Classical Mechanics
Phy 235, Lecture 05.

Frank L. H. Wolfs
Department of Physics and Astronomy
University of Rochester

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The last KLM 747 landing on St. Maarten.

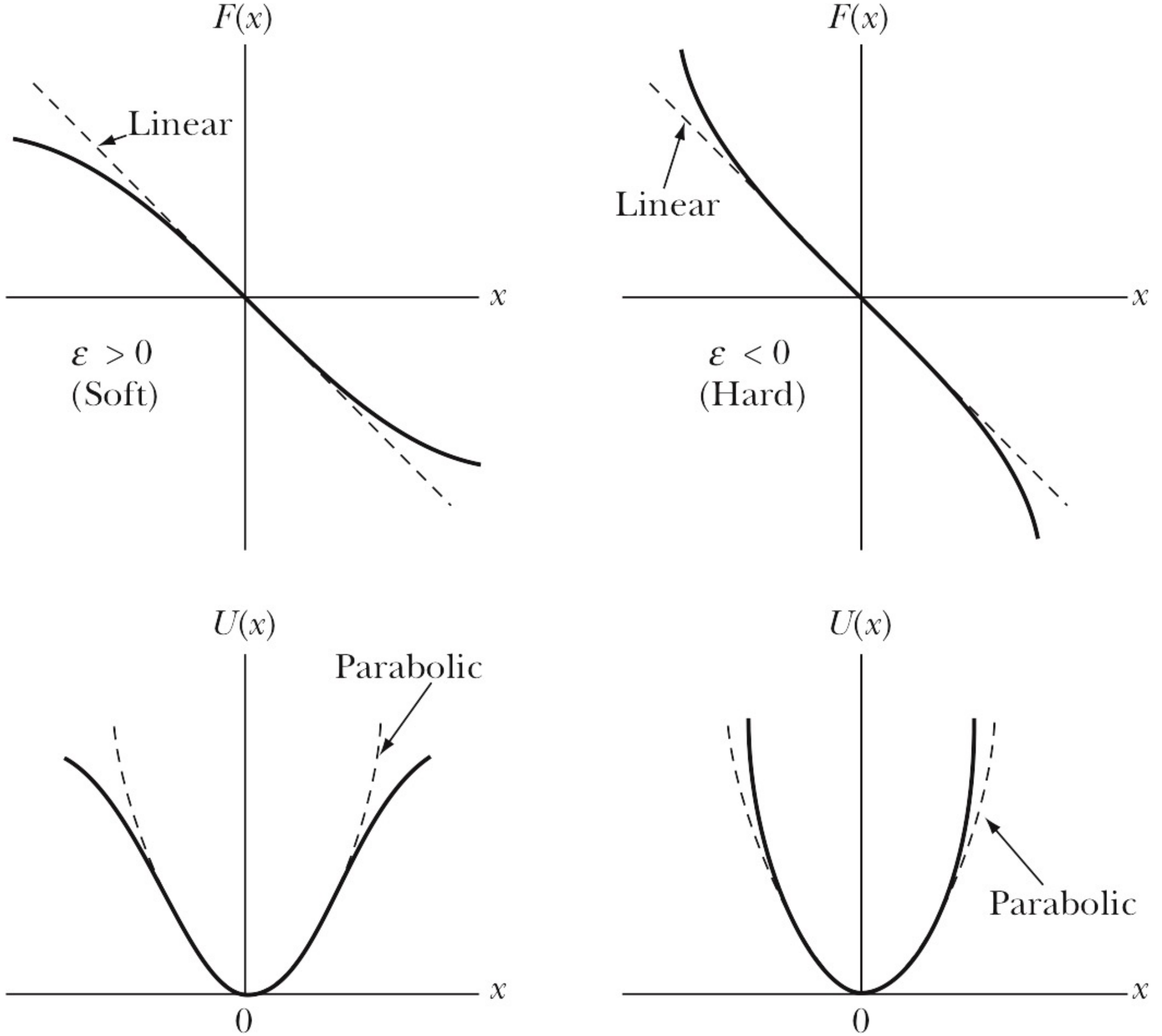
Non-linear oscillations.

