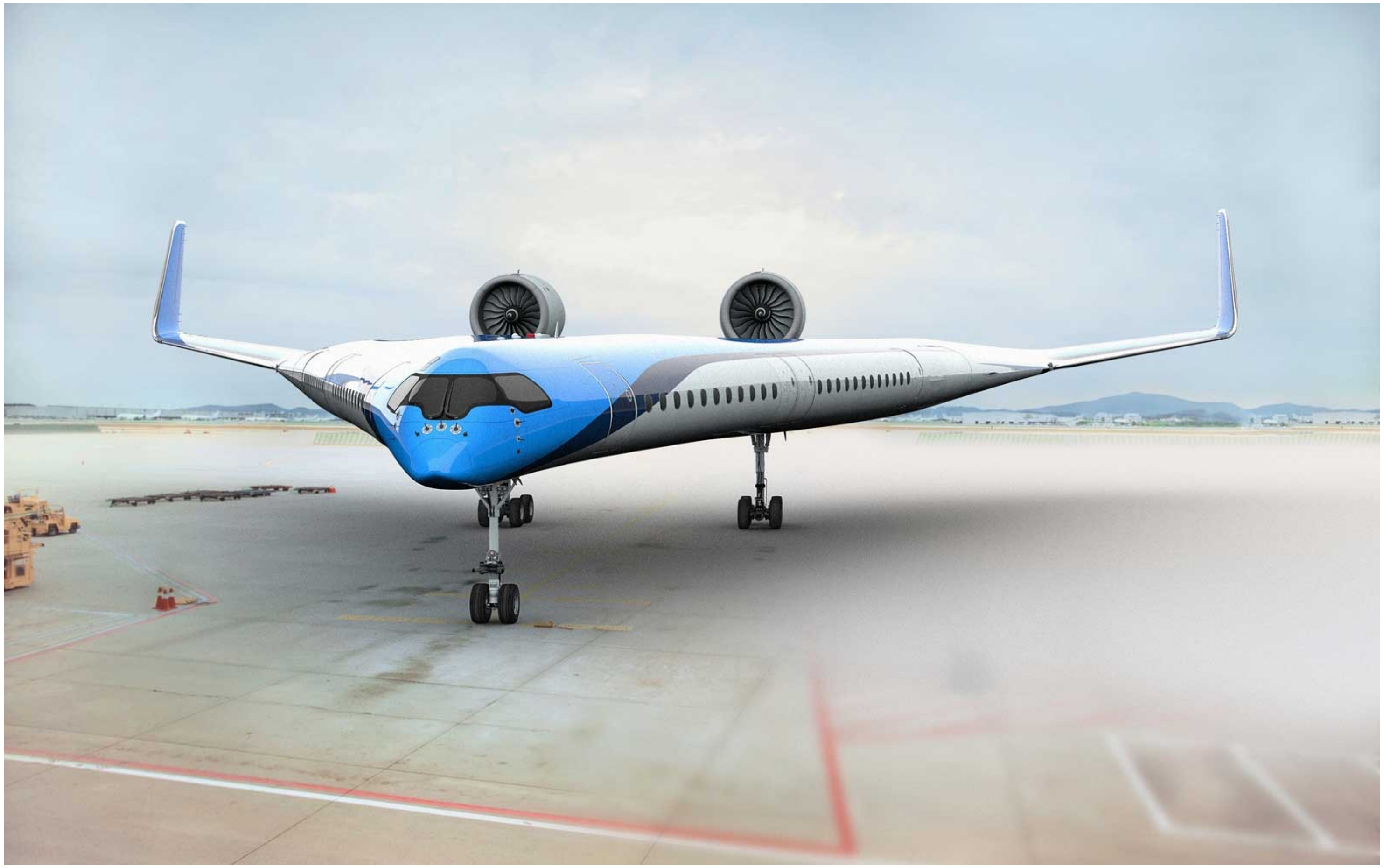
KLM:
Working on the future of aviation.



Course Comments

- Review of the material covered on Exam 2 is scheduled for Wednesday (10/20).
- The next homework set is due on 10/22.
- Details about the term paper can be found on the web at:

  http://teacher.pas.rochester.edu/PHY235/CourseInformation/TermPaper.htm
- Note:
  - 37 days until your paper is due!!!
  - Before that: 1) you need to submit a proposal and 2) you need to review your draft with the writing fellows. Now is the time to schedule your meeting!!!

Two-Body Central-Force Motion.

