Physics 235, Midterm Exam # 2

October 26, 2021: 8.00 am - 9.30 am

Do not turn the pages of the exam until you are instructed to do so.

Exam rules: You may use *only* a writing instrument while taking this test. You may *not* consult any calculators, computers, books, nor each other.

Problems 1 and 2 must be answered in exam booklet 1. Problems 3 and 4 must be answered in exam booklet 2. The answers need to be well motivated and expressed in terms of the variables used in the problem. You will receive partial credit where appropriate, but only when we can read your solution. Answers that are not motivated will not receive any credit, even if correct.

At the end of the exam, you need to hand in your exam, the blue exam booklets, and the equation sheet. All items must be clearly labeled with your name, your student ID number, and the day/time of your recitation. **If any of these items are missing, we will not grade your exam, and you will receive a score of 0 points.**

You are required to complete the following *Honor Pledge for Exams*. Copy and sign the pledge before starting your exam.

"I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own."

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Name: ____

Signature: ____

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Useful Relations:

$$\cos(30°)=\frac{1}{2}\sqrt{3} \quad \sin(30°)=\frac{1}{2} \quad \tan(30°)=\frac{1}{3}\sqrt{3}$$
$$\cos(45°)=\frac{1}{2}\sqrt{2} \quad \sin(45°)=\frac{1}{2}\sqrt{2} \quad \tan(45°)=1$$
$$\cos(60°)=\frac{1}{2} \quad \sin(60°)=\frac{1}{2}\sqrt{3} \quad \tan(60°)=\sqrt{3}$$

$$\cos\left(\frac{1}{2}\pi-\theta\right)=\sin(\theta) \quad \sin\left(\frac{1}{2}\pi-\theta\right)=\cos(\theta)$$
$$\cos(2\theta)=1-2\sin^2(\theta) \quad \sin(2\theta)=2\sin(\theta)\cos(\theta)$$

	Circle	Sphere
circumference	$2\pi r$	
(surface) area	πr^2	$4\pi r^2$
volume		$\frac{4}{3}\pi r^3$

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Good Luck !

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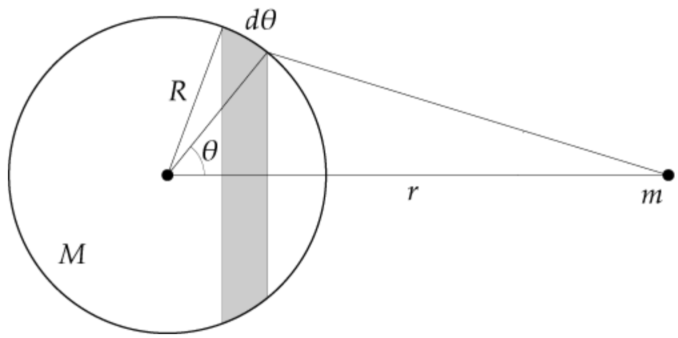
PROBLEM 1 (25 POINTS) **ANSWER IN BOOK 1**

Consider a thin uniform spherical shell of mass M and radius R. The center of the shell is located at the origin of our coordinate system.

a. **(10 points)** Consider a thin ring on the shell at a polar angle θ and with an angular width $d\theta$, as shown in the Figure below.



Find the gravitational potential Φ due to this ring at a distance r from the center of the spherical shell. Assume $r > R$.

b. **(5 points)** Find the total graviational potential Φ at a distance r from the center of the spherical shell by integrating the expression obtained in part a) over all possible values of θ. Assume $r > R$.

c. **(5 points)** Use the result obtained in b) to determine the gravitational field (magnitude and direction) of the spherical shell at a distance r from its center. Assume $r > R$.

d. **(5 points)** Use the expression of the gravitational field obtained in part c) to determine the magnitude and the direction of the gravitational force on point mass m at a distance r from the center of the spherical shell. Assume $r > R$.

Your answers must be well motivated and expressed in terms of the variables provided.

NOTE: YOU CANNOT USE THE SHELL THEOREM TO SOLVE THIS PROBLEM.

