Physics 235, Midterm Exam # 1

September 27, 2018: 8.00 am - 9.30 am

Do not turn the pages of the exam until you are instructed to do so.

Exam rules: You may use *only* a writing instrument while taking this test. You may *not* consult any calculators, computers, books, nor each other.

Problems 1 and 2 must be answered in exam booklet 1. Problems 3 and 4 must be answered in exam booklet 2. The answers need to be well motivated and expressed in terms of the variables used in the problem. You will receive partial credit where appropriate, but only when we can read your solution. Answers that are not motivated will not receive any credit, even if correct.

At the end of the exam, you need to hand in your exam, the blue exam booklets, and the equation sheet. All items must be clearly labeled with your name, your student ID number, and the day/time of your recitation. **If any of these items are missing, we will not grade your exam, and you will receive a score of 0 points.**

You are required to complete the following *Honor Pledge for Exams*. Copy and sign the pledge before starting your exam.

"I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own."

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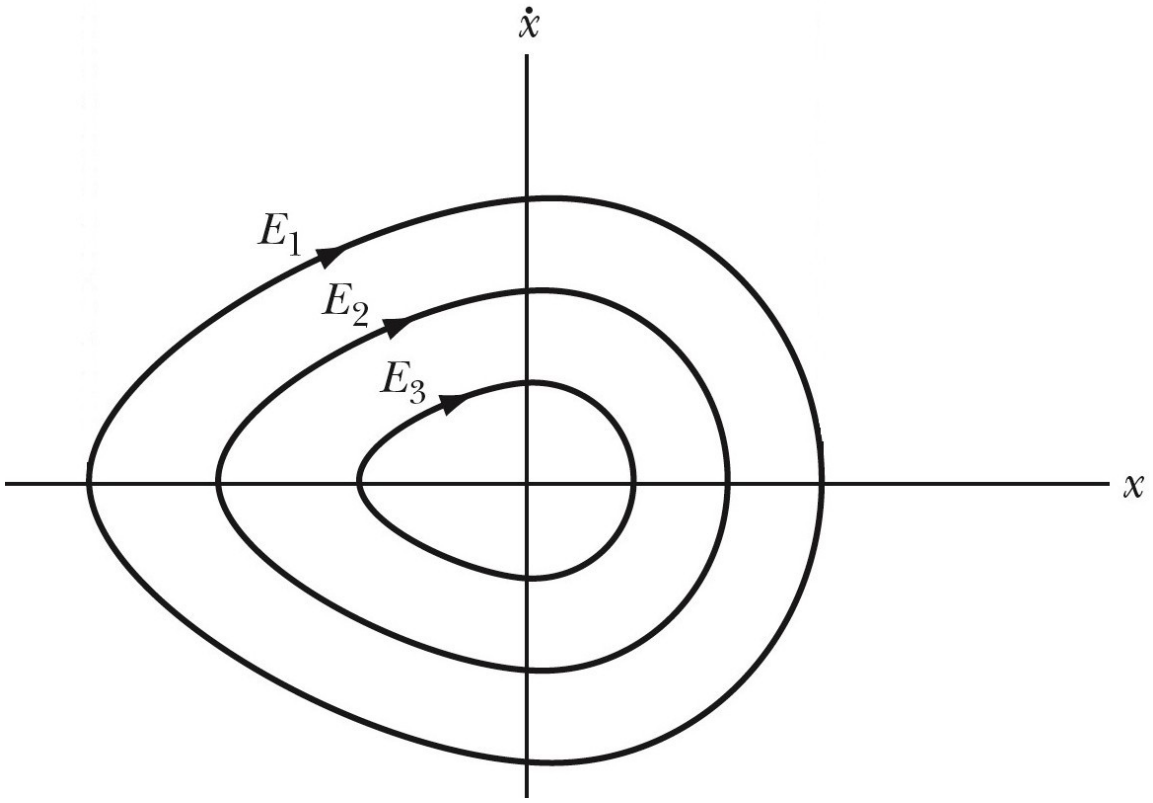
Name: _____

Signature: _____

Problem 1 (25 points)

a. Consider the following phase diagram that describes the one-dimensional motion of an object of mass m.