- For two-body central force problems, we found:
  - The problem can be reduced to a one-body problem.
  - Angular momentum was a conserved quantity.
  - Kepler's second law is a direct consequence of conservation of angular momentum.
  - Since the Lagrangian does not depend explicitly on time, energy is conserved.

$$E = T + U = \frac{1}{2}\mu\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + U(r) = \frac{1}{2}\mu\left(\dot{r}^2 + r^2\left(\frac{l}{\mu r^2}\right)^2\right) + U(r) =$$

$$= \frac{1}{2}\mu\dot{r}^2 + \frac{1}{2}\frac{l^2}{\mu r^2} + U(r)$$

↑ Modification to the potential energy.

Orbital motion: orbital properties.

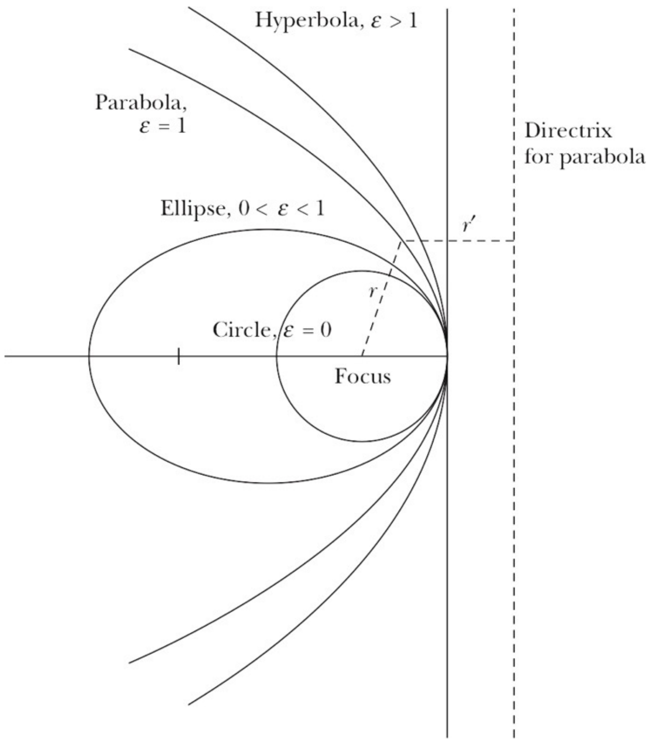
- Properties of the orbits can be found by solving the following integrals:

$$t = \int dt = \pm \int \frac{1}{\sqrt{\frac{2}{\mu}\left(E - U(r)\right) - \frac{l^2}{\mu^2 r^2}}} dr$$

$$\theta(r) = \int \frac{\dot{\theta}}{\dot{r}} dr = \pm \int \frac{l}{r^2 \sqrt{2\mu\left(E - U - \frac{l^2}{2\mu r^2}\right)}} dr$$

Orbital motion: orbital properties (shape).

- Properties:
  - $E > 0$: Hyperbola
  - $E = 0$: Parabola
  - $V_{min} < E < 0$: Ellipse
  - $E = V_{min}$: Circle

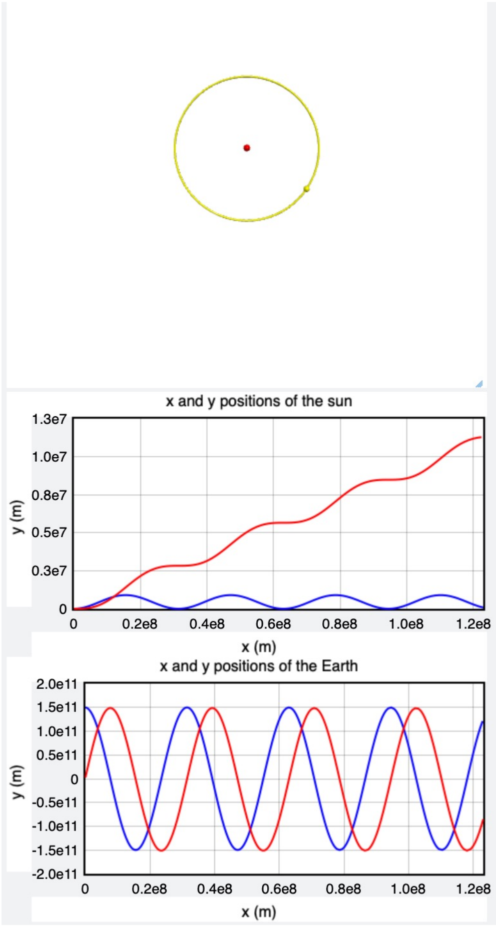


Simulations are a good way to study orbital motion.

