Do not turn the pages of the exam until you are instructed to do so.

Exam rules: You may use only a writing instrument while taking this test. You may not consult any calculators, computers, books, nor each other.

Problems 1 and 2 must be answered in exam booklet 1. Problems 3 and 4 must be answered in exam booklet 2. The answers need to be well motivated and expressed in terms of the variables used in the problem. You will receive partial credit where appropriate, but only when we can read your solution. Answers that are not motivated will not receive any credit, even if correct.

At the end of the exam, you need to hand in your exam, the blue exam booklets, and the equation sheet. All items must be clearly labeled with your name, your student ID number, and the day/time of your recitation. If any of these items are missing, we will not grade your exam, and you will receive a score of 0 points.

You are required to complete the following Honor Pledge for Exams. Copy and sign the pledge before starting your exam.

“I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.”

____________________________________________________________________________

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Name: ________________________________________________________________

Signature: _____________________________________________________________
Good Luck!
PROBLEM 1 (25 POINTS)

Consider a thin uniform disk of mass $M$ and radius $a$. The center of the disk is located at the origin of our coordinate system, and the disk is located in the $xy$ plane.

a. Find the potential $\phi$ on the $z$ axis, a distance $z$ from the center of the disk.

b. Find the gravitational field on the $z$ axis, a distance $z$ from the center of the disk. Indicate the direction and magnitude of the gravitational field.

c. Find the gravitational force on a mass $m$ located at this position. Indicate the direction and magnitude of the gravitational force.

Your answers must be well motivated and expressed in terms of the variables provided.
PROBLEM 2 (25 POINTS)  

Consider a spherical pendulum of mass \( m \) and length \( b \), as shown in the Figure below. For a spherical pendulum, both \( \theta \) and \( \phi \) can depend on time.

a. Express the kinetic energy and the potential energy of the pendulum in terms of the generalized coordinates \( \theta \) and \( \phi \).

b. Calculate are the generalized momenta.

c. Determine the Hamiltonian for this system.

d. Determine the equations of motion.

e. Which of the generalized coordinates or momenta are constant?

Your answers must be well motivated and expressed in terms of the variables provided.
Consider a particle travelling in a constant force field (e.g. the gravitational field), directed along the x axis. The particle starts from rest at (0,0) and moves to a position \((x_2, y_2)\).

a. How does the velocity of the particle depends on \(x\)?

b. Express the time required for the particle to move from the initial position to the final position, following a path \(y(x)\), in terms of an integral over \(x\) between \(x = 0\) and \(x = x_2\).

c. Find the path that allows the particle to accomplish this movement in the least amount of time.

Your answers must be well motivated and expressed in terms of the variables provided.
PROBLEM 4 (25 POINTS)

a. Consider a disk rolling without slipping on an inclined plane, as shown in the Figure below. Consider the coordinates $y$ and $\theta$.

What is the equation of constraint you would use for this system?
b. Consider the orbital speed of the observed mass as function of the distance from the center of the Andromeda galaxy, shown in the following Figure.

![Graph showing orbital speed vs. radius from galactic center.](image)

The dashed line represents the $1/\sqrt{R}$ dependence expected from the Keplerian result of Newton’s laws. What can account for the difference between the expected orbital speed and the observed orbital speed?
c. Consider light passing from one medium with index of refraction $n_1$ into another medium with index of refraction $n_2$, as shown in the following Figure.

What parameter do we minimize when we derive the law of refraction?

d. What two conditions must be met for the Hamiltonian $H$ to be equal to the total energy $E$?
e. What is the favorite color of your instructor?

A. Black
B. Red
C. Green
D. Yellow
E. Blue
F. Orange
G. Pink
H. White
I. Yankees
J. The Netherlands