- Linear differential equations:
  - Terms are proportional to acceleration, velocity, and position:

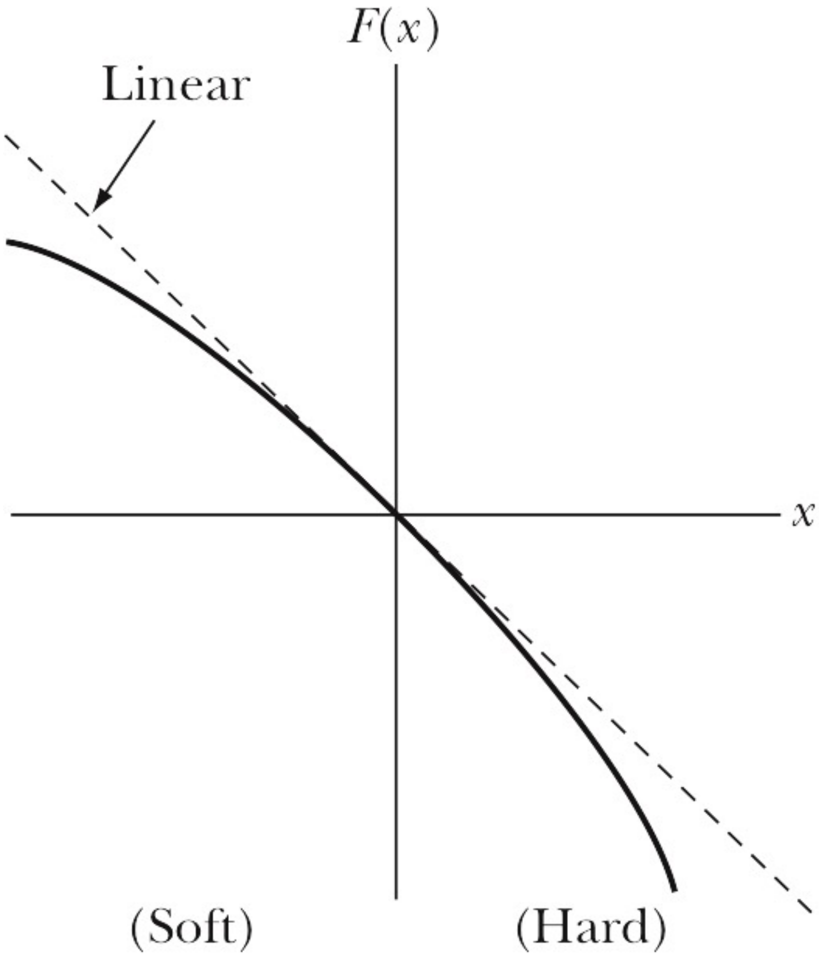
$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = f(x)$$

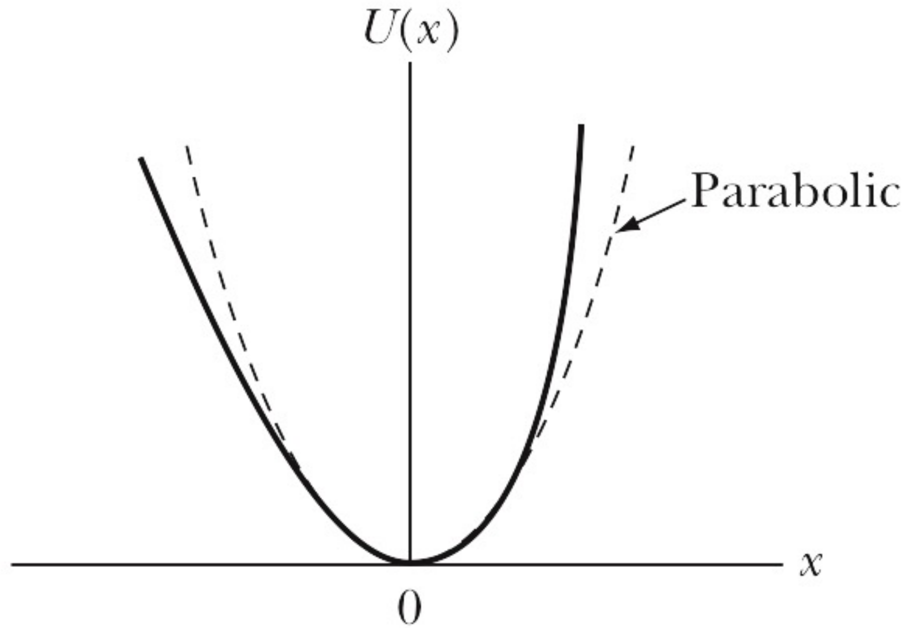
- Non-linear differential equations:
  - Include terms that non-linear in term of acceleration, velocity, and position.
  - Non-linear terms are divided in two groups:
    - Symmetric around the equilibrium position. This requires terms proportional to εr^3. If $\varepsilon>0$: soft system. If $\varepsilon<0$: hard system.
    - Asymmetric around the equilibrium position. This requires terms proportional to r^2.