- The following links point towards two interesting simulations of orbital motion using glowscript.

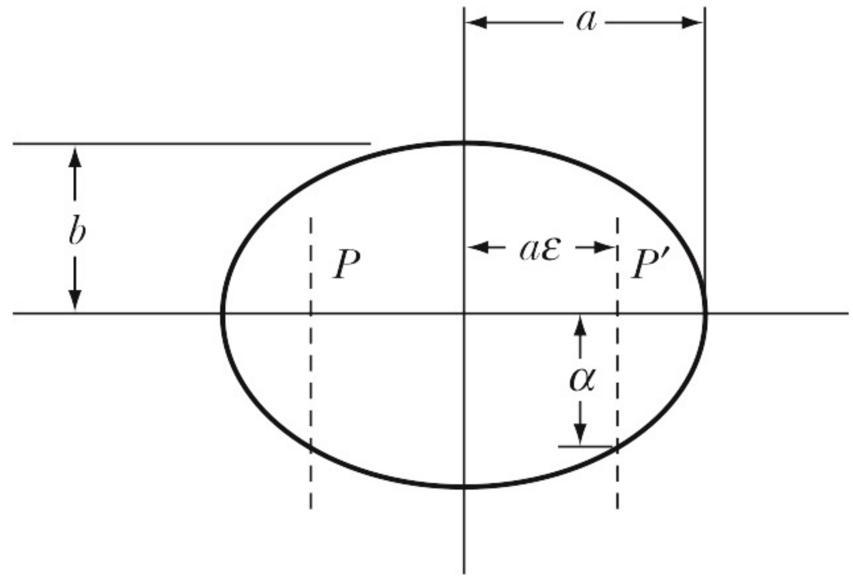
- Sun-Earth system:
  - https://www.glowscript.org/#/user/wolfs/folder/Public/program/OrbitalMotion

- Double star system:
  - https://www.glowscript.org/#/user/wolfs/folder/Public/program/DoubleStarSystem



Orbital motion: orbital properties (period).

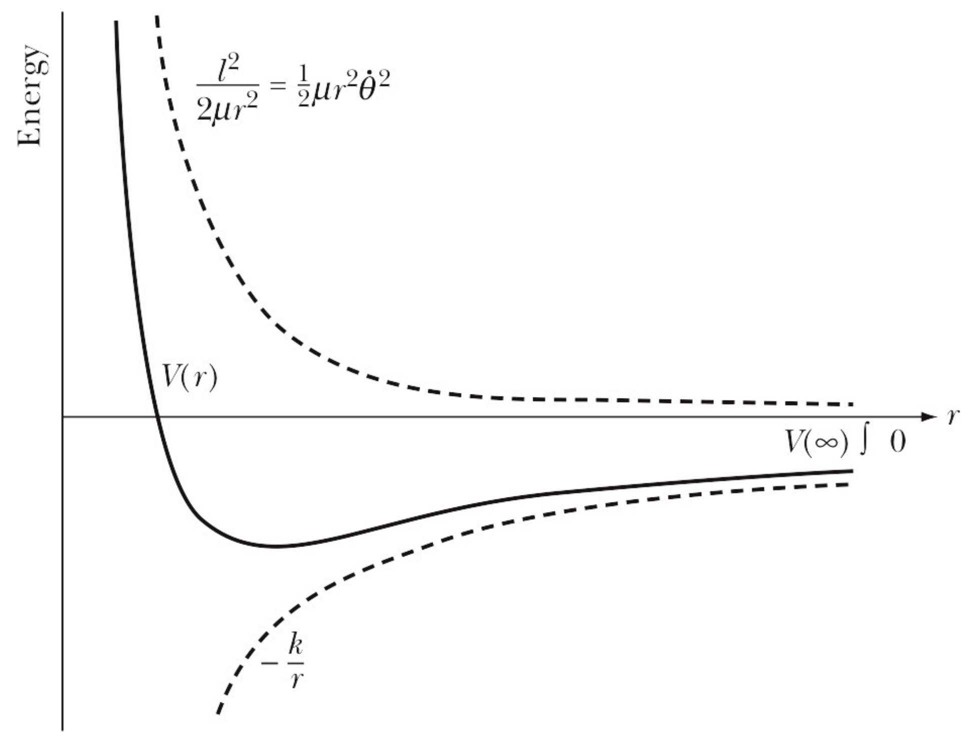
- To find the period of the orbit we use:
  - The known area velocity: $dt = (2\mu/l)\, dA$.
  - The known area of an ellipse: πab.
- We can now determined the orbital period:



$$\tau = \int dt = \frac{2\mu}{l}\int dA = \frac{2\mu}{l}(\pi ab) = \frac{2\mu}{l}\left(\pi \frac{k}{2|E|}\frac{l}{\sqrt{2\mu|E|}}\right) = \pi k\sqrt{\frac{\mu}{2}}|E|^{-3/2}$$