PROBLEM 2 (25 POINTS) **ANSWER IN BOOK 1**

a. **(15 points)** Find the curve $y(x)$ that passes through the endpoints (0, 0) and (1, 1) and minimizes the following path integral

$$I[y] = \int_0^1 \left[\left(\frac{dy}{dx} \right)^2 - y^2 \right] dx$$

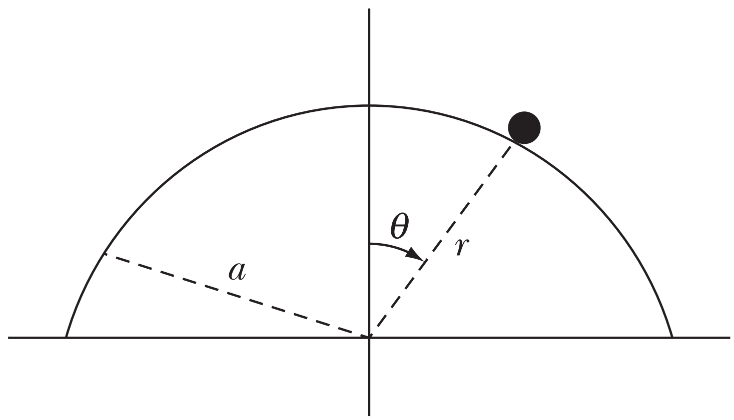
b. **(5 points)** What is the minimum value of the path integral?

c. **(5 points)** Evaluate $I[y]$ for a straight line $y = x$ between the points (0, 0) and (1, 1).

Your answers must be well motivated and expressed in terms of the variables provided.

PROBLEM 3 (25 POINTS) **ANSWER IN BOOK 2**

A particle of mass m starts at rest on top of a smooth fixed hemisphere of radius a.



We will express the motion of the particle in terms of the generalized coordinates r and θ.

a. **(2.5 points)** What is the equation of constraint when the particle moves across the surface?

b. **(5 points)** What is the Lagrangian describing this motion? Express L in terms of the generalized coordinates r and θ.

c. **(5 points)** Find the Lagrange equations of motion.

d. **(5 points)** Use the equations of motion found in part c) and the equation of constraint found in part a) to find the differential equation for θ.

e. **(5 points)** Find the Lagrange undetermined multiplier(s). You can express the Lagrange undetermined multiplier(s) in terms of θ and $\dot{\theta}$.

f. **(2.5 points)** Explain how you use the result of part e) to determine the angle θ at which the particle leaves the hemisphere.

Your answers must be well motivated and expressed in terms of the variables provided.

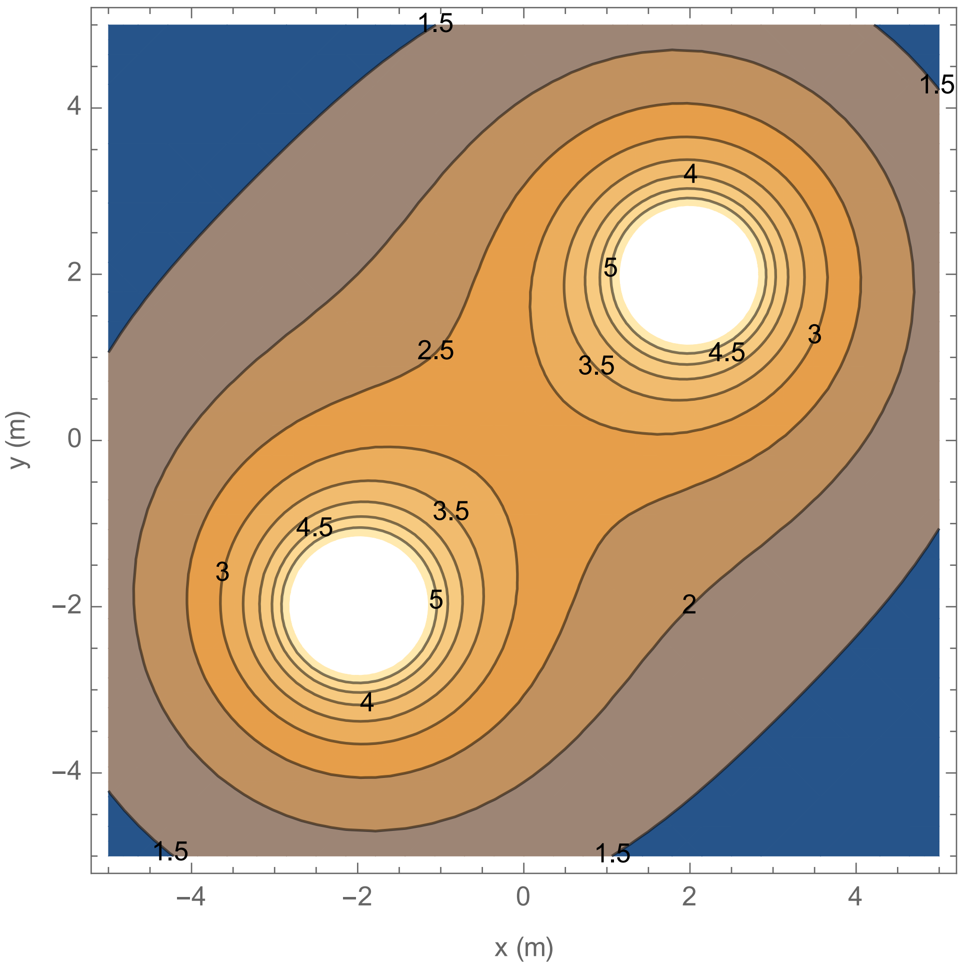
PROBLEM 4 (25 POINTS) **ANSWER IN BOOK 2**

a. **(5 points)** Consider the great-circle path between Amsterdam and Rochester, shown in the Figure below.

