Physics 235, Midterm Exam # 1

September 23, 2021: 8.00 am - 9.30 am

Do not turn the pages of the exam until you are instructed to do so.

Exam rules: You may use *only* a writing instrument while taking this test. You may *not* consult any calculators, computers, books, nor each other.

Problems 1 and 2 must be answered in exam booklet 1. Problems 3 and 4 must be answered in exam booklet 2. The answers need to be well motivated and expressed in terms of the variables used in the problem. You will receive partial credit where appropriate, but only when we can read your solution. Answers that are not motivated will not receive any credit, even if correct.

At the end of the exam, you need to hand in your exam, the blue exam booklets, and the equation sheet. All items must be clearly labeled with your name, your student ID number, and the day/time of your recitation. **If any of these items are missing, we will not grade your exam, and you will receive a score of 0 points.**

You are required to complete the following *Honor Pledge for Exams*. Copy and sign the pledge before starting your exam.

"I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own."

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Name: _____

Signature: _____

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Useful Relations:

$$\cos(30°)=\frac{1}{2}\sqrt{3} \quad \sin(30°)=\frac{1}{2} \quad \tan(30°)=\frac{1}{3}\sqrt{3}$$
$$\cos(45°)=\frac{1}{2}\sqrt{2} \quad \sin(45°)=\frac{1}{2}\sqrt{2} \quad \tan(45°)=1$$
$$\cos(60°)=\frac{1}{2} \quad \sin(60°)=\frac{1}{2}\sqrt{3} \quad \tan(60°)=\sqrt{3}$$

$$\cos\left(\frac{1}{2}\pi-\theta\right)=\sin(\theta) \quad \sin\left(\frac{1}{2}\pi-\theta\right)=\cos(\theta)$$
$$\cos(2\theta)=1-2\sin^2(\theta) \quad \sin(2\theta)=2\sin(\theta)\cos(\theta)$$

	Circle	Sphere
circumference	$2\pi r$	
(surface) area	πr^2	$4\pi r^2$
volume		$\frac{4}{3}\pi r^3$

$$\int_0^L \sin\left(\frac{r\pi x}{L}\right)\sin\left(\frac{s\pi x}{L}\right)dx=\frac{L}{2}\delta_{rs}$$

$$\int x\sin(x)\,dx=\sin(x)-x\cos(x)+\text{constant}$$

$$\int x^2\sin(x)\,dx=-x^2\cos(x)+2x\sin(x)+2\cos(x)$$

$$\frac{8}{L^3}\int_0^L x(L-x)\sin\left(\frac{r\pi x}{L}\right)dx=\frac{16}{r^3\pi^3}\left[1-(-1)^r\right]$$

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Good Luck !

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Problem 1 (25 points) **BOOK 1**

Consider an object of mass m undergoing vertical motion close to the surface of the Earth. The air through which the object is moving, generates a drag force that is proportional to the velocity of the object, but pointing in the opposite direction. The magnitude of the drag force is $km|v|$, where k is a constant and v is the velocity of the object. The initial velocity of the object at time $t = 0$ s is v_0.

a. **(5 points)** Make a sketch of the forces acting on this object. Make sure you clearly indicate their directions and the coordinate system you are using to solve this problem.

b. **(5 points)** What is the equation of motion for this system? Express the equation of motion in terms of the velocity (v and dv/dt) instead of the accelation a.

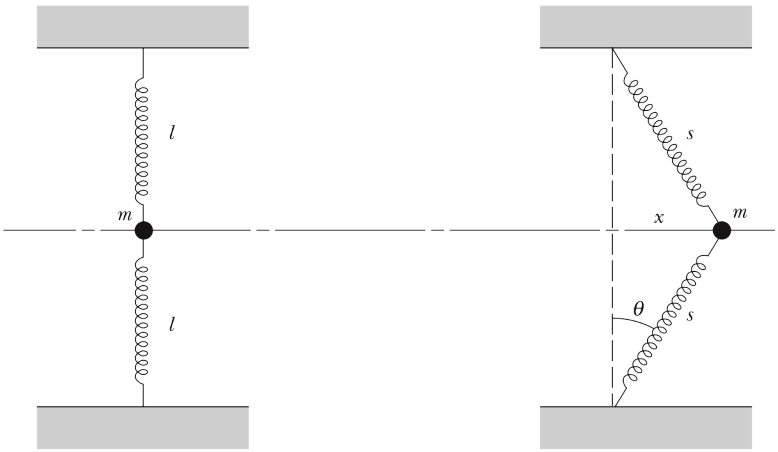
c. **(10 points)** Determine the velocity of the object as function of time.

