Physics 235, Final Exam

December 14, 2021: 12.30 pm – 3.30 pm

Do not turn the pages of the exam until you are instructed to do so.

Exam rules: You may use *only* a writing instrument while taking this test. You may *not* consult any calculators, computers, books, call phones, nor each other.

The answers need to be well motivated and expressed in terms of the variables used in the problem. You will receive partial credit where appropriate, but only when we can read your solution. Answers that are not motivated will not receive any credit, even if correct.

At the end of the exam, you need to hand in your exam, the blue exam booklets, and the equation sheet. All items must be clearly labeled with your name, your student ID number, and the day/time of your recitation. **If any of these items are missing, we will not grade your exam, and you will receive a score of 0 points.**

<u>Answer questions 1 - 3 in exam book 1, questions 4 – 6 in exam book 2, and questions 7 - 8 in exam book 3.</u>

You are required to complete the following *Honor Pledge for Exams*. Copy and sign the pledge before starting your exam.

"I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own."

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Name: ____

Signature: ____

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Useful Relations:

$$\cos(30°)=\frac{1}{2}\sqrt{3} \quad \sin(30°)=\frac{1}{2} \quad \tan(30°)=\frac{1}{3}\sqrt{3}$$
$$\cos(45°)=\frac{1}{2}\sqrt{2} \quad \sin(45°)=\frac{1}{2}\sqrt{2} \quad \tan(45°)=1$$
$$\cos(60°)=\frac{1}{2} \quad \sin(60°)=\frac{1}{2}\sqrt{3} \quad \tan(60°)=\sqrt{3}$$

$$\cos\left(\frac{1}{2}\pi-\theta\right)=\sin(\theta) \quad \sin\left(\frac{1}{2}\pi-\theta\right)=\cos(\theta)$$
$$\cos(2\theta)=1-2\sin^2(\theta) \quad \sin(2\theta)=2\sin(\theta)\cos(\theta)$$

	Circle	Sphere
circumference	$2\pi r$	
(surface) area	πr^2	$4\pi r^2$
volume		$\frac{4}{3}\pi r^3$

$$\int_0^L \sin\left(\frac{r\pi x}{L}\right)\sin\left(\frac{s\pi x}{L}\right)dx=\frac{L}{2}\delta_{rs}$$

$$\int x\sin(x)dx=\sin(x)-x\cos(x)+\text{constant}$$

$$\int x^2\sin(x)dx=-x^2\cos(x)+2x\sin(x)+2\cos(x)$$

$$\frac{8}{L^3}\int_0^L x(L-x)\sin\left(\frac{r\pi x}{L}\right)dx=\frac{16}{r^3\pi^3}\left[1-(-1)^r\right]$$

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Good Luck !

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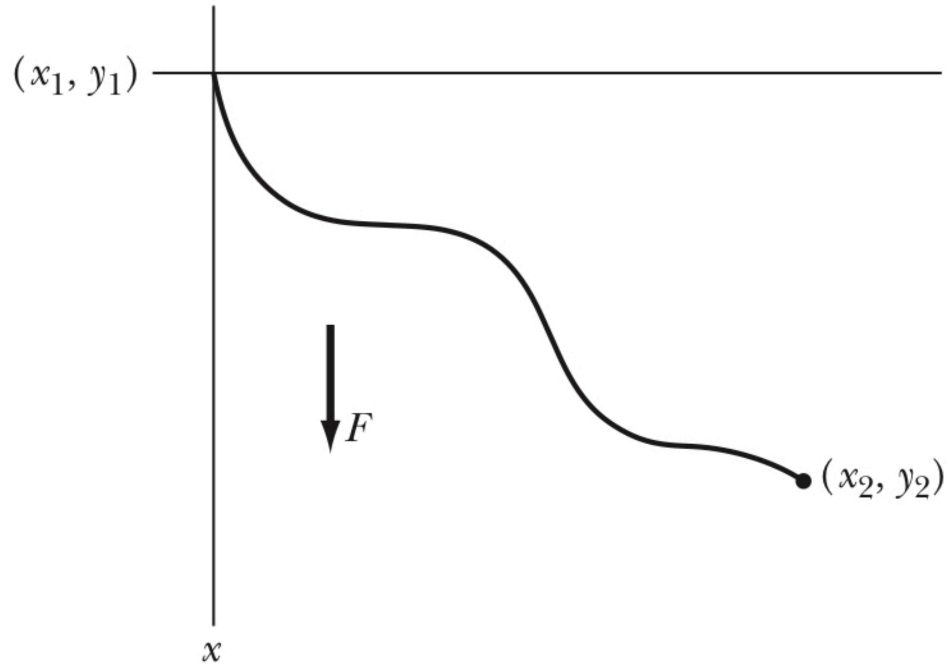
PROBLEM 1 (25 POINTS) **ANSWER IN BOOK 1**