Non-linear Forces: εx^3.



Non-linear Forces: λx^2.





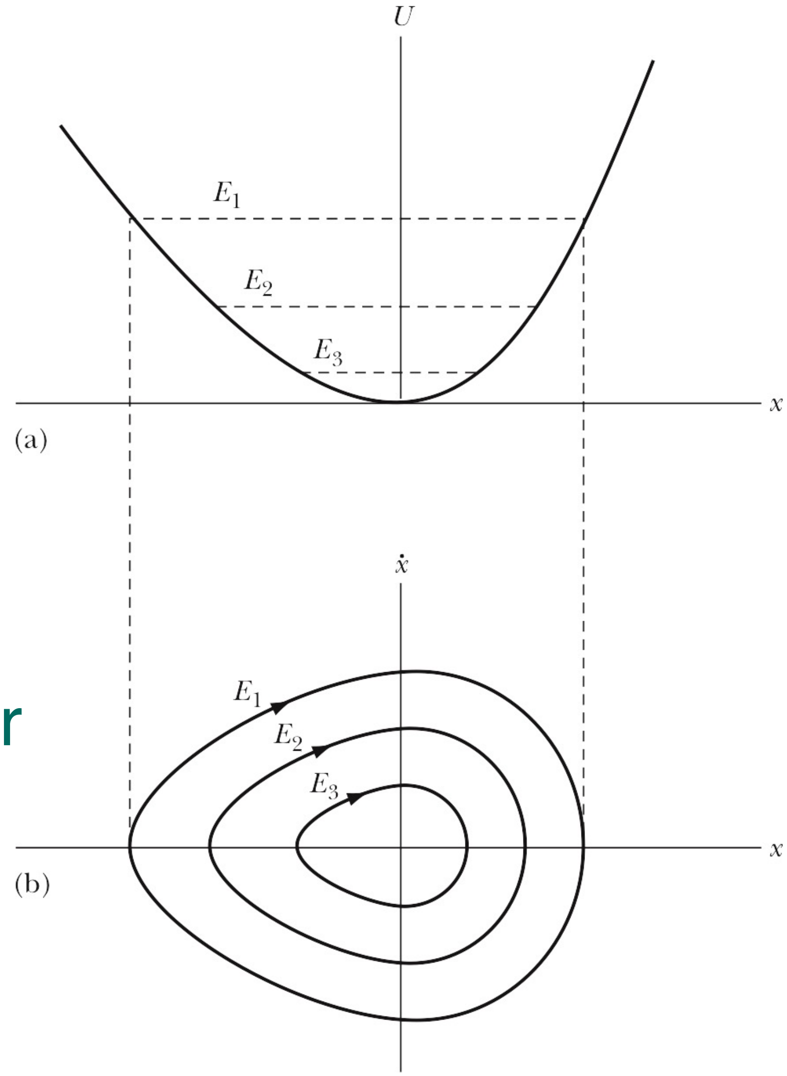
Numerical studies.

- Using tools such as VPython, it is easy to explore what happens when we add these additional components to the restoring force.

- Let us have a look:

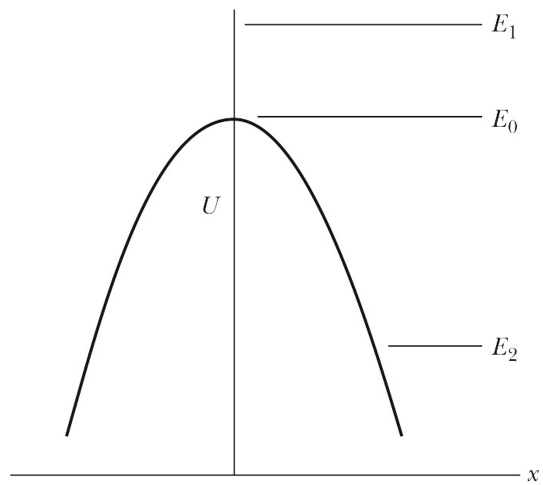
http://www.glowscript.org/#/user/wolfs/folder/Public/program/HardandSoftMotion

Phase Diagrams.
Asymmetric for asymmetric potentials.

