Problem 1 (2.5 points) **Answer on Scantron form**

What did **NOT** happen on December 5, 2023?

Figure 1: Sinterklaas.

1. Good Dutch children received presents.
2. Professor Wolfs taught a lecture and told us about Sinterklaas.
3. **Sinterklaas celebrated his birthday**.
4. Naughty Dutch children were put in a bag.

Option 3 is the correct answer. The birthday of Sinterklaas is December 6.

Problem 2 (2.5 points) **Answer on Scantron form**

If all three collisions shown in Fig. 2 are totally inelastic, which collisions cause(s) the most damage?

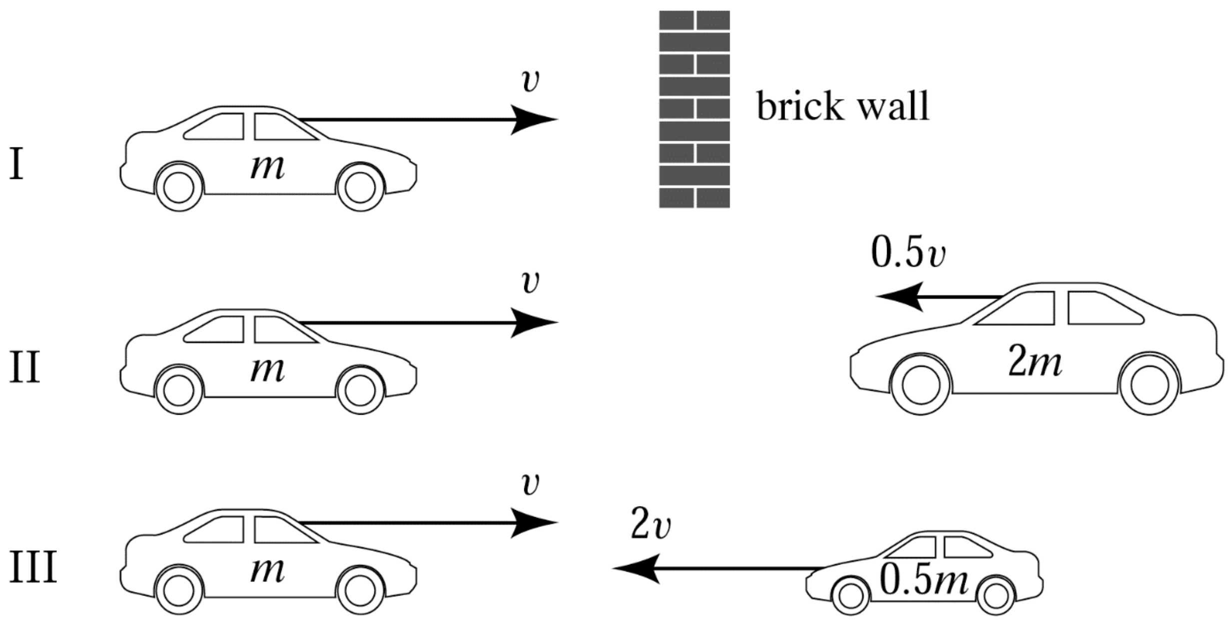

Figure 2: Various car collisions.

1. I.
2. II.
3. **III**.
4. I and II.
5. I and III.
6. II and III.
7. All three.

Option 3 is the correct answer.

Problem 3 (2.5 points) **Answer on Scantron form**

Which of the following statements is **false**?

1. The buoyant force in a liquid is much larger than the buoyant force in air.
2. The buoyant force is a result of small differences in the molecular density of the air/liquid across the surface of the object.
3. **The buoyant force is a result of differences in the average molecular velocity of the air/liquid molecules across the surface of the object**.
4. The magnitude of the buoyant force increases with increasing temperature, due to the corresponding increase in the average molecular velocities.

Option 3 is the correct answer.