Consider a particle travelling in a constant force field (e.g. the gravitational field), directed along the x axis. The particle starts from rest at (0,0) and moves to a position (x_2, y_2).



a) **(5 points)** How does the velocity of the particle depends on x?

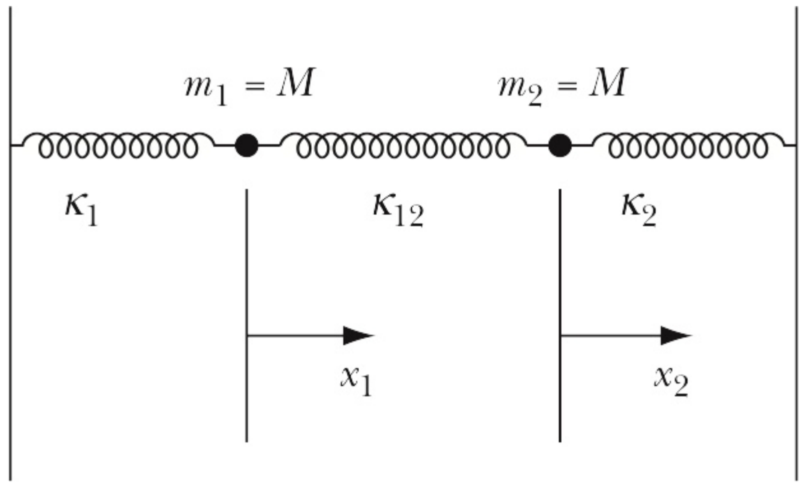
b) **(10 points)** Express the time required for the particle to move from the initial position to the final position, following a path $y(x)$, in terms of an integral over x between $x = 0$ and $x = x_2$.

c) **(10 points)** Find the path that allows the particle to accomplish this movement in the least amount of time.

Your answers must be well motivated and expressed in terms of the variables provided.

PROBLEM 2 (25 POINTS) **ANSWER IN BOOK 1**

Consider a system of two objects of mass M. The two objects are attached to two springs with different spring constants κ_1 and κ_2 as shown in the Figure below. The interaction force between the masses is represented by a third spring with spring constant κ_{12}, which connects the two masses.



a) **(5 points)** What is the equation of motion of mass m_1, in terms of its mass and the position of masses m_1 and m_2?

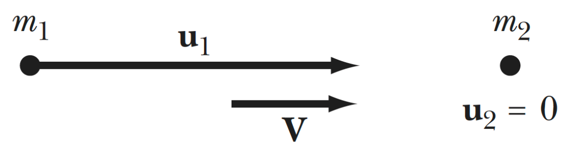
b) **(5 points)** What is the equation of motion of mass m_2, in terms of its mass and the position of masses m_1 and m_2?

c) **(15 points)** What are the eigen frequencies of the system?

Your answers must be well motivated and expressed in terms of the variables provided.