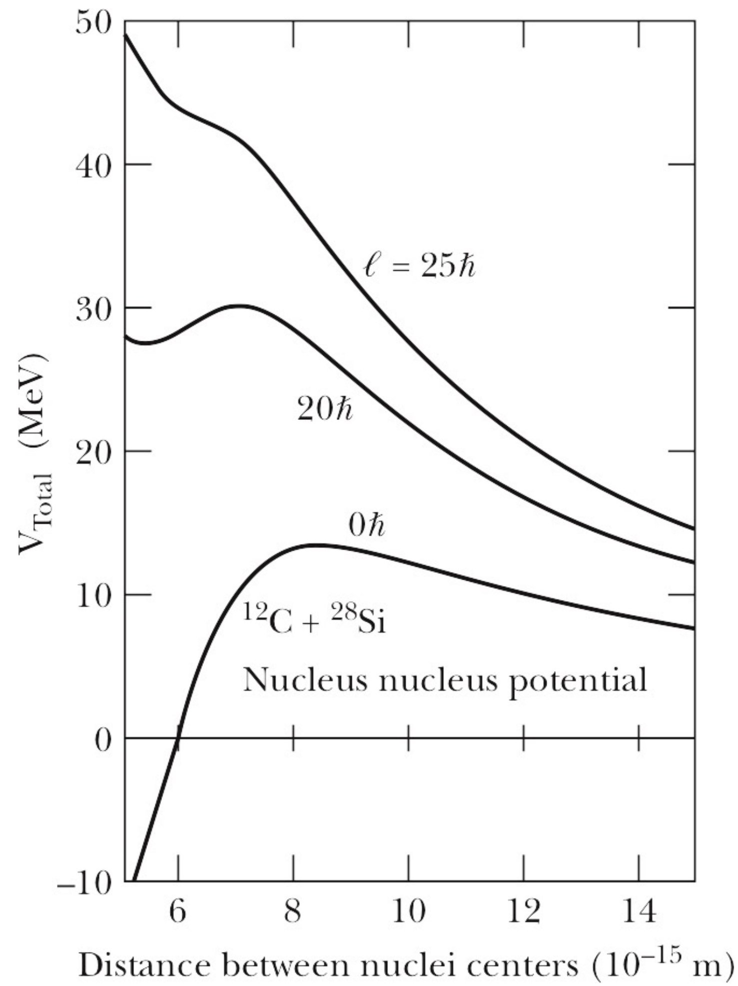
The "effective" potential.

- The effective potential is composed of the real potential and the centrifugal potential energy.
- Observations:
  - The effective potential may show a dip that indicates that for certain energies, the orbit is bound.
  - For small distances, the effective force becomes repulsive.



The “effective” potential for nuclear reactions.

- The effective potential for nuclear reactions includes the nuclear potential, the Coulomb potential, and the centrifugal potential.
- The dynamics of nuclear interactions can be understood of we look at the effective potential:
  - If $l = 0$, nuclear fusion can occur if $E > 15$ MeV.
  - For large l, the centrifugal barrier prevents nuclear fusion.



55 second pre-intermission.

