

## Physics 235, Homework Set 07

**Write the following text on the front cover of your homework assignment and sign it. If the text is missing, 20 points will be subtracted from your homework grade.**

### Honor Pledge for Graded Assignments

"I affirm that I have not given or received any unauthorized help on this assignment, and that this work is my own."

---

---

---

Signature \_\_\_\_\_

1. Show that the areal velocity is constant for a particle moving under the influence of an attractive force, given by  $F(r) = -kr$ . Calculate the time average of the kinetic and the potential energy.
2. A communication satellite is in a circular orbit of radius  $R$  around the earth. Its velocity is  $v$ . Its engine accidentally fires, giving the satellite an outward radial velocity  $v$  in addition to its original velocity.
  - a. Calculate the ratio of the new energy and angular momentum to the old.
  - b. Describe the subsequent motion of the satellite and plot  $T(r)$ ,  $U(r)$ , and  $E(r)$  after the engine fires.

Note: for a circular orbit,  $T = -U/2$ .

3. A particle of unit mass moves from infinity along a straight line that, if continued, would allow it to pass a distance  $b\sqrt{2}$  from a point  $P$ . If the particle is attracted toward  $P$  with a force varying as  $k/r^5$ , and if the angular momentum about  $P$  is  $(\sqrt{k})/b$ , show that the trajectory is given by

$$r = b \coth\left(\frac{\theta}{\sqrt{2}}\right)$$

4. A particle moves in an elliptical orbit in an inverse-square-law central-force field. If the ratio of the maximum angular velocity to the minimum angular velocity of the particle in its orbit is  $n$ , then show that the eccentricity of the orbit

$$\varepsilon = \frac{\sqrt{n} - 1}{\sqrt{n} + 1}$$

5. Consider the family of orbits in a central potential for which the total energy is a constant. Show that if a stable circular orbit exists, the angular momentum associated with this orbit is larger than that for any other orbit in the family.