PROBLEM 3 (25 POINTS) **ANSWER IN BOOK 1**

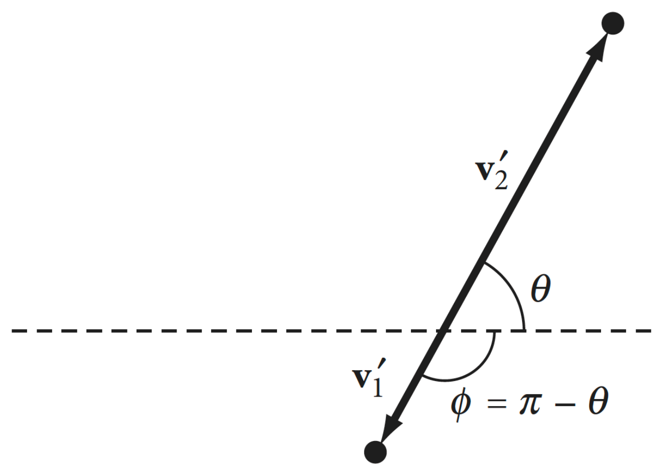
Consider an elastic collision of two particles with massesm_1 and m_2. Mass m_2 is initially at rest in the laboratory system. The initial configuration is shown in the following figure.



a. **(5 points)** What is the velocity V of the center of mass of the system?

b. **(5 points)** What are the linear momenta of particles 1 and 2 before the collision in the center-of-mass frame?

After the collision, particle 2 scatters with an angle θ, measured in the center-of-mass frame with respect to the direction of particle 1 before the collision, as shown in the following figure.



c. **(5 points)** What is the linear momentum of particle 2 after the collision in the center-of-mass frame?

d. **(10 points)** What are the laboratory scattering angle and the velocity of particle 2 after the collision?

Your answers must be well motivated and expressed in terms of the variables provided.

PROBLEM 4 (25 POINTS) **ANSWER IN BOOK 2**

Two particles of mass m_1 and m_2 are moving under the influence of their mutual gravitational force in perfect circular orbits with a period τ. The masses are suddenly stopped in their orbits and allowed to gravitate toward each other. **Note: look at the motion of these two particles in the center of mass frame.**

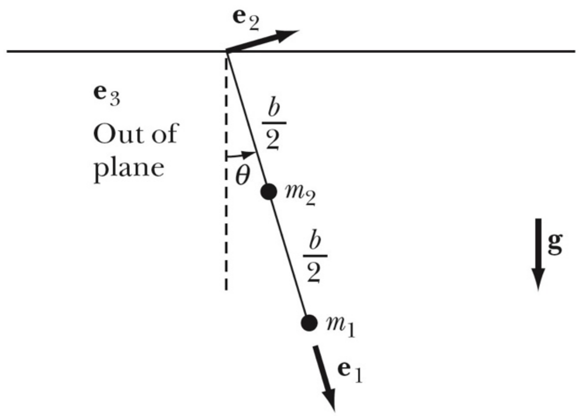
a) **(15 points)** What is the distance between the particles when they are orbiting in their circular orbits?

b) **(10 points)** What is the time interval between the moment the particles are stopped in their orbits and the moment they collide? **Note: if you obtain the correct integral for *t*, you will get full credit for this part of the problem. You do not need to solve the integral.**

Your answers must be well motivated and expressed in terms of the variables provided.

PROBLEM 5 (25 POINTS) **ANSWER IN BOOK 2**

Consider a pendulum, shown in the Figure below, composed of a rigid rod of length b with a mass m_1 at its end. Another mass m_2 is placed halfway down the rod. Consider the motion of the pendulum when the oscillations are small, and assume that the pendulum swings in the plane of the Figure.



a) **(5 points)** What is the inertia tensor?

b) **(5 points)** Express the angular momentum of the pendulum in terms of its angular velocity.