2 Minute 36 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 2 minute 36 second intermission.

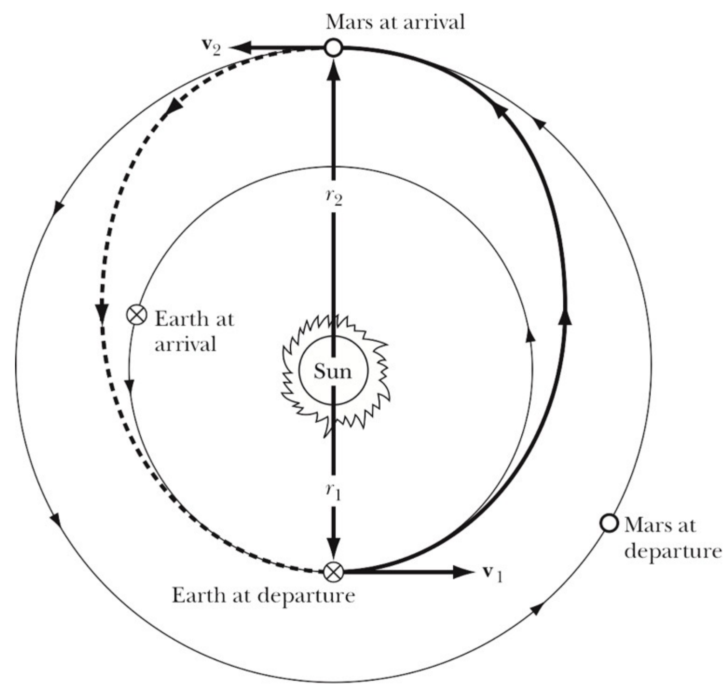
- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.

Problem 8.22

- Discuss the motion of a particle moving in an attractive central-force field described by $F(r) = -k/r^3$. Sketch some of the orbits for different values of the total energy.

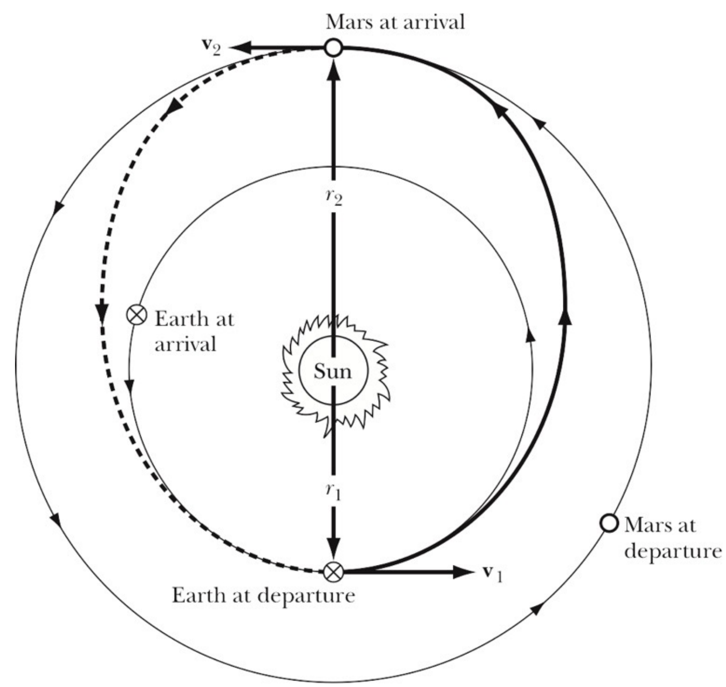
Orbital motion. Travelling to Mars.

- By changing the energy of a spaceship, we change the shape of its orbit.
- When we launch a spaceship from Earth, we can increase (or decrease) its total energy and change its orbit from circular to elliptical.
- If the proper conditions exist, we can use this to travel to Mars.



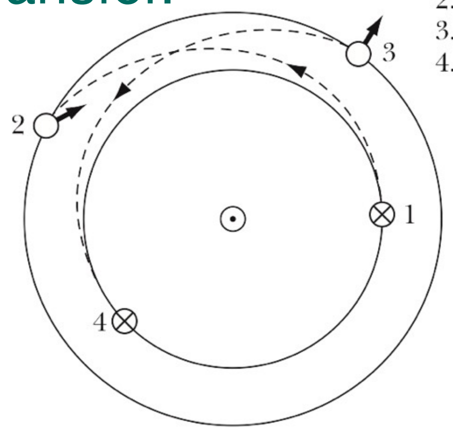
Orbital motion. Travelling to Mars.

- This mode of travelling to Mars minimizes the amount of energy required.
- It is called the **Hohmann transfer**.
- Comments:
  - This transfer minimizes energy.
  - It does not minimize time.
  - This mode of transport only works if the conditions are correct (Mars needs to be in the proper location).



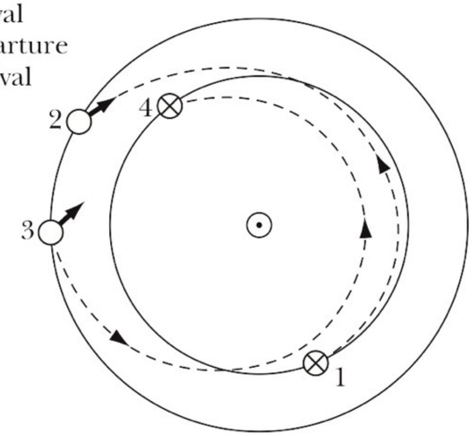
Other ways to get to Mars.