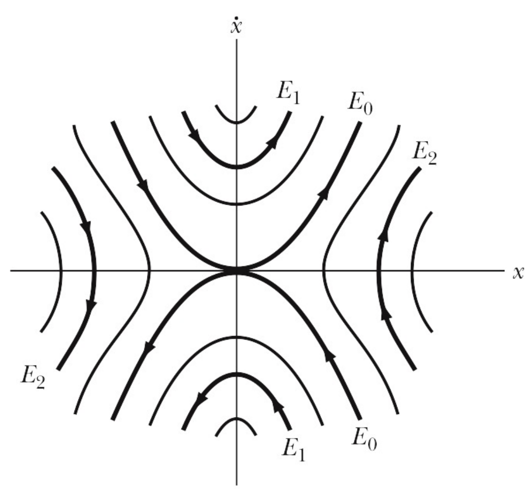


Closed contours for motion around stable equilibrium positions.

Phase diagrams.



(a)



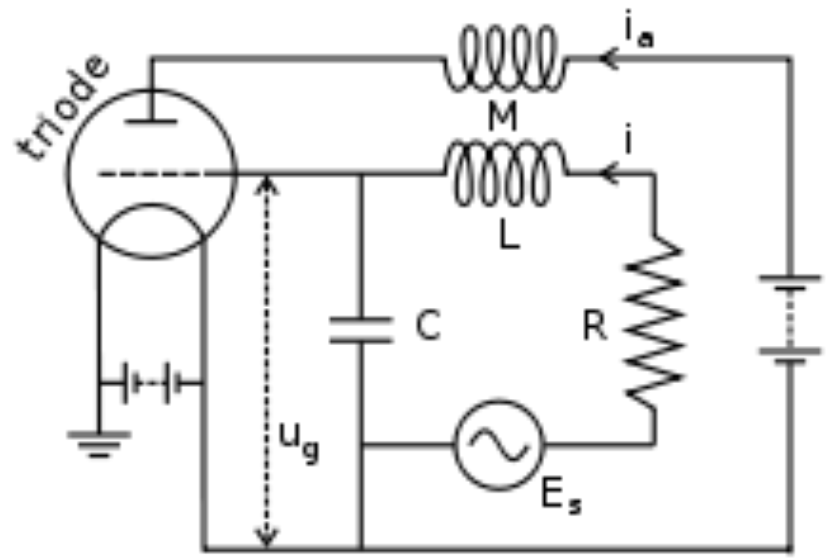
(b)

Open contours for motion around unstable equilibrium positions.

Van der Pol Equation.

- Used to describe non-linear oscillations in circuits containing vacuum tubes.
- Important facts:
  - Van der Pol was a Dutch physicist.
  - He studied physics in Utrecht and received his PhD in 1920.
- Van der Pol equation:

$$\ddot{x} + \mu\left(x^2 - a^2\right)\dot{x} + \omega_0^2 x = 0$$



Using Mathematic to solve and display the solution of differential equations.

```
In[268]:= (* Set the values of the various parameters *)
mu = 0.05;
a = 1.0;
w0 = 1;

(* Solve the differential equations with the given set of initial conditions. *)
sol = NDSolve[
  {x'[t] == v[t],
   v'[t] == -mu * (x[t] * x[t] - a * a) * v[t] - w0 * w0 * x[t],
   x[0] == 1,
   v[0] == 0},
  {x, v}, {t, 0, 50 * 2 * Pi}, MaxSteps → 20 000];

(* Plot the solution of the differential equations *)
ParametricPlot[Evaluate[{x[t], v[t]} /. sol], {t, 0, 50 * 2 * Pi},
 PlotRange → All,
 AxesLabel → {"x (rad)", "v (rad/s)"}]
```

Initial parameters.

Diff. equation.