Problem 4 (2.5 points) **Answer on Scantron form**

The coefficient of performance of a Carnot engine operated as a heat pump is

1. $1 - T_L/T_H$
2. $(1 - T_L/T_H)^{-1}$
3. $(T_H/T_L - 1)^{-1}$
4. $T_H/T_L - 1$

In these equations, T_H is the temperature of the hot reservoir and T_L is the temperature of the cold reservoir.
Option 2 is the correct answer.

Problem 5 (2.5 points) **Answer on Scantron form**

You have two identical springs, connected in parallel. When you hang a mass m from this system, as shown in Fig. 3, the new equilibrium position of the system is a distance d below the equilibrium position when no mass is connected to the system.

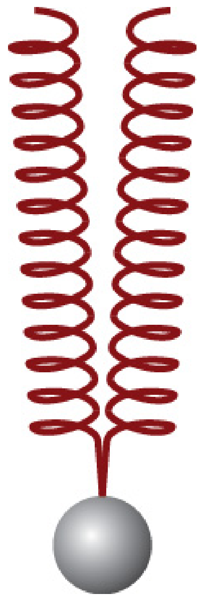

Figure 3: A parallel spring system.

Now you connect the two springs in series. The system is in equilibrium when you connect mass m to the end of the lower spring. What is the displacement of mass m when it has reached its new equilibrium position?

1. $4d$
2. $2d$
3. $\sqrt{2}d$
4. d
5. $d/\sqrt{2}$
6. $d/2$
7. $d/4$

Option 1 is the correct answer.

Problem 6 (2.5 points) **Answer on Scantron form**

A cart rolls with low friction on a track. A fan is mounted on the cart. When the fan is turned on, there is a constant force acting on the cart. Three different experiments are performed:

a) Fan off: The cart is originally at rest. You give it a brief push, and it coasts a long distance along the $+x$ direction, slowly coming to a stop.

b) Fan forward: The fan is turned on and you hold the cart stationary. You then take your hand away and the cart moves forward in the $+x$ direction. After travelling a long distance along the track, you quickly stop and hold the cart.

c) Fan backward: The fan is turned on facing the "wrong" direction and you hold the cart stationary. You give it a brief push and the cart moves in the $+x$ direction, slowing down, turning around, and returning to the starting position where you quickly stop and hold the cart.

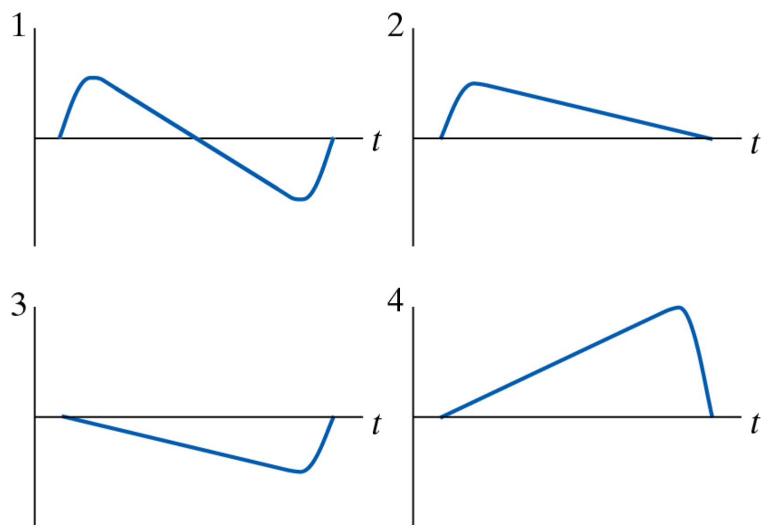

Figure 4: Linear momentum versus time.

Figure 4 shows the x component of the linear momentum of the car as a function of time. Match the three experiments with the correct graphs.

1. a = 1, b = 2, c = 3
2. a = 1, b = 3, c = 2
3. a = 1, b = 4, c = 2
4. a = 2, b = 1, c = 4
5. a = 2, b = 3, c = 1
6. a = 2, b = 4, c = 1
7. a = 3, b = 4, c = 1
8. a = 3, b = 1, c = 4
9. a = 4, b = 2, c = 3
10. a = 4, b = 2, c = 1

Option 6 is the correct answer.