Hohmann transfer.



(a)

1. Earth departure
2. Mars arrival
3. Mars departure
4. Earth arrival

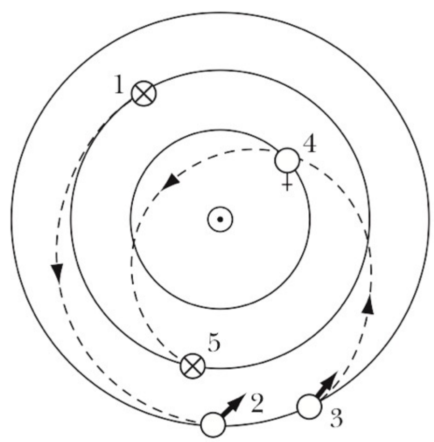


(b)

A shorter mission:

- More fuel required.
- Closer to the sun

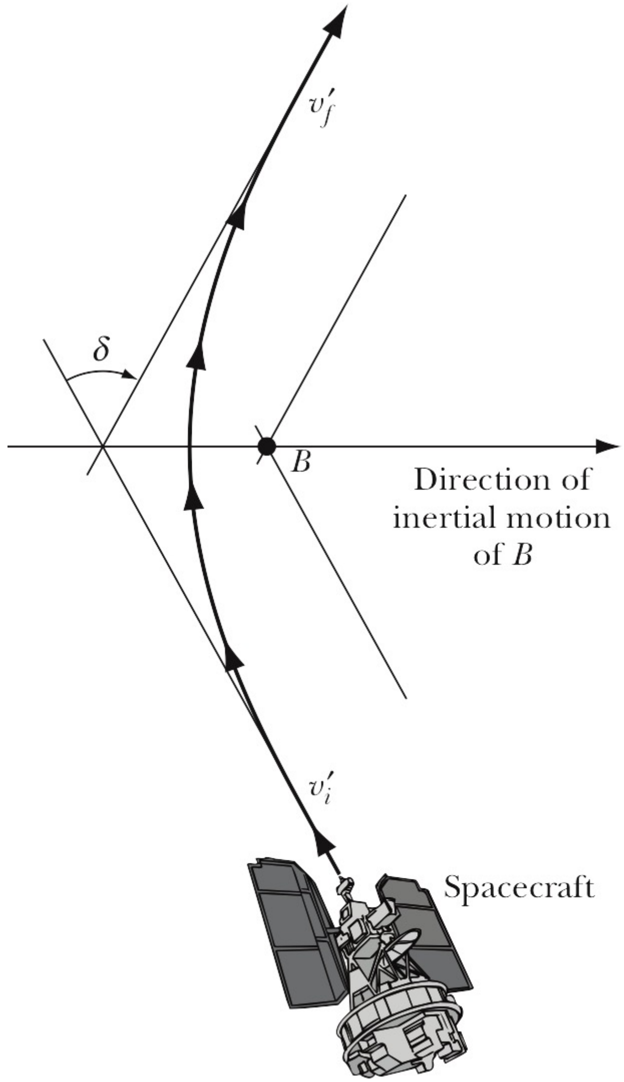
1. Earth departure
2. Mars arrival
3. Mars departure
4. Venus passage
5. Earth arrival