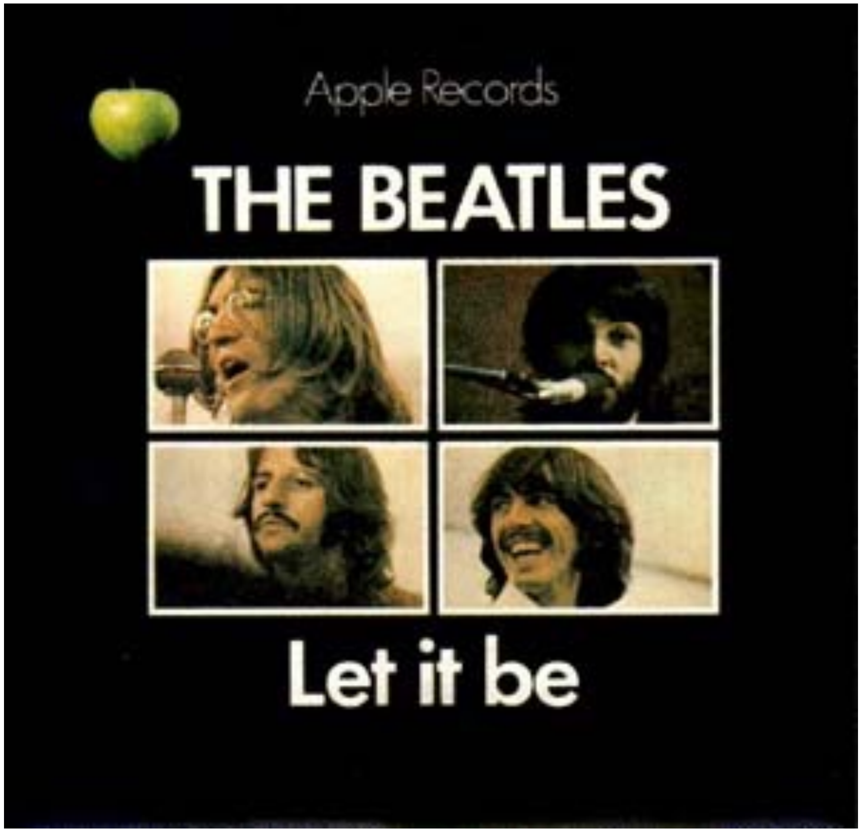
Boundary conditions (*x* and *v* at $t = 0$).

Display the solution.

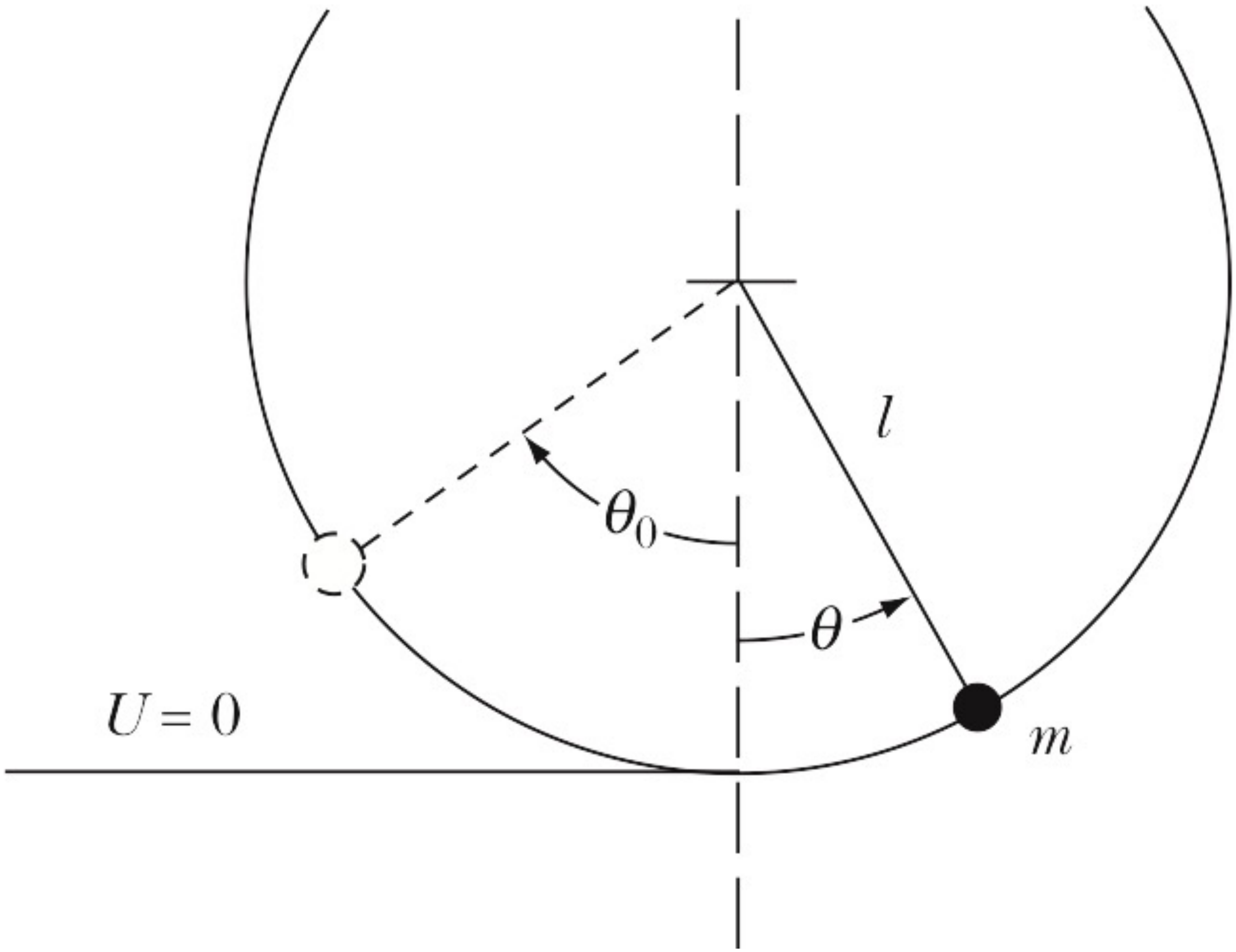
3 Minute 52 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 3 minute 52 second intermission.