d. **(5 points)** What is the terminal speed of this object?

Your answers needs to be well motivated and expressed in terms of the variables provided.

Problem 2 (25 points) **BOOK 1**

Consider a particle of mass *m* suspended between two identical stretched springs, as shown in the Figure below. Assume the rest length of each spring is ℓ_0. We ignore the gravitational force in this problem.



(a) Equilibrium position (b) Extended position

a. **(10 points)** Calculate the net force on mass *m* in terms of the position coordinate *x*.

b. **(5 points)** Use one of the expansions you can find in Appendix D (see equation sheet) to express the expresion you found for the net force in powers of *x*. You can ignore terms proporional to x^4 and above.

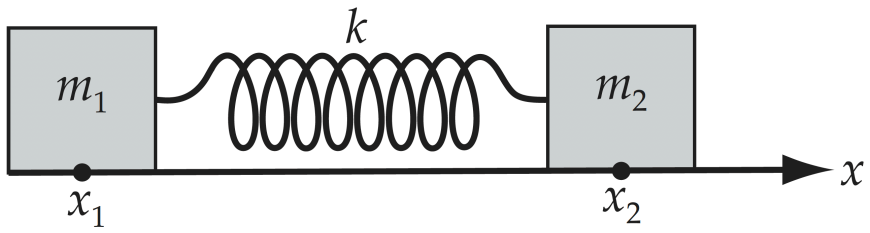
c. **(5 points)** Is this a non-linear system? If so, why? If so, is it a soft or a hard system?

d. **(5 points)** Use the force obtained in part b) to determine the potential energy *U* in which the particle moves.

Your answers needs to be well motivated and expressed in terms of the variables provided.

Problem 3 (25 points) **BOOK 2**

Two masses m_1 and m_2 slide freely on a horizontal frictionless track. They are connected by a spring whose force constant is k. The spring has a rest length of L.



a. **(5 points)** What is the equation of motion of mass m_1, in terms of the mass and the position of masses m_1 and m_2, the spring constant k, and the rest length L.

b. **(5 points)** What is the equation of motion of mass m_2, in terms of the mass and the position of masses m_1 and m_2, the spring constant k, and the rest length L.

c. **(5 points)** Uncouple the two equations of motion obtained in parts a) and b) to determine an equation of motion of mass m_2, in terms of the position of mass m_2 and the masses m_1 and m_2, the spring constant k, and the rest length L.