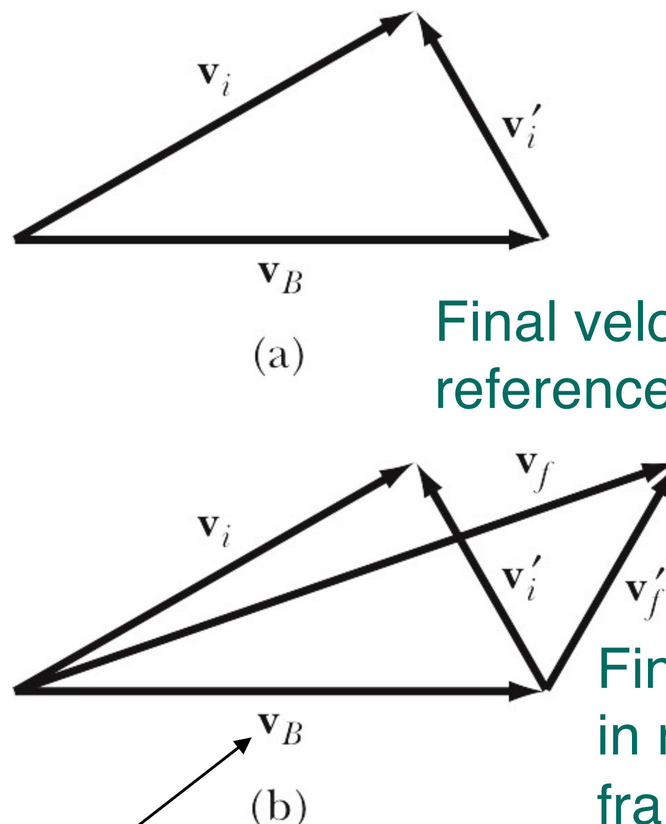


(c)

Reduce fuel consumption if Venus is in the proper position.

Fuel consumption can be reduced: gravity assists.



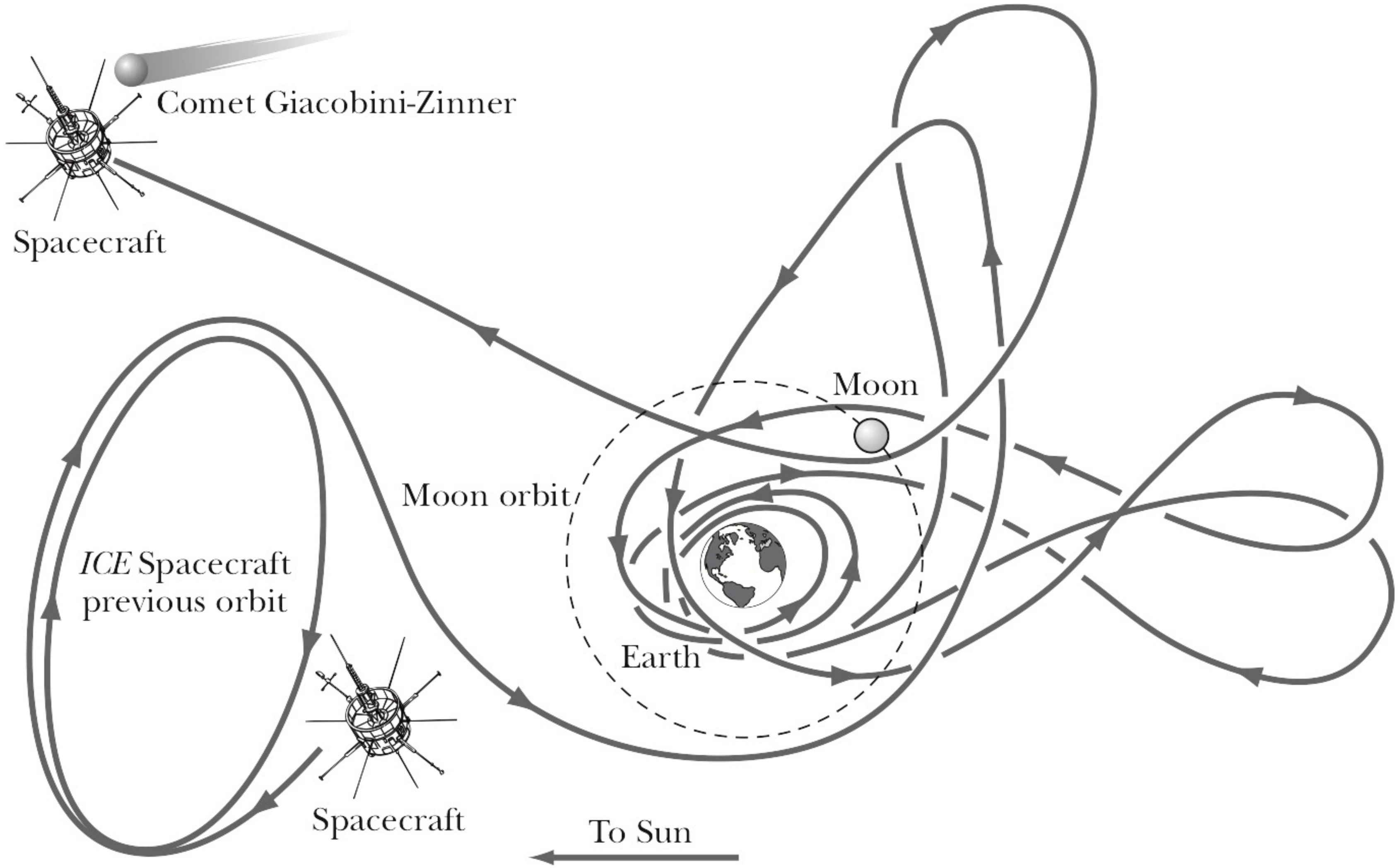


Final velocity in our reference frame.