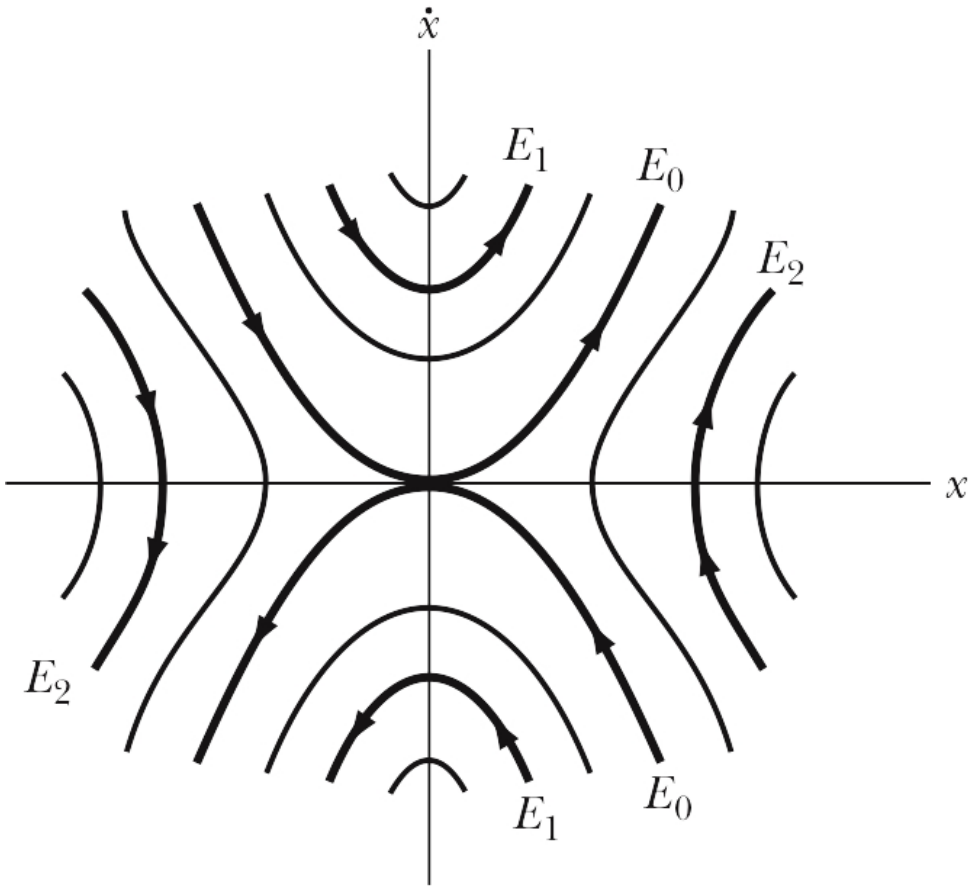


For this phase diagram answer the following questions:

1. What is the location of the equilibrium position? You need to motivate your answer!
2. Is the equilibrium a stable or an unstable equilibrium? You need to motivate your answer!
3. Rank the three energies, E_1, E_2, and E_3, from largest to smallest.

Sketch the potential associated with the phase diagram. In particular, pay attention to the symmetry of the potential around the equilibrium position.

b. Consider the following phase diagram that describes the one dimentional motion of an object of mass m.