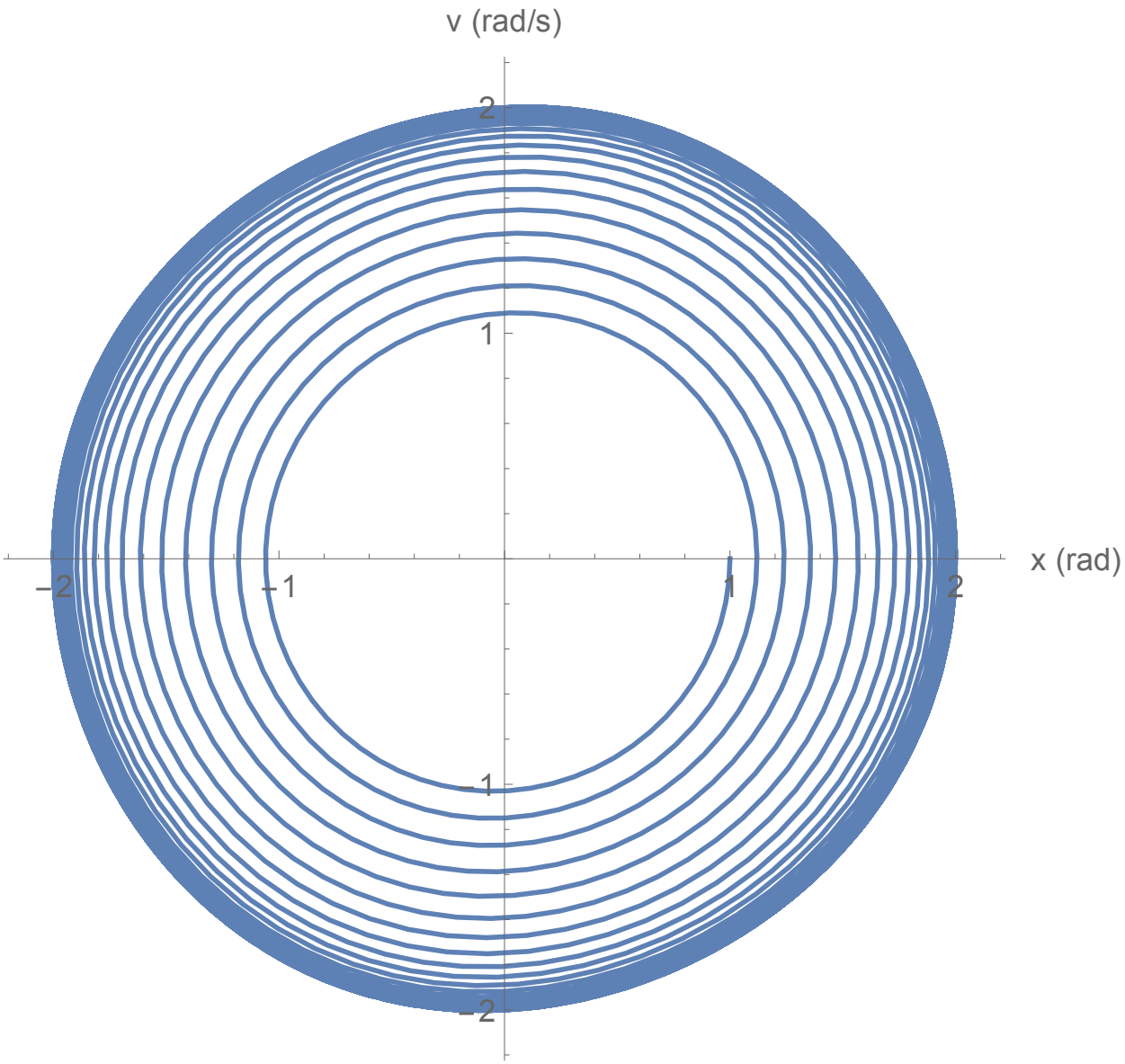
d. **(5 points)** Show that simple harmonic motion is one possible solution to the uncoupled equation obtained in part c). What is the frequency of the harmonic motion?

e. **(5 points)** Show that linear motion of mass m_2 is also a solution to the uncoupled equation obtained in part c).

Your answers needs to be well motivated and expressed in terms of the variables provided.

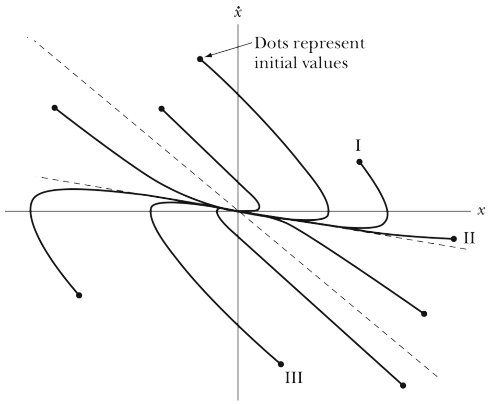
Problem 4 (25 points) **BOOK 2**

a. **(5 points)** Consider the following phase diagram that corresponds to a solution of the van der Pol equation. The initial condition of this solution has an initial angular position of 1 rad and an initial angular velocity of 0 rad/s.