https://www.gpsvisualizer.com/map?output_calculators

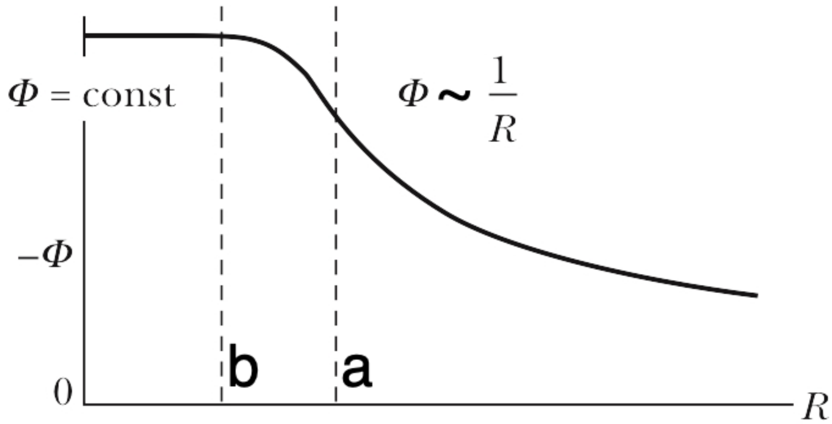
If we describe the surface of the Earth as a perfect sphere of radius R, what would the equations of constraint be in 1) Cartesian coordinates and in 2) spherical coordinates?

b. **(5 points)** Consider the contour diagram of the gravitational potential V, generated by two point masses, shown as function of x and y in the following Figure.



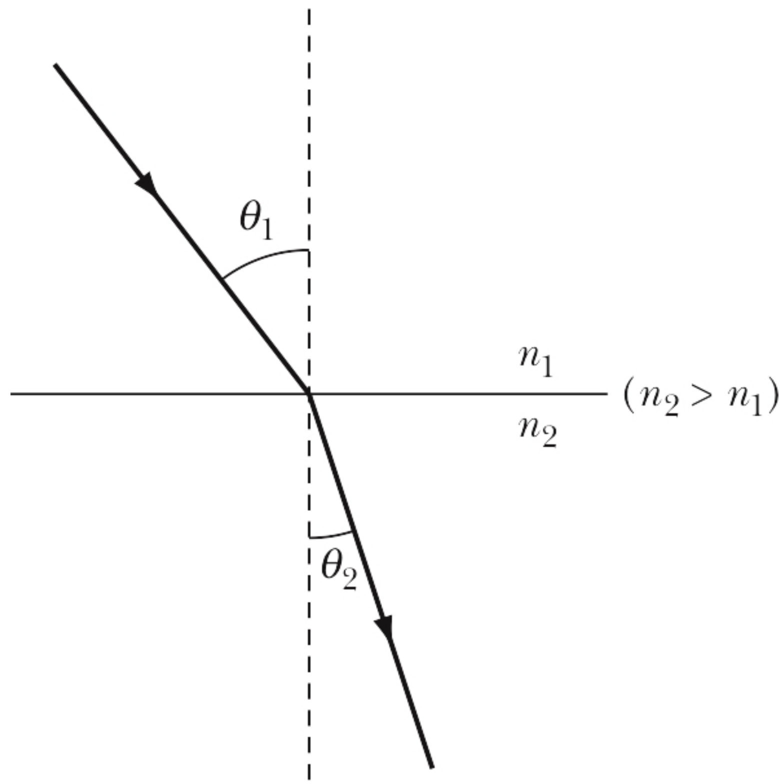
Where are the two masses located (specify their x and y coordinates) and what is their mass ratio?

c. **(5 points)** Consider the gravitational potential distribution shown in the following Figure.



Make a sketch of the magnitude of the corresponding gravitational force. What type of mass distribution would generate such a gravitational potential distribution?

d. **(5 points)** Consider light passing from one medium with index of refraction n_1 into another medium with index of refraction n_2, as shown in the following Figure.



The *x* axis is defined by the interface between the two media and the *y* axis is perpendicular to this surface (represented by the dashed line in the Figure).

What would you use as your independent variable when you derive the law of refraction?

e. **(5 points)** How old is the KLM? Express your answer in units of years.

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