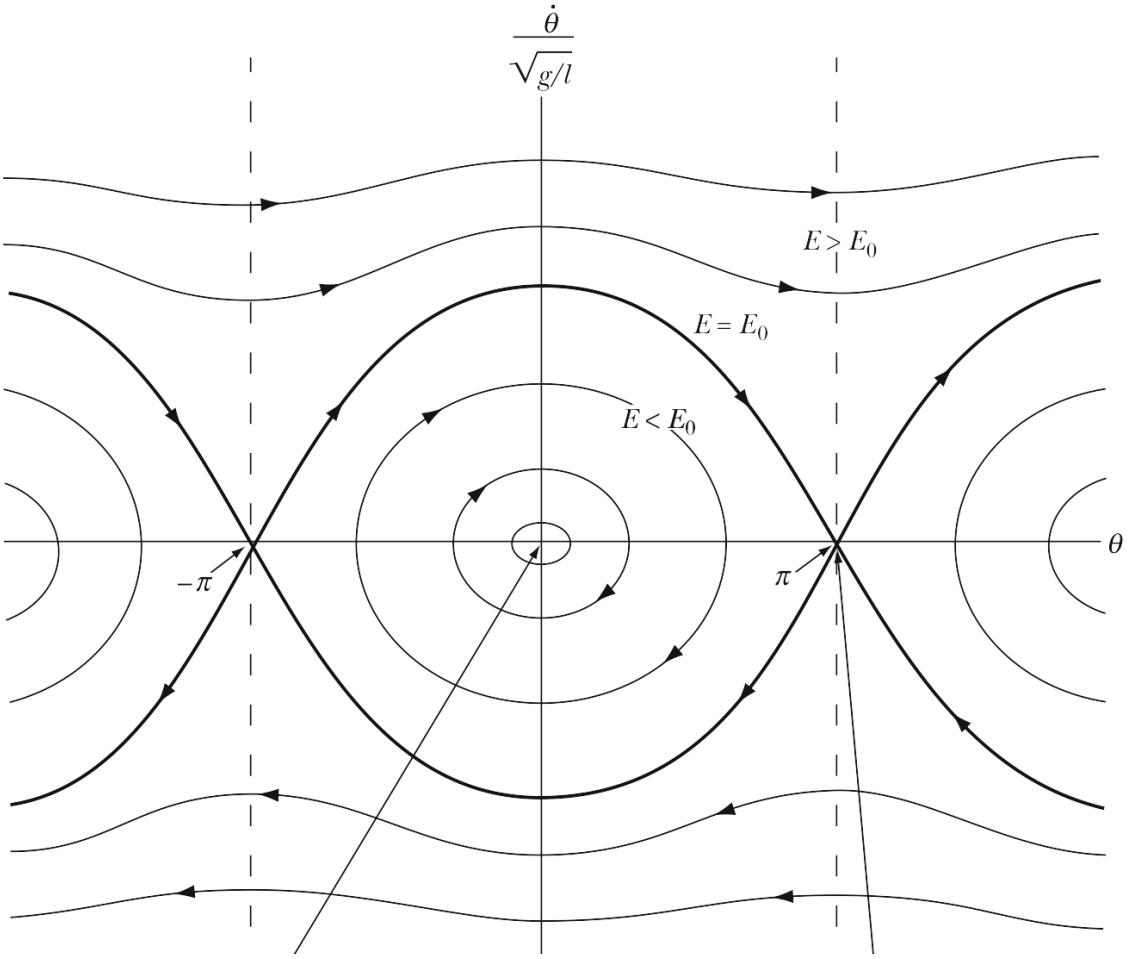


For this phase diagram answer the following questions:

1. What is the location of the equilibrium position? You need to motivate your answer!
2. Is the equilibrium a stable or an unstable equilibrium? You need to motivate your answer!
3. Rank the three energies, E_0, E_1, and E_2, from largest to smallest.

Sketch the potential associated with the phase diagram. In particular, pay attention to the symmetry of the potential around the equilibrium position.