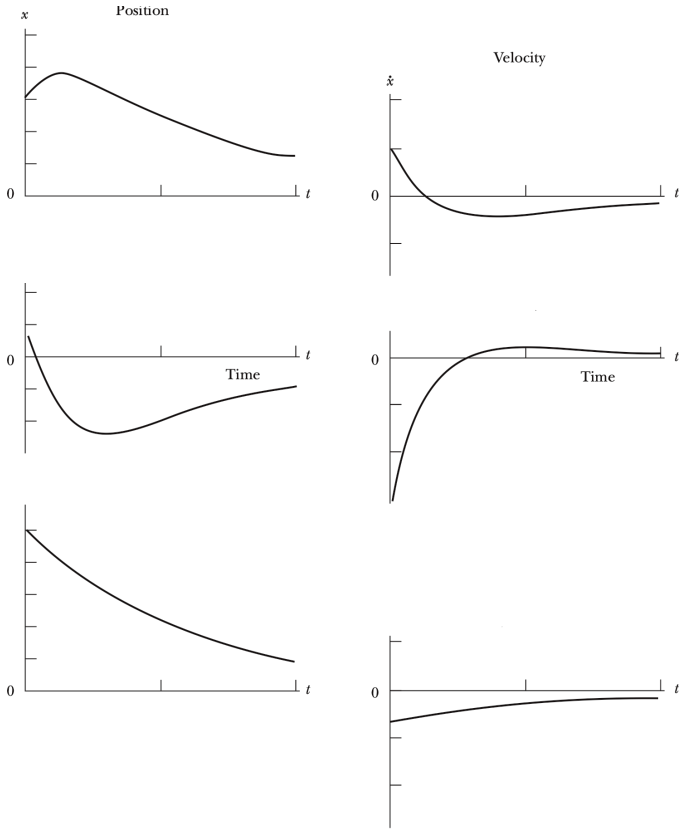


What is the value of *a* in the van der Pol equation that produces this phase diagram?

b. **(5 points)** Consider the following phase diagram that shows phase paths for overdamped motion.



Consider the three paths labelled in the phase diagram as path I, path II, and path III. Identify which position and velocity functions, shown as function of time below, match the motion for these three phase paths:



In your blue book please write:

The top position and velocity graphs correspond to path …

The middle position and velocity graphs correspond to path …

The bottom position and velocity graphs correspond to path …

c. **(5 points)** The following two figures show the logistic equation map for two different values of α.