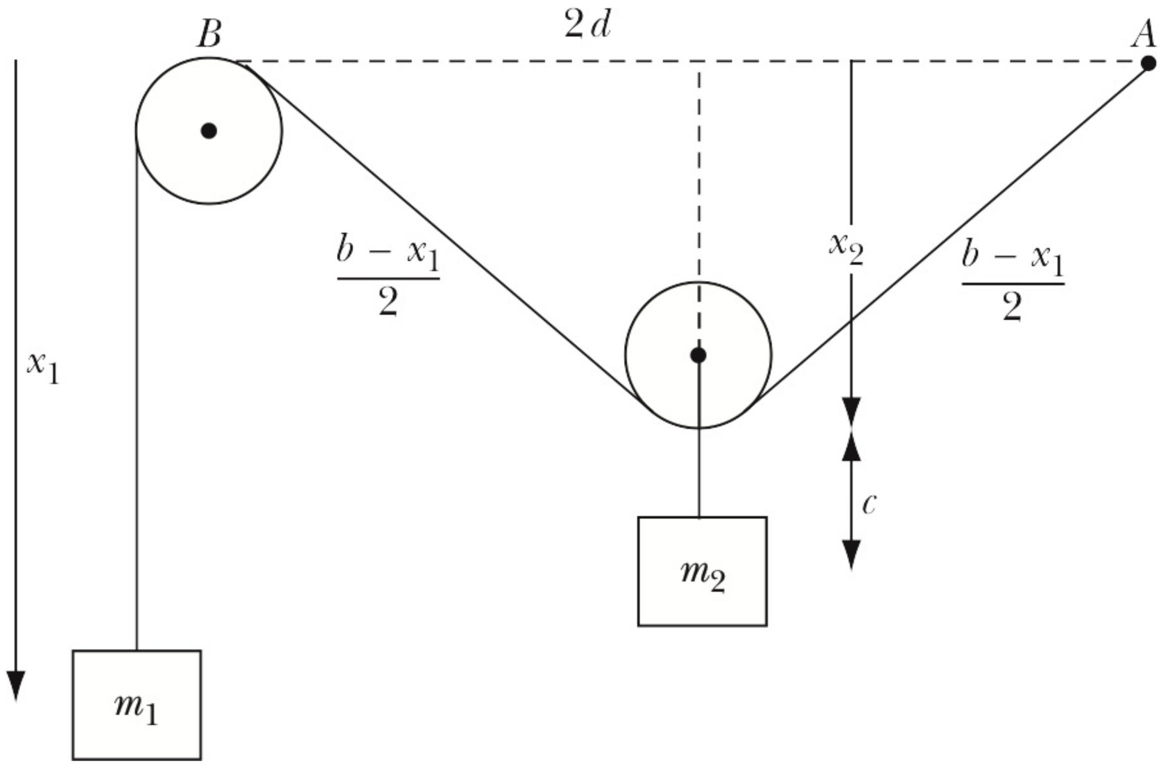
c) **(10 points)** Use the external torque acting on the pendulum to determine the equation of motion for the angle θ.

d) **(5 points)** What is the angular frequency of the system?

Your answers must be well motivated and expressed in terms of the variables provided.

PROBLEM 6 (25 POINTS) **ANSWER IN BOOK 2**

Consider the system of pulleys, masses, and string shown in the Figure below.



A massless string of length b is attached to point A, passes over a pulley at point B located a distance $2d$ away, and finally attaches to mass m_1. Another pulley with mass m_2 attached, passes over the string, pulling it down between A and B. Assume that the radius of the pulley holding mass m_2 is small so that we can neglect its size. Assume also that the pulleys are massless. The distance c is constant.

a) **(10 points)** What is the potential energy of the system in this configuration (when mass m_1 is located a distance x_1 below the level defined by A and B)?

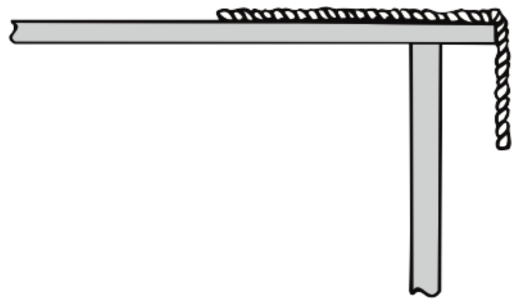
b) **(10 points)** Determine the position of mass m_1 for which the system will be in equilibrium.

c) **(5 points)** Determine if the equilibrium position obtained in part b) is a stable or an unstable equilibrium position.

Your answers must be well motivated and expressed in terms of the variables provided.

PROBLEM 7 (25 POINTS) **ANSWER IN BOOK 3**

A flexible cord of length L and mass M slides from a frictionless table top as shown in the Figure below.