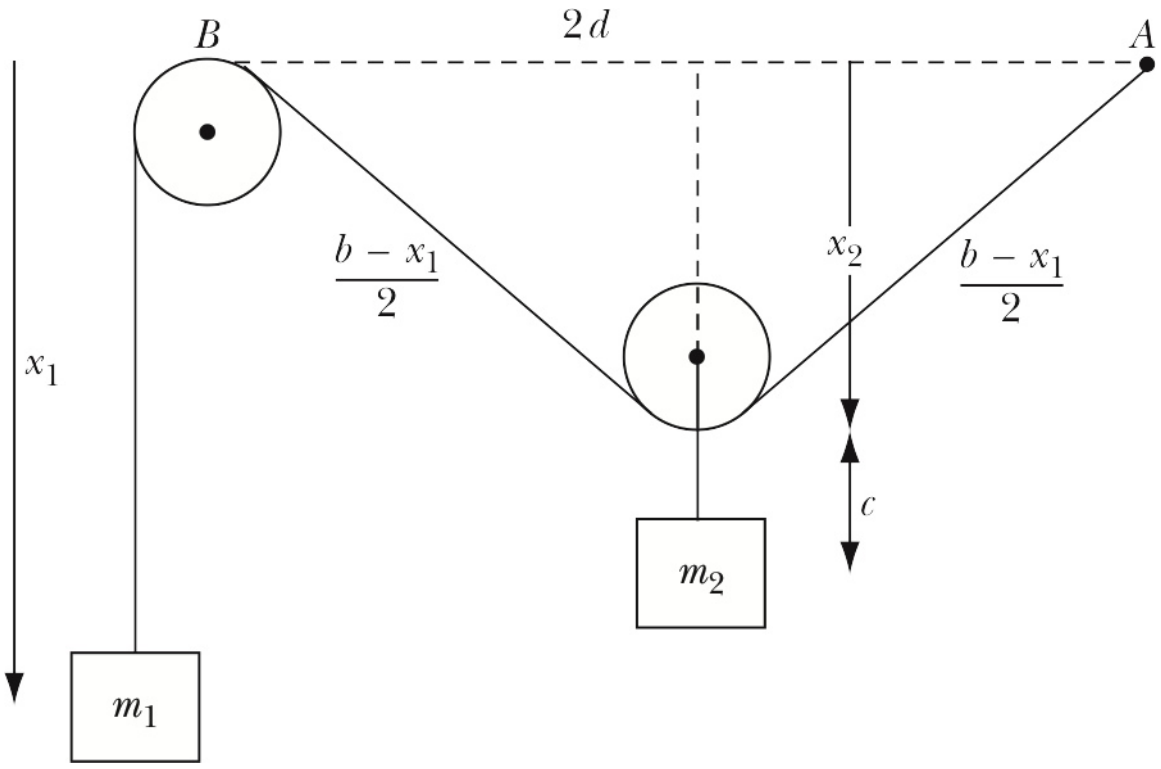
c. Consider the following phase diagram that describes the motion of a plane pendulum of mass m and length l.



Derive the expression for the phase path of the plane pendulum (angular velocity as function of angle) if the total energy E exceeds $E_0 = 2mgl$.

Problem 2 (25 points)

Consider the system of pulleys, masses, and string shown in the Figure below.



A massless string of length b is attached to point A, passes over a pulley at point B located a distance $2d$ away, and finally attaches to mass m_1. Another pulley with mass m_2 attached, passes over the string, pulling it down between A and B. Assume that the radius of the pulley holding mass m_2 is small so that we can neglect it. Assume also that the pulleys are massless. The distance c is constant.

a) What is the potential energy of the system in this configuration (when mass m_1 is located a distance x_1 below the level defined by A and B)?
b) Determine the position of mass m_1 for which the system will be in equilibrium.
c) Determine if the equilibrium position obtained in part b) is a stable or an unstable equilibrium position.

Problem 3 (25 points)

A particle of mass m is at rest at the end of a spring with spring constant k, hanging from a fixed support. At time $t = 0$, a constant downward force F is applied to the mass and acts until time $t = t_0$.

a) Write down the equation of motion for times between $t = 0$ and $t = t_0$.

b) Write down the equation of motion for times $t \geq t_0$.

c) What is the general solution for times between $t = 0$ and $t = t_0$. Note: you do not yet need to determine the two integration constants in this solution.