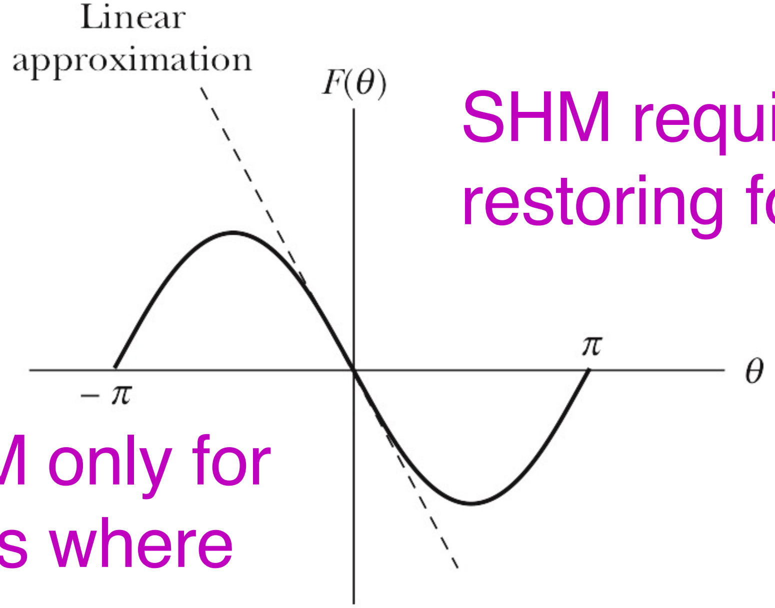
- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.



Plane Pendulum.

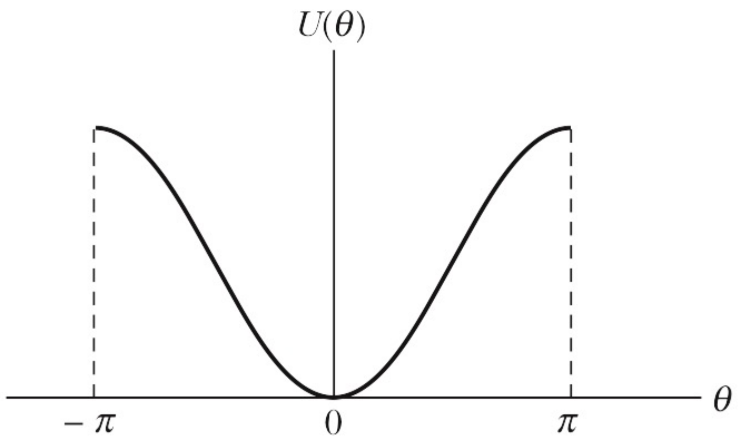


Plane Pendulum.