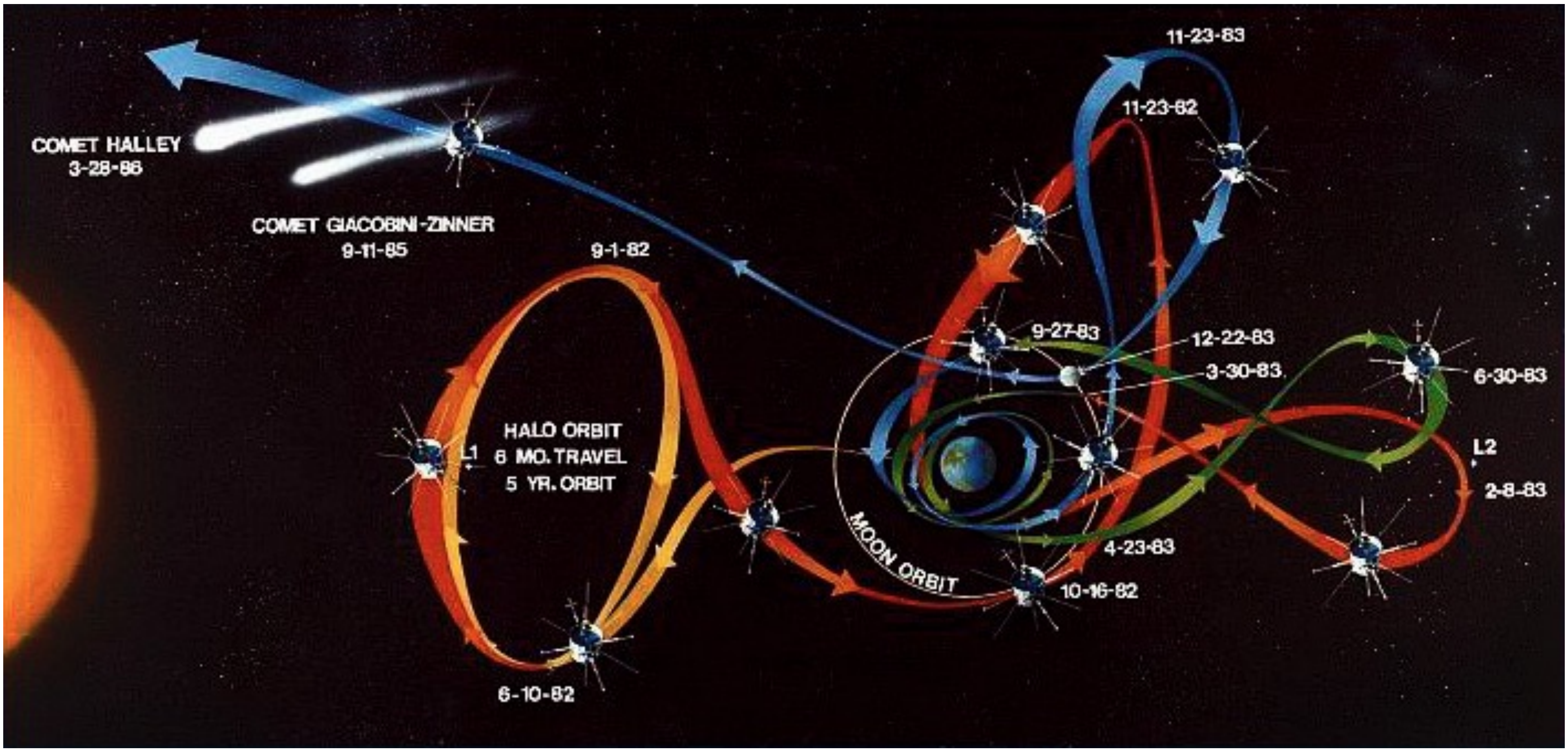
Final velocity in reference frame B.

Velocity of B in our reference frame.

Gravity assists



How long did it take?
A great application of numerical calculations!



ENOUGH FOR TODAY?