The rope is released when a length y_0 is hanging over the edge of the table.

a) **(10 points)** What is the Lagrangian of this system when a length y is hanging over the edge of the table.

b) **(5 points)** Use the Lagrangian to obtain the equation of motion.

c) **(10 points)** At what time will the left end of the cord reach the edge of the table.

Your answers must be well motivated and expressed in terms of the variables provided.

PROBLEM 8 (25 POINTS) **ANSWER IN BOOK 3**

a) **(5 points)** The following two figures show the logistic equation map for two different values of α.

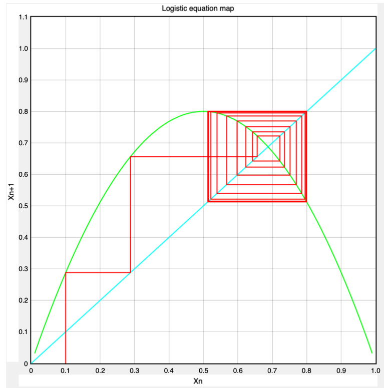


Figure a

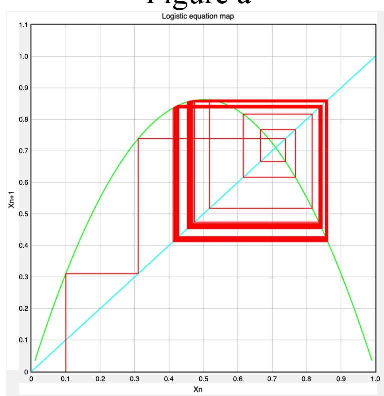
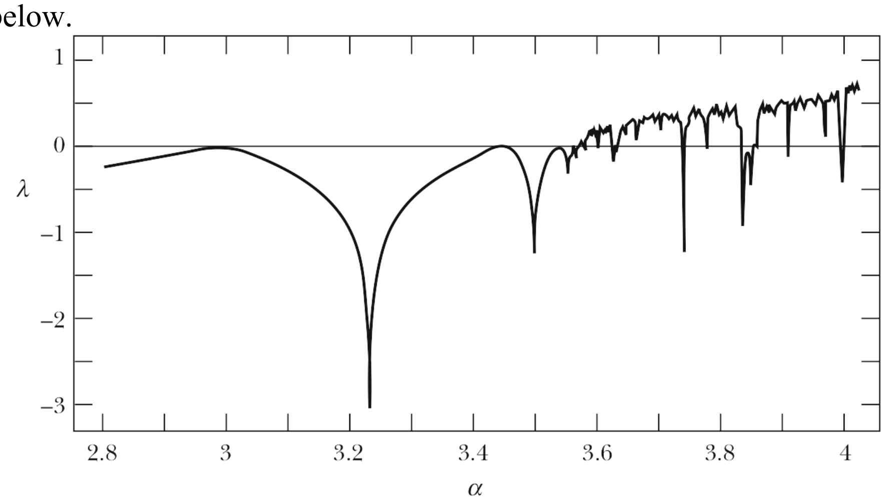


Figure b

The corresponding Lyapunov exponent for this logistic equation as a function of α is shown in the Figure below.



What are the ranges of possible value of α for Figures (a) and (b) in the two Figures shown at the top of this page?