Problem 7 (2.5 points) **Answer on Scantron form**

Two masses M_1 and M_2 sit on a table connected by a rope, as shown in Fig. 5. A second rope is attached to the opposite side of M_2. Both masses are pulled along the table with the tension in the second rope equal to T_2.

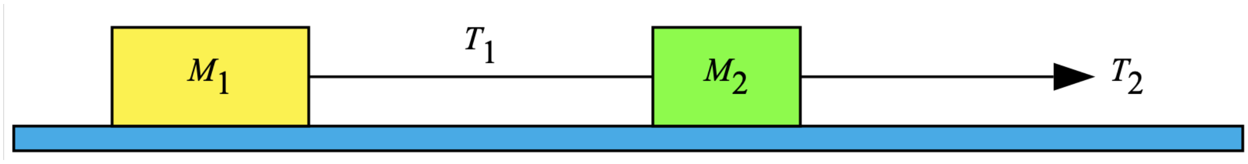

Figure 5: Two masses being pulled.

Let T_1 denote the tension in the rope connecting the two masses. Which of the following statements is true?

1. $T_1 = T_2$
2. $T_1 > T_2$
3. $T_1 < T_2$
4. We need to know the relative values of M_1 and M_2 to answer this question.

Option 3 is the correct answer.

Problem 8 (2.5 points) **Answer on Scantron form**

The linear density of a long thin rod of cross-sectional area A and length L decreases linearly from a value of ρ_0 at the left end to zero at the right end. How far from the left end is the rod's center-of-mass located?

1. $(1/5)L$
2. $(1/4)L$
3. $\mathbf{(1/3)}L$
4. $(2/5)L$
5. $(1/2)L$
6. $(3/5)L$
7. $(2/3)L$
8. $(3/4)L$
9. $(4/5)L$

Option 3 is the correct answer.

Problem 9 (2.5 points) **Answer on Scantron form**

The specific heat capacity of a diatomic gas as function of temperature is shown in Fig. 6.

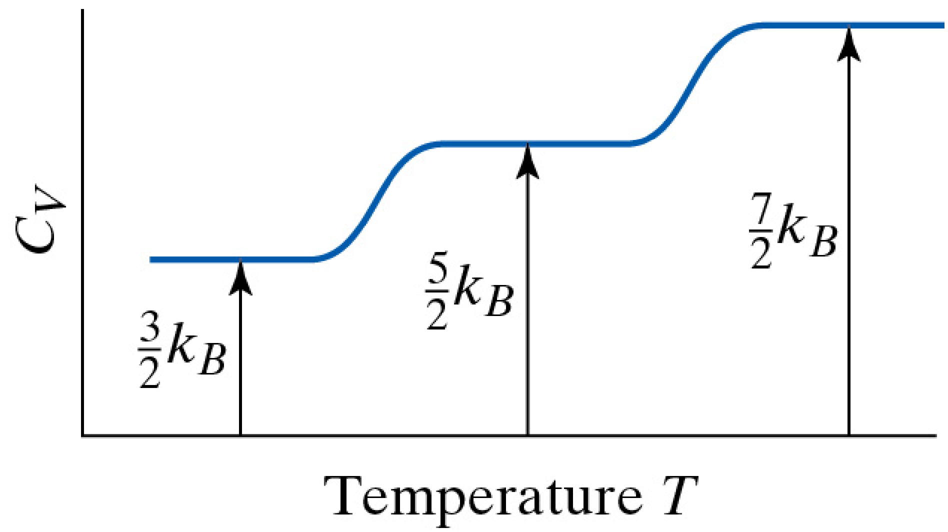

Figure 6: Heat capacity C_V of a diatomic gas as function of temperature T.

At a certain temperature T, the specific heat capacity C_V of the diatomic gas is measured to be $(5/2)k_B$. What degrees of freedom are accessible at this temperature?

1. **Translational and rotational**.
2. Translational and vibrational.
3. Rotational and vibrational.
4. Translational and electronic.
5. Electronic and rotational.
6. Electronic and vibrational.
7. Translational, electronic, rotational, and vibrational.
8. Electronic, rotational, and vibrational

Option 1 is the correct answer.