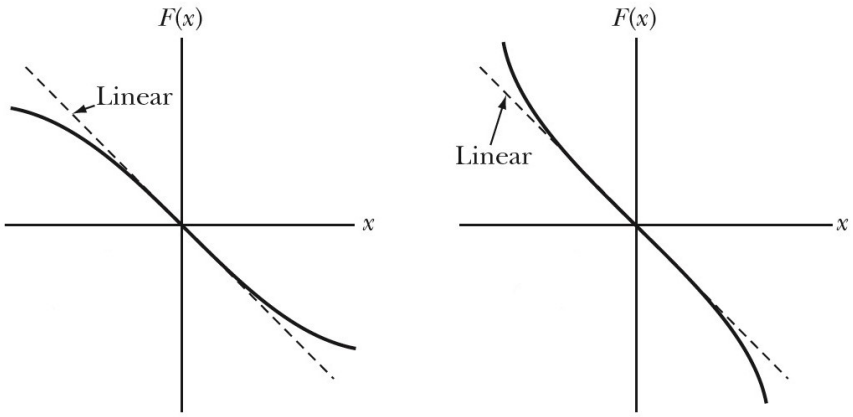
d) What is the general solution for times $t \geq t_0$. Note: you do not yet need to determine the two integration constants in this solution.

e) What are the initial conditions at time $t = 0$? Use these intial conditions to determine the integration constants in the general solution found in 3c for times between $t = 0$ and $t = t_0$.

f) Find the displacement of the mass as function of time after the force F has been removed ($t \geq t_0$).

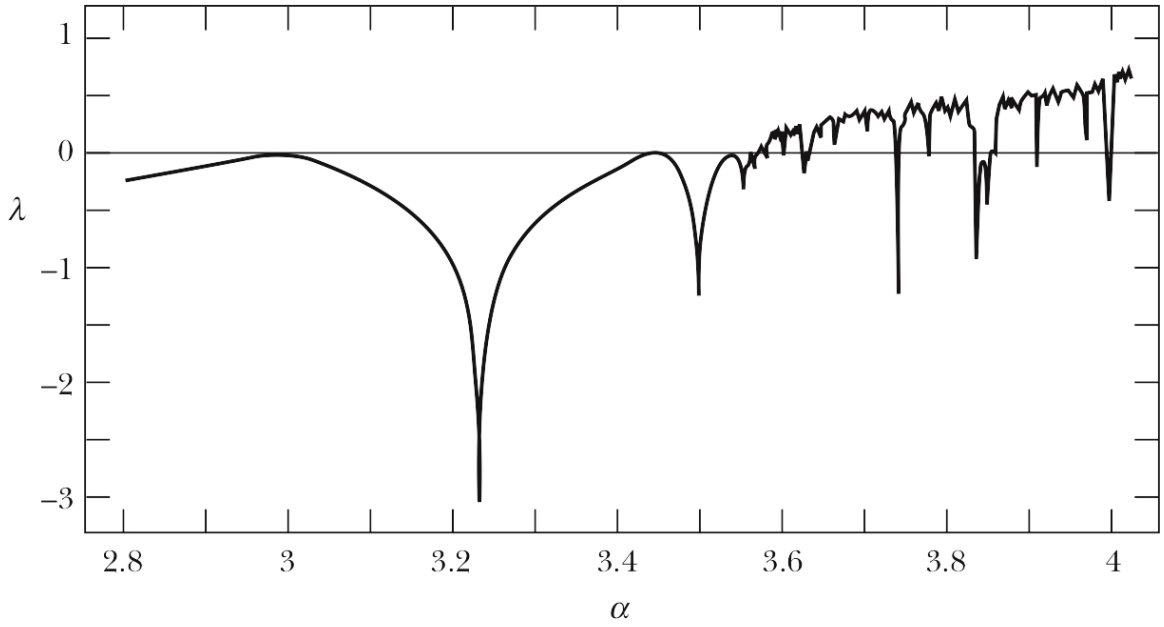
Problem 4 (25 points)

a. Consider two systems, each consisting of a particle of mass m. The forces acting on these particles are shown in the following figure. The forces deviate from a linear dependence on x at large $|x|$. Which of the two forces will produce a soft system?



