Classical Mechanics
Phy 235, Lecture 12.

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1

A Boeing 747 of the KLM, landing at Schiphol.

2

Course Comments

- The next exam will take place on Tuesday October 26.
  - Material covered: Chapters 5 – 7.
  - Material will be reviewed on Wednesday next week.
  - There will be extra office hours before the exams (details to be announced via email).
- You must submit a written proposal of the topic to be covered in your term paper by Friday October 29, 2021, at noon. Details about the term paper can be found on the web at:

  http://teacher.pas.rochester.edu/PHY235/CourseInformation/TermPaper.htm

3

Overview.

- Central force problems are important problems in physics.
- In Chapter 8 we focus on the motion associated with central forces.
- Today we will solve the problem of orbital motion.
- What is different today compared to your introductory course?
  - We do not assume that one object is more massive than the other object.
  - We do not assume that the more massive object is at rest.
  - We determine the general properties of the orbit and not restrict ourselves to circular motion.

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4

Chapter 8. Central-Force Motion.

- Many important problems in physics involve the motion of two bodies with a central force acting between them.
- Assume the potential depends on the position between the two objects.
- The Lagrangian can be written in terms of the coordinates of the two masses:

$$L = \frac{1}{2}m_1\left|\dot{\vec{r}}_1\right|^2 + \frac{1}{2}m_2\left|\dot{\vec{r}}_2\right|^2 - U\left(\vec{r}_1 - \vec{r}_2\right)$$

- In terms of their relative position:

$$L = \frac{1}{2}\mu\left|\dot{\vec{r}}\right|^2 - U\left(\vec{r}\right)$$

- Note: the two-body problem has been reduced to a one-body problem.

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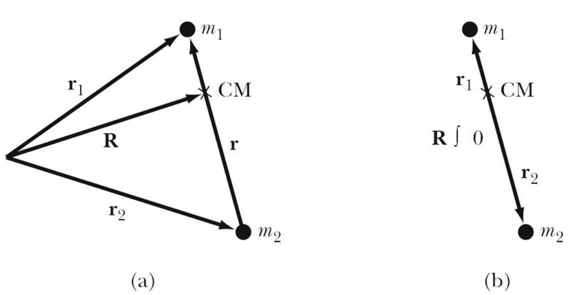
5

Changing a 2-body problem into a 1-body problem.



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6

Conservation of angular momentum. Spherical symmetry: U only depends on r.

Starting from the Lagrangian:

$$L = \frac{1}{2}\mu\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - U(r)$$

we define the generalized momenta:

$$p_r = \frac{\partial L}{\partial \dot{r}} = \mu \dot{r}$$

$$p_\theta = \frac{\partial L}{\partial \dot{\theta}} = \mu r^2 \dot{\theta}$$

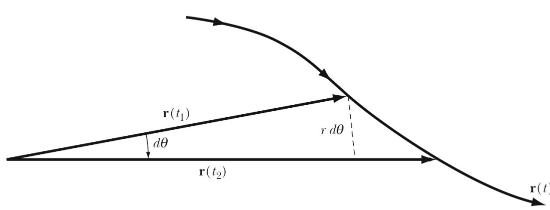
The time derivatives of the generalized momenta are:

$$\dot{p}_r = \frac{d}{dt}\frac{\partial L}{\partial \dot{r}} = \frac{\partial L}{\partial r} = \mu r \dot{\theta}^2 - \frac{\partial U}{\partial r}$$

$$\dot{p}_\theta = \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = \frac{\partial L}{\partial \theta} = 0$$

7

Areal Velocity.



8

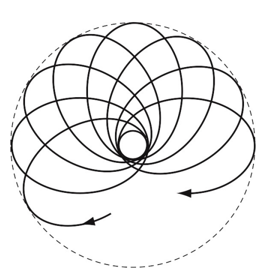
Orbital Motion.

- Equation of motion:

$$\frac{dr}{dt} = \pm\sqrt{\frac{2}{\mu}(E-U) - \frac{l^2}{\mu^2 r^2}}$$

- Change in polar angle during one period:

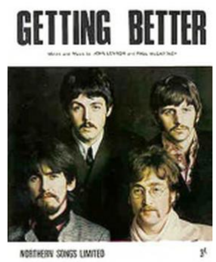
$$\Delta\theta = 2\int_{r_{\min}}^{r_{\max}} \frac{l}{r^2\sqrt{2\mu\left(E - U - \frac{l^2}{2\mu r^2}\right)}}\,dr$$



9

2 Minute 47 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 2 minute 47 second intermission.
- You can/you must complete the TA/TI survey: https://webapps.pas.rochester.edu/secure/phpQ/fillsurvey.php?sid=80



10

Problem 8.8

- Investigate the motion of a particle repelled by a force center according to the law $F(r) = kr$. Show that the orbit can only be hyperbolic.

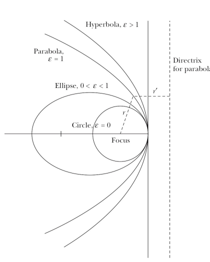
11

Solving the Orbital Equation.

$$\theta(r) = \pm\int \frac{1}{\sqrt{-u^2 + 2\mu\frac{k}{l}u + 2\mu E}}du = \pm\sin^{-1}\left(\frac{-2u + 2\mu\frac{k}{l}}{\sqrt{\left(2\mu\frac{k}{l}\right)^2 + 8\mu E}}\right) + C =$$

$$= \pm\sin^{-1}\left(\frac{\mu\frac{k}{l} - u}{\sqrt{\left(\mu\frac{k}{l}\right)^2 + 2\mu E}}\right) + C = \pm\sin^{-1}\left(\frac{\mu\frac{k}{l} - \frac{l}{r}}{\sqrt{\left(\mu\frac{k}{l}\right)^2 + 2\mu E}}\right) + C =$$

$$= \pm\sin^{-1}\left(\frac{\mu k - \frac{l^2}{r}}{\sqrt{(\mu k)^2 + 2\mu l^2 E}}\right) + C$$



12

ENOUGH FOR TODAY?

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13