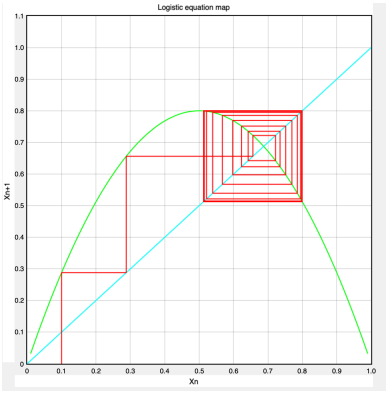


Figure a

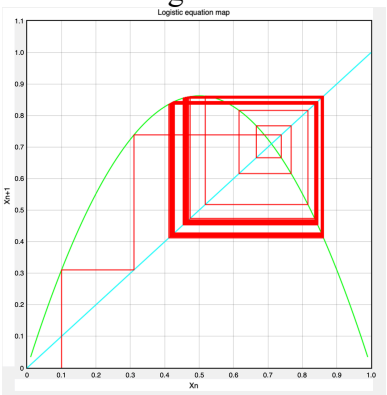
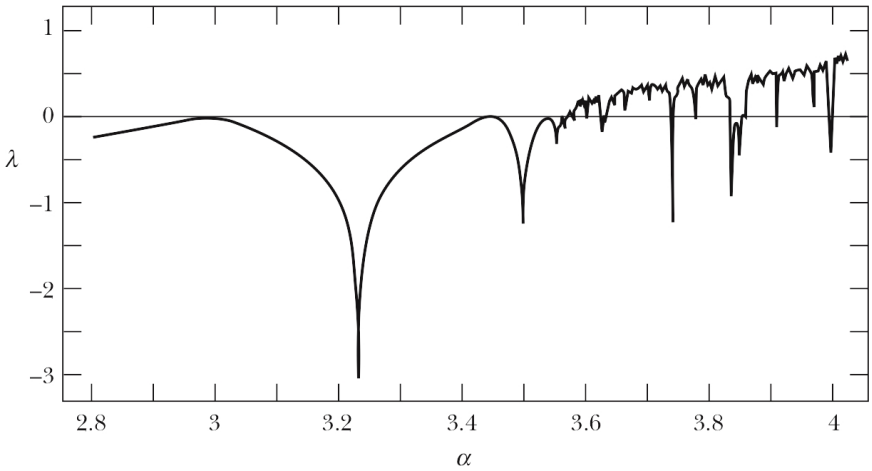


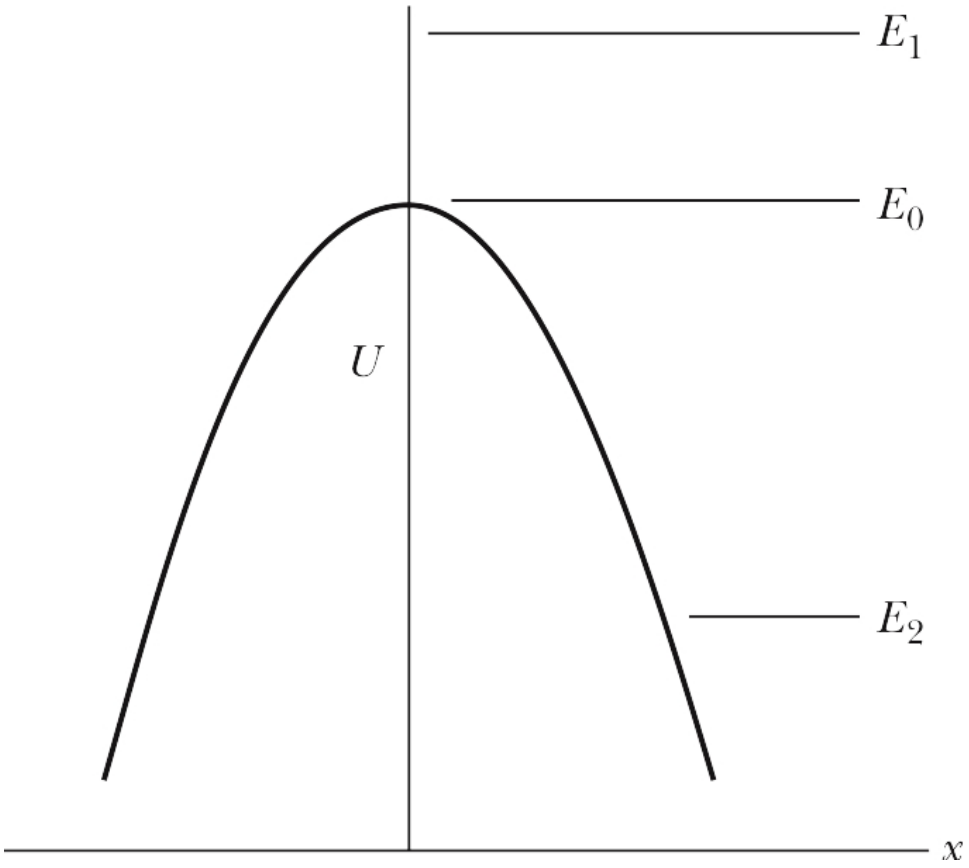
Figure b:

The corresponding Lyapunov exponent for this logistic equation as a function of α is shown in the Figure below.