b. For each of the two forces shown in the figure above, sketch the potential energy as function of x. In the same figures, also sketch expected dependence of the potential energy on x for a linear system.

c. Consider the Lyapunov exponent as a function of α for the logistic equation map, shown in the Figure below. How many solutions are possible when $\alpha = 3.5$? What is the smallest value of α for which chaotic motion will occur?



d. Consider a system that can be described by a second order differential equation. Consider the following three second order differential equations:

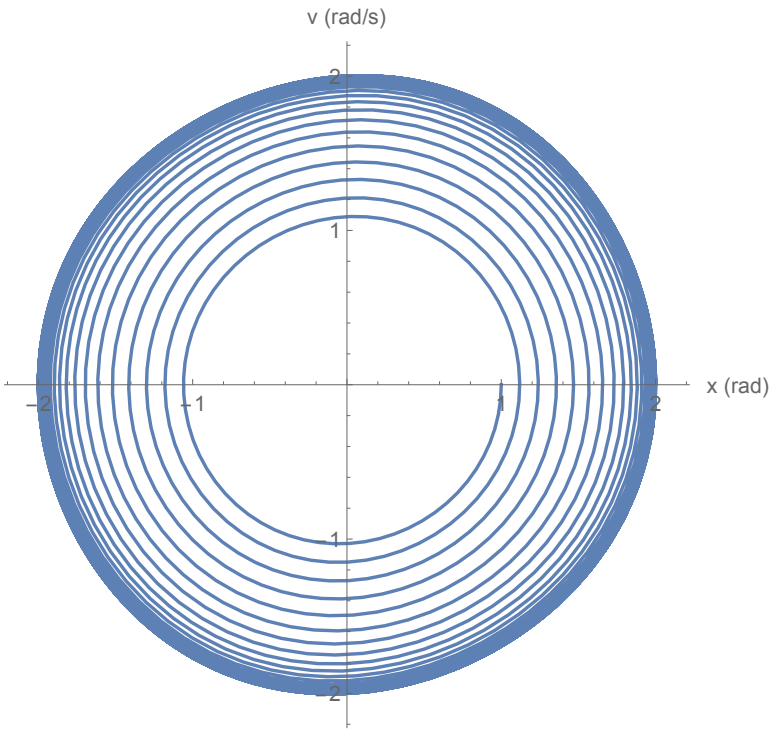
$$Eq.\,1: \quad \ddot{x} + \mu\left(x^2 - a^2\right)\dot{x} + \omega_0^2 x = 0$$