Problem 10 (2.5 points) **Answer on Scantron form**

A gas is made up of diatomic molecules. At temperature T, the ratio of the number of molecules in vibrational energy state 2 to the number of molecules in the ground state is measured to be 0.35. The difference in energy between state 2 and the ground state is ΔE. Which of the following conclusions is correct?

1. $\Delta E \approx k_B T$
2. $\Delta E \ll k_B T$
3. $\Delta E \gg k_B T$

Option 1 is the correct answer.

Problem 11 (25 points) **Answer in booklet 1**

A cylinder with cross sectional area A contains N molecules of nitrogen gas at pressure p_0. The gas is in thermal equilibrium with a heat bath of temperature T_0. A piston confines the gas inside a region of volume V_0. The entire system is contained in a vacuum vessel, and only the nitrogen gas exerts a pressure on the piston.

a) You quickly pull up the piston to increase the volume of the gas to V_f. What is the temperature T_f of the gas immediately after you finish pulling up the piston? What approximations did you make?

Since the piston is pulled up quickly we can assume that this is an adiabatic expansion. During an adiabatic expansion pV^γ = constant. Using the ideal gas law we can rewrite this in terms of the temperature T and the volume V: $TV^{\gamma-1}$ = constant. Applying this relation to the expansion we conclude that

$$T_0 {V_0}^{\gamma-1} = T_f {V_f}^{\gamma-1} \tag{1}$$

or

$$T_f = T_0 \frac{{V_0}^{\gamma-1}}{{V_f}^{\gamma-1}} \tag{2}$$

b) What is the work done by the gas during this expansion?

The work done by the gas during the expansion can be found by determining the area under the p versus V curve:

$$\begin{aligned} W &= \int_{V_0}^{V_f} p dV = \int_{V_0}^{V_f} \left(\frac{p_0 {V_0}^\gamma}{V^\gamma}\right) dV = \\ &= p_0 {V_0}^\gamma \int_{V_0}^{V_f} V^{-\gamma} dV = p_0 {V_0}^\gamma \left\{ \left.\frac{V^{-\gamma+1}}{-\gamma+1}\right|_{V_0}^{V_f} \right\} = \\ &= \frac{p_0 {V_0}^\gamma}{\gamma-1} \left\{ {V_0}^{-\gamma+1} - {V_f}^{-\gamma+1} \right\} = \frac{p_0 V_0}{\gamma-1} \left\{ 1 - \frac{{V_0}^{\gamma-1}}{{V_f}^{\gamma-1}} \right\} \end{aligned} \tag{3}$$

c) What is the force you must exert on the piston, immediately after you finish pulling it up, in order to hold it into its final position?

Since pV^γ = constant we can easily calculate the pressure of the gas right after the adiabatic expansion:

$$p_f = p_0 \frac{{V_0}^\gamma}{{V_f}^\gamma} \tag{4}$$

The force on the piston is thus equal to

$$F_f = p_f A = p_0 \frac{V_0{}^{\gamma}}{V_f{}^{\gamma}} A \tag{5}$$

Note: we could have also used the ideal gas law and the temperature calculated in part a) to determine the final pressure:

$$p_f = \frac{NkT_f}{V_f} = \frac{Nk}{V_f} T_0 \frac{V_0{}^{\gamma-1}}{V_f{}^{\gamma-1}} = \frac{NkT_0}{V_0} \frac{V_0{}^{\gamma}}{V_f{}^{\gamma}} = p_0 \frac{V_0{}^{\gamma}}{V_f{}^{\gamma}} \tag{6}$$

d) You wait while the nitrogen returns back to its original temperature T_0. What is now the force you must exert on the piston in order to hold it into its final position?

We can use the ideal gas law to determine the pressure of the gas when it has returned to its original temperature T_0:

$$p = \frac{NkT_0}{V_f} \tag{7}$$

The force on the piston at this point is thus equal to

$$F = pA = \frac{NkT_0}{V_f} A \tag{8}$$

e) You now very slowly move the piston back to its original position such that the gas is contained again in a volume V_0. How much work must you do to move the piston back to this position? What approximations did you make?