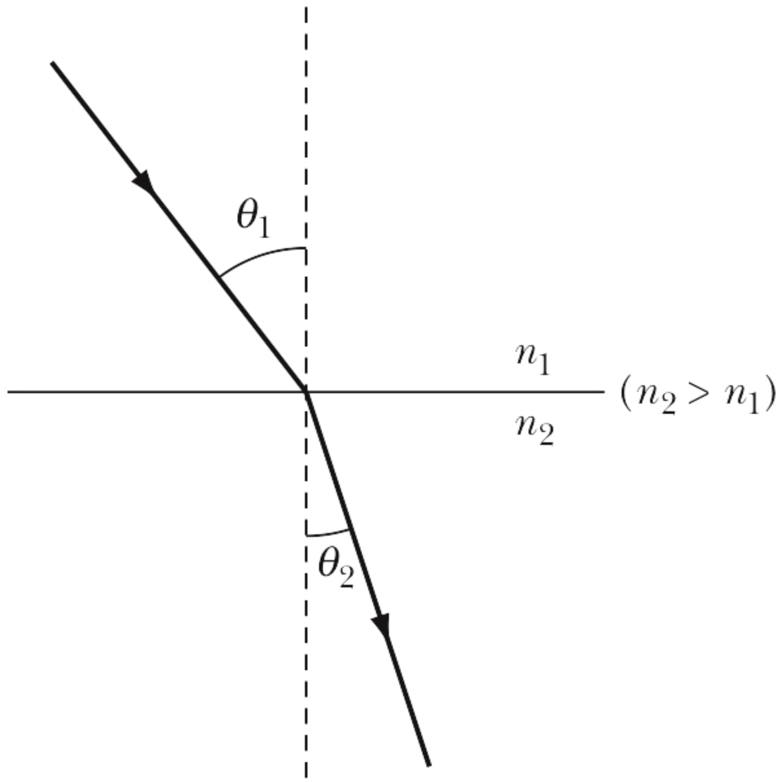
PROBLEM 8 (CONTINUED) **ANSWER IN BOOK 3**

b) **(5 points)** Consider a large pendulum in its equilibrium position on the Northern hemisphere.

Due to the rotation of the Earth, the equilibrium position of the pendulum in your laboratory does not coincide with the vertical direction in your laboratory reference frame. What is the direction of the deflection you observe? Specify: North, East, South, or West.

PROBLEM 8 (CONTINUED) **ANSWER IN BOOK 3**

c) **(5 points)** Consider light passing from one medium with index of refraction n_1 into another medium with index of refraction n_2, as shown in the following Figure.

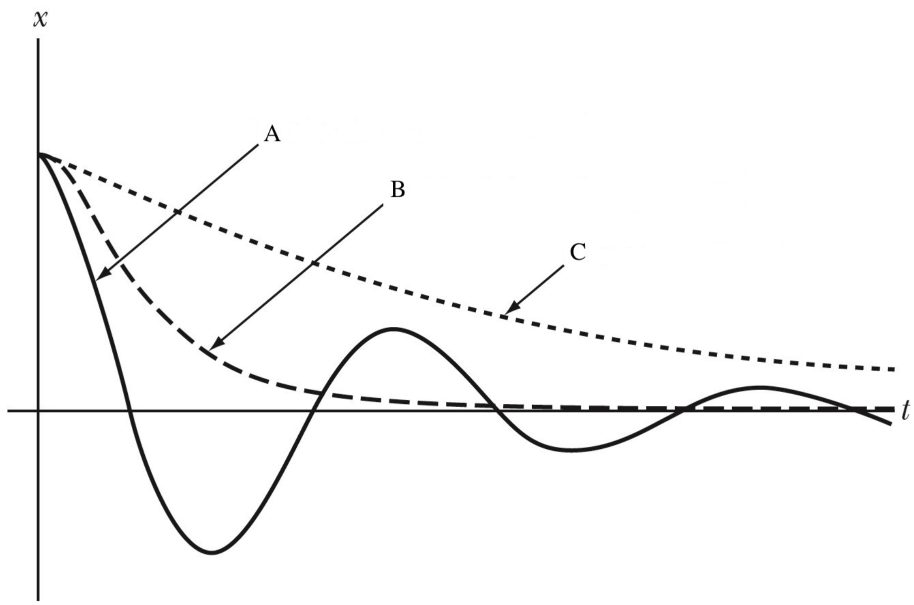


The x axis is defined by the interface between the two media and the y axis is perpendicular to this surface (represented by the dashed line in the Figure).

What would you use as your independent variable when you derive the law of refraction?

PROBLEM 8 (CONTINUED) **ANSWER IN BOOK 3**

d) **(5 points)**Which of the curves in the following figure shows the time dependence of the amplitude of an over-damped oscillator?



PROBLEM 8 (CONTINUED) **ANSWER IN BOOK 3**

e) **(5 points)** At what airport was the following photograph made

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