$$Eq.\,2: \quad \ddot{x} + \frac{b}{m}\dot{x} + \omega_0^2 x = 0$$

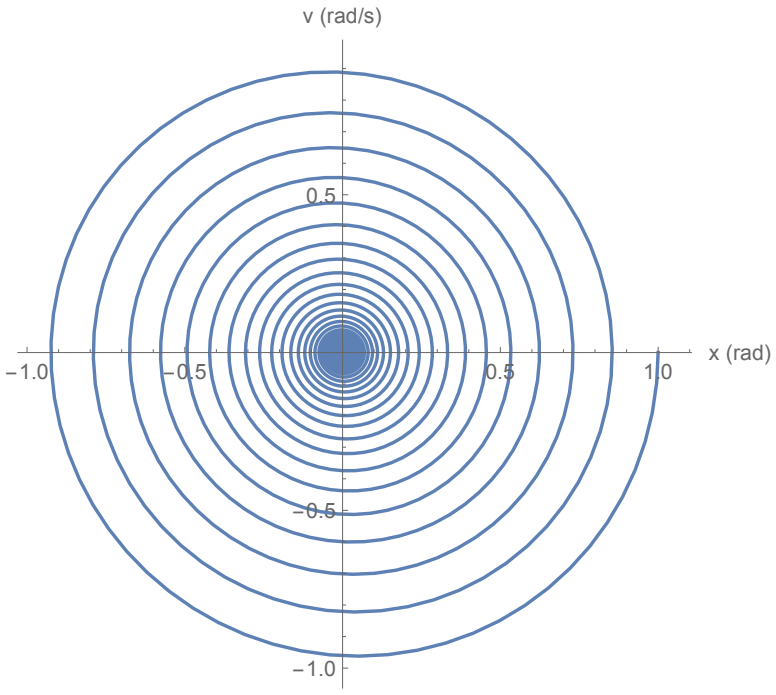
$$Eq.\,3: \quad \ddot{x} - \frac{b}{m}\dot{x} + \omega_0^2 x = 0$$

The constants in these differential equations are positive constants. The mass m is at rest at time $t = 0$ s and located at $x = 1$ m.

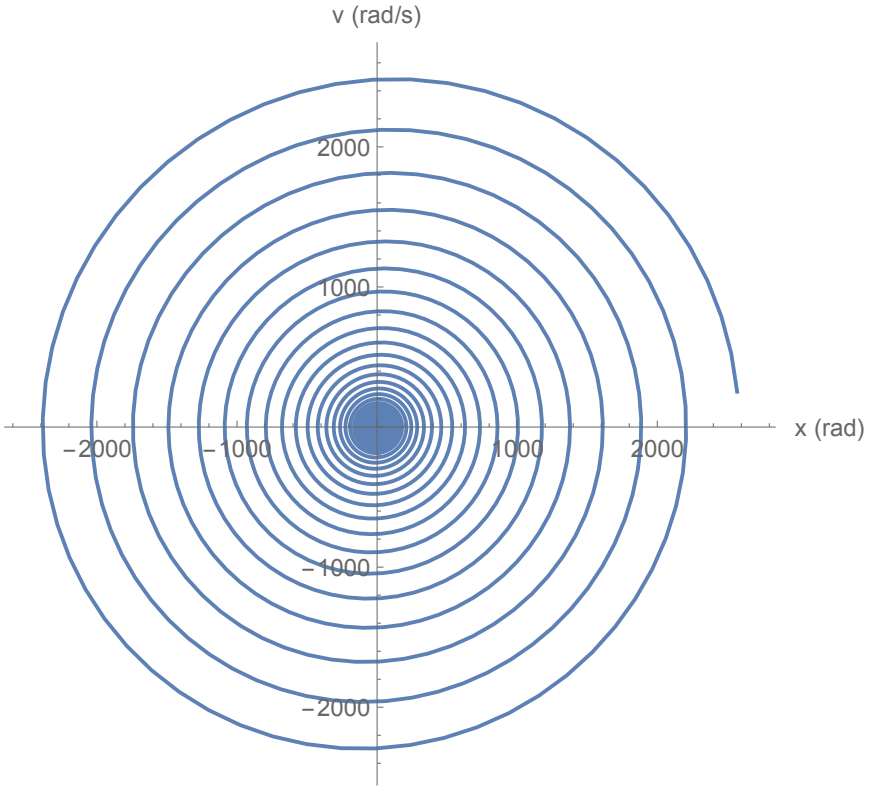
Consider the following phase diagrams. Match each phase diagram to one of these three differential equations.



Phase diagram # 1.



Phase diagram # 2.



Phase diagram # 3.

e. At what airport was this Boeing 747 of the Koninklijke Luchtvaart Maatschappij landing?

___ Amsterdam
___ Rochester
___ Orlando
___ JFK
___ Toronto
___ Chicago
___ Dalfsen
___ Atlanta
___ Boston
___ Montreal