Final Exam, Physics 217
December 20, 2001, 12.30 pm – 3.30 pm

1. Answer questions 1, 2, and 3 in exam book # 1.
   Answer questions 4, 5, and 6 in exam book # 2.

2. Each question is worth 20 points: Yes, your algebra is correct, and the total points you can score is 120 (a 20 point bonus if you answer all 6 questions).

3. Each answer needs to be well motivated. You will not receive any credit for just the answer (even if it is correct) if no motivation is provided.

4. Here are some useful relations:

$$\delta(\bar{r}) = \frac{1}{4\pi} \bar{\nabla} \bullet \left(\frac{1}{r^2} \hat{r} \right)$$

$$\int_0^\infty r e^{-\lambda r} dr = \frac{1}{\lambda^2}$$

The general solution of the following differential equation

$$\frac{df}{dr} = \frac{1}{r}$$

is

$$f(r) = \ln\left(\frac{r}{a} \right)$$

where a is a constant.

5. The grades will be distributed via email on or before 12/24.

6. Have a good and save holiday, and best wishes for 2002.

7. All complaints/comments/questions about the exam and the course should be directed to the instructor.

Problem 1 (20 points)

Find the force on the charge $+q$, shown in Figure 1, which is located a distance of $3d$ above an infinitely large grounded conducting plane, which is located in the xy plane.

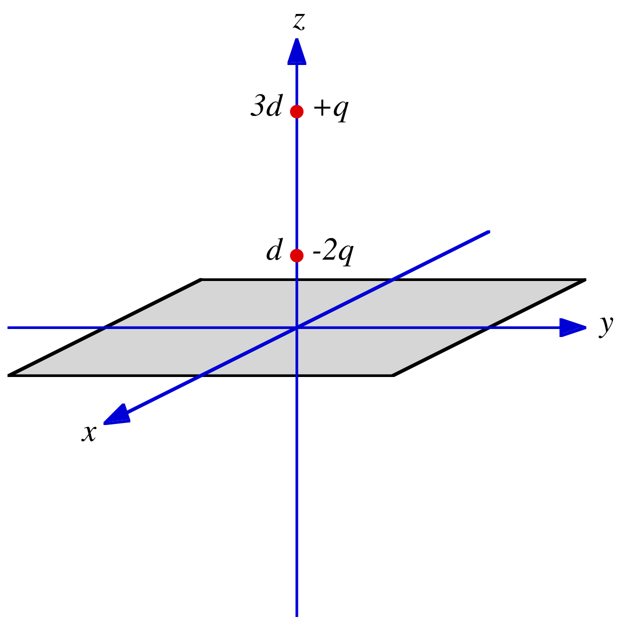

Figure 1. Problem 1.

Express your answer in terms of d and q. Make sure you specify both the magnitude and the direction of the force.

Problem 2 (20 points)

Consider an infinitely long straight wire of radius R, directed along the z axis, which carries a current I.

a) What is the magnetic field $\overline{B}(\overline{r})$ at a distance r from the center of the wire ($r > R$)?

b) Find the vector potential $\overline{A}(\overline{r})$ at a distance r from the center of the wire ($r > R$).

c) Show that your solution in part b) is correct by verifying that $\overline{\nabla} \bullet \overline{A} = 0$.

d) Show that your solution in part b) is correct by verifying that $\overline{\nabla} \times \overline{A} = \overline{B}$.

Express all your answers in terms of R, I, and r.

Problem 3 (20 points)

The electrostatic potential of some charge configuration is given by the expression

$$V(\bar{r}) = A\frac{e^{-\lambda r}}{r}$$

where A and λ are constants.

a) Find the electric field $\bar{E}(\bar{r})$.

b) Find the charge density $\rho(\bar{r})$.

c) Find the total charge Q.

Express all your answers in terms of A and λ.

Problem 4 (20 points)

A point charge q is imbedded at the center of a sphere of linear dielectric material (with susceptibility χ_e and radius R).

a) Find the electric field $\overline{E}(\bar{r})$ inside the sphere ($r < R$).

b) Find the polarization $\overline{P}(\bar{r})$ inside the sphere ($r < R$).

c) Find the bound volume and surface charge densities ρ_b and σ_b.

d) What is the total bound surface charge on the surface of the sphere?

e) Where is the opposing bound volume charge located?

Express all your answers in terms of q, χ_e, and R

Problem 5 (20 points)

A certain transmission line in constructed from two thin metal “ribbons” of width w, a very small distance $h << w$ apart (see Figure 2). A current I travels down one strip and back along the other. In each case, the current density across the ribbon is uniform.

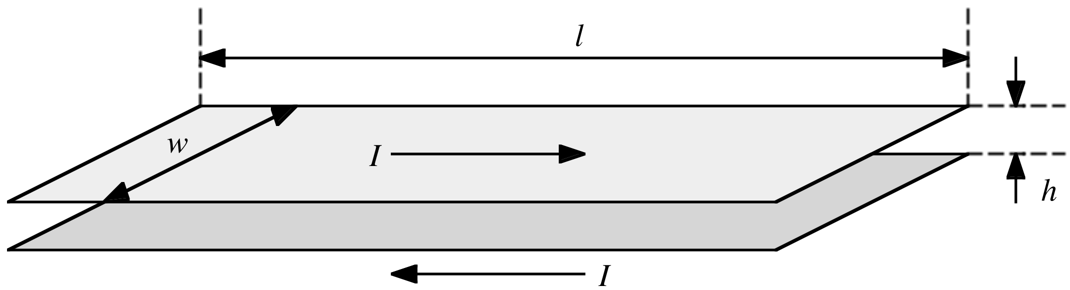

Figure 2. Problem 5.

a) Find the capacitance per unit length C.

b) Find the inductance per unit length L.

c) What is the product of C and L?

d) If the strips are insulated from one another by a non-conducting material of permittivity ε and permeability μ, what will the product of C and L be?

Express your answers in terms of h, w, ε, and μ.

Problem 6 (20 points)

An infinitely long cylinder, of radius R, carries a "frozen-in" magnetization, parallel to the axis,

$$\overline{M} = kr\,\hat{k}$$

where k is a constant and r is the distance from the axis (there is no free current anywhere). Find the magnetic field inside and outside the cylinder by two different methods:

a) Locate all the bound currents, and calculate the field they produce.

b) Use Ampere's law to find $\overline{H}$, and then get $\overline{B}$.

Express you answers in terms of k and R.