SHM requires a linear restoring force.

Expect SHM only for small angles where $\sin(\theta) = \theta$.

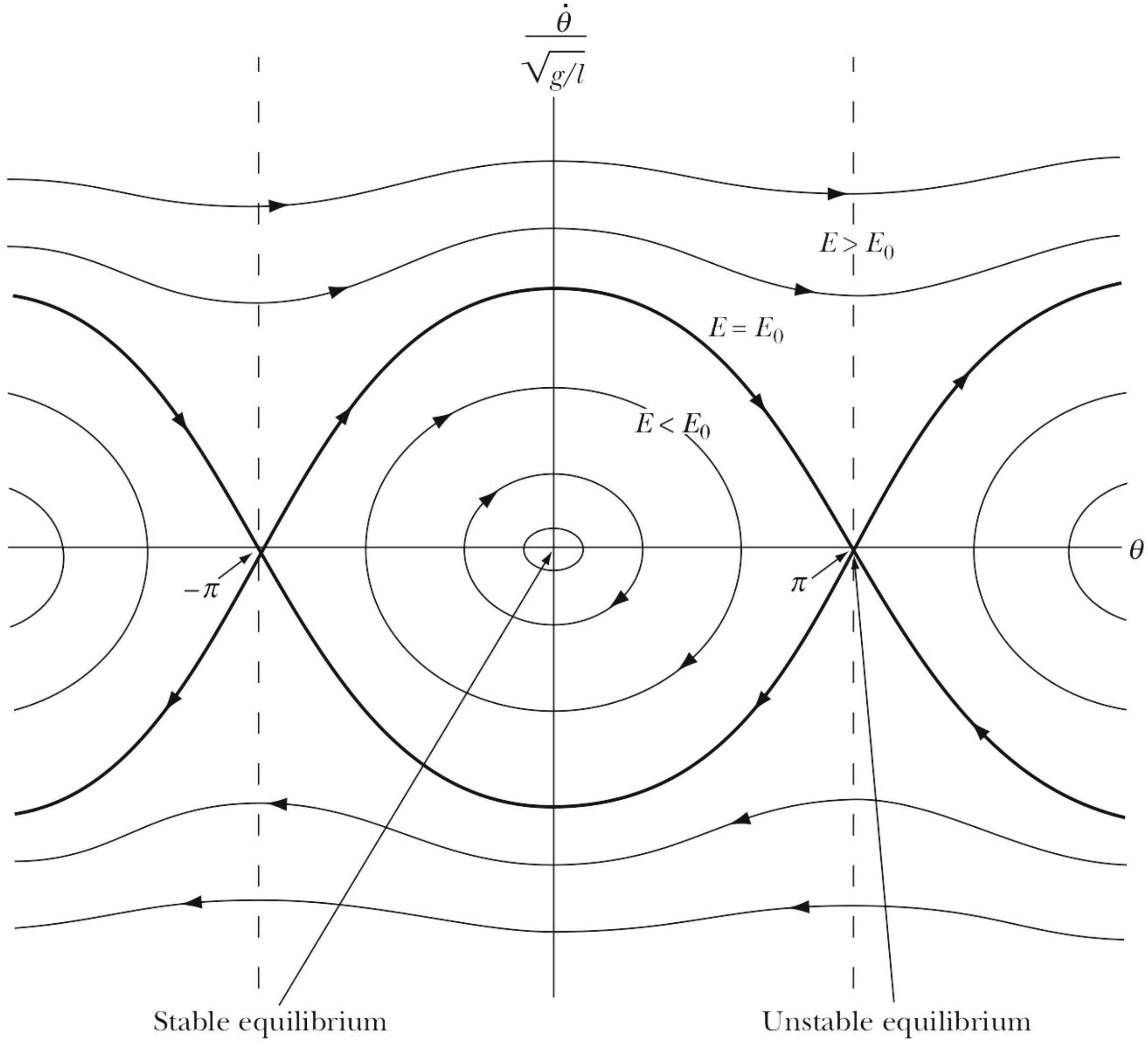


Numerical studies.