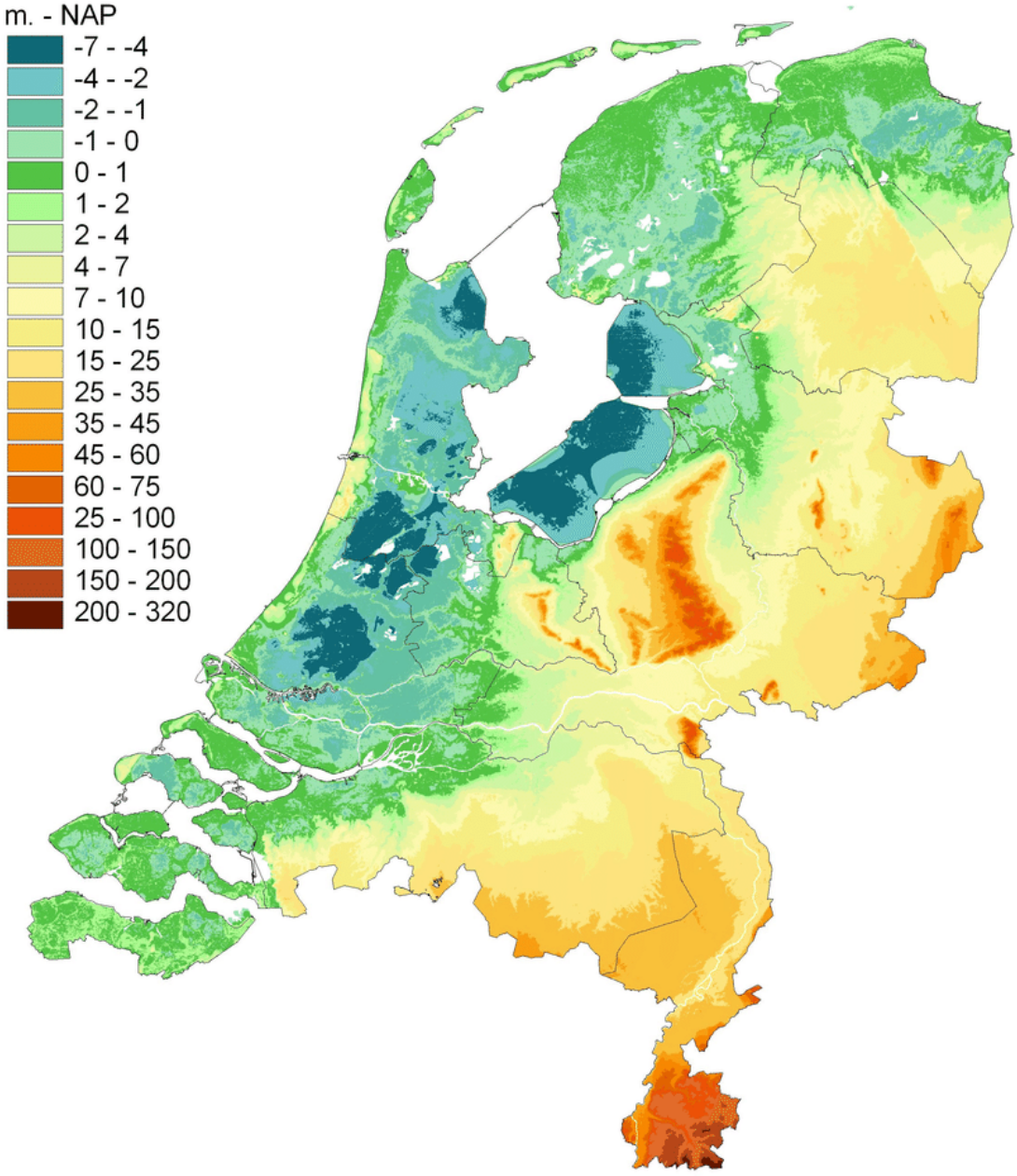
What are the ranges of possible value of α for Figures (a) and (b) in the two Figures shown at the top of this page?

d. **(5 points)** Consider the following potential energy distribution for a one-dimentsional system.



Sketch the curves in a phase diagram that correspond to energy E_0, energy E_1, and energy E_2.

e. **(5 points)** What does NAP stand for?



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