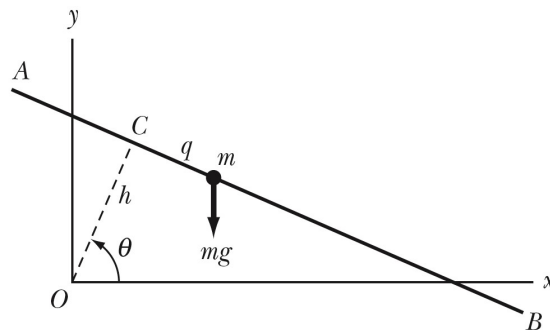


## Physics 235, Homework Set 06

1. A double pendulum consists of two simple pendula, with one pendulum suspended from the bob of the other. If the two pendula have equal lengths, have bobs of equal mass, and are confined to move in the same plane, find Lagrange's equations of motion for the system. Do not assume small angles.
2. A particle of mass  $m$  can slide freely along a wire  $AB$  whose perpendicular distance to the origin  $O$  is  $h$  (see Figure below).



The line  $OC$  rotates about the origin at a constant rate  $d\theta/dt = \omega$ . The position of the particle can be described in terms of the angle  $\theta$  and the distance  $q$  to the point  $C$ . If the particle is subject to a gravitational force, and if the initial conditions are

$$\theta(0) = 0 \quad q(0) = 0 \quad \dot{q}(0) = 0$$

show that the time dependence of the coordinate  $q$  is

$$q(t) = \frac{g}{2\omega^2} (\cosh \omega t - \cos \omega t)$$

Sketch the result. Compute the Hamiltonian for the system, and compare it with the total energy. Is the total energy conserved?

3. Two masses  $m_1$  and  $m_2$  ( $m_1 \neq m_2$ ) are connected by a rigid rod of length  $d$  and of negligible mass. An extensionless string of length  $l_1$  is attached to  $m_1$  and connected to a fixed point of support  $P$ . Similarly, a string  $l_2$  ( $l_1 \neq l_2$ ) connects  $m_2$  to  $P$ . Obtain the equation describing the motion of this system in the plane of  $m_1$ ,  $m_2$ , and  $P$ , and find the frequency of small oscillations around the equilibrium position.

4. A particle is constrained to move (without friction) on a circular wire rotating with a constant angular speed  $\omega$  about a vertical diameter. Find the equilibrium position of the particle, and calculate the frequency of small oscillations around this position. Find and interpret physically a critical angular velocity  $\omega = \omega_c$  that divides the particle's motion into two distinct types. Construct a phase diagram for the two cases  $\omega < \omega_c$  and  $\omega > \omega_c$ .
  
5. A particle of mass  $m$  moves under the influence of gravity along the helix  $z = k\theta$ ,  $r = \text{constant}$ , where  $k$  is a constant and  $z$  is the coordinate along the vertical axis. Obtain the Hamiltonian equations of motion.