- Using tools such as VPython, it is easy to explore what happens for large angles.
- Let us have a look:

http://www.glowscript.org/#/user/wolfs/folder/Public/program/PlanePendulum

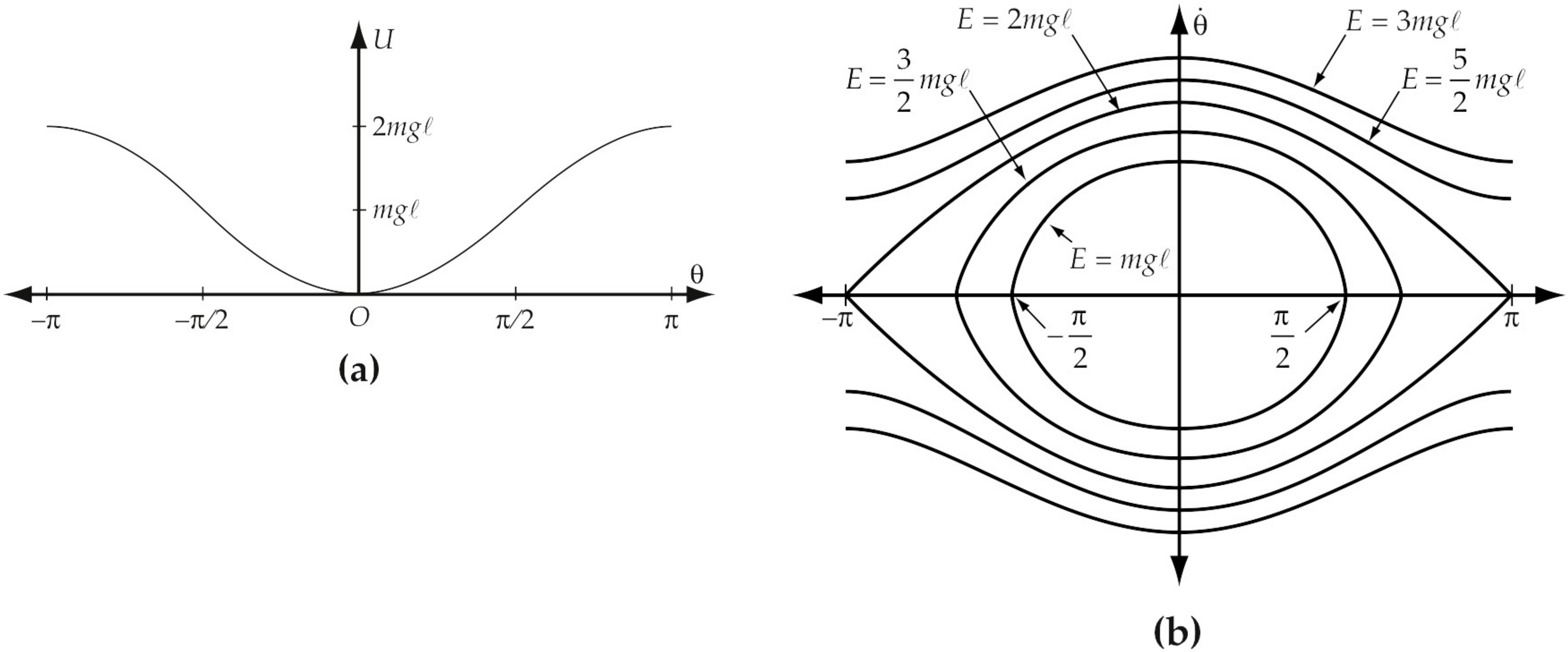
Phase diagram for the plane pendulum.



Problem 4.6.

- Derive the expression for the phase paths of the plane pendulum if the total energy $E > 2mgl$. Note that this is just the case of a particle moving in a periodic potential $U(\theta) = mgl(1 - \cos\theta)$.

Problem 4.6.



U
$2mg\ell$
$mg\ell$
θ
$-\pi$
$-\pi/2$
O
$\pi/2$
π
(a)
$E = 2mg\ell$
$\dot{\theta}$
$E = 3mg\ell$
$E = \frac{3}{2}mg\ell$
$E = \frac{5}{2}mg\ell$
$E = mg\ell$
θ
$-\pi$
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
(b)

ENOUGH FOR TODAY?