The compression we now carry out is an isothermal compression. Since this compression is slow, there is ample of time to assure that the gas is maintained at a temperature T_0. The work done by the gas, which is opposite to the work that you need to do, can be calculated using the ideal gas law:

$$W = \int_{V_f}^{V_0} p dV = NkT_0 \int_{V_f}^{V_0} \frac{1}{V} dV = NkT_0 \left\{ \ln V \Big|_{V_f}^{V_0} \right\} = -NkT_0 \ln\left(\frac{V_f}{V_i}\right) \tag{9}$$

The work calculate in part b) can be rewritten as

$$W = \frac{3}{2} p_0 V_0 \left\{ 1 - \frac{V_0{}^{2/3}}{V_f{}^{2/3}} \right\} = \frac{3}{2} NkT_0 \left\{ 1 - \frac{V_0{}^{2/3}}{V_f{}^{2/3}} \right\} \tag{10}$$

To compare the work done by these two processes, we really compare the work done during an isothermal process with the work done during an adiabatic process. If we compare the area under the pV curve for an adiabatic process with the work done under the pV curve for an isothermal process with the same starting conditions (same p, same V) we see that the work done on the gas during an adiabatic compression is less than the work done by the gas during an isothermal expansion (see Fig. 7).

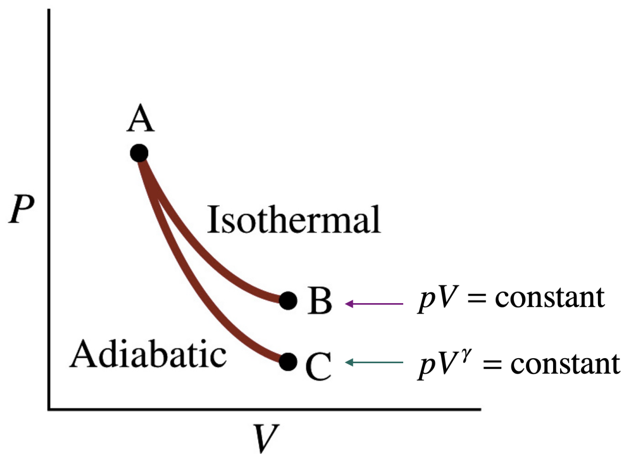

Figure 7: pV curves for an ideal gas.

Problem 12 (25 points) **Answer in booklet 1**

Three bodies of identical mass m rotate in circular orbits of radius r around their center of mass, as shown in Fig. 8. The system is held together by their mutual gravitational forces.

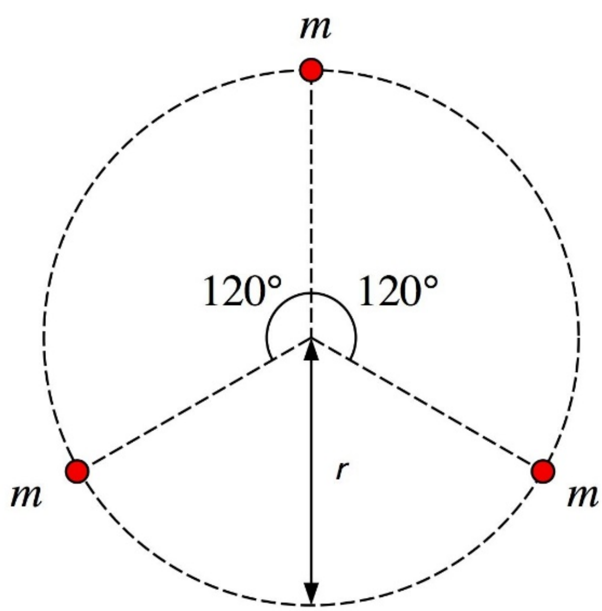

Figure 8: A three-body system.

a) Using a diagram, indicate the direction of the net force acting on each body.

The net force on each body is directed in the radial direction, as in indicated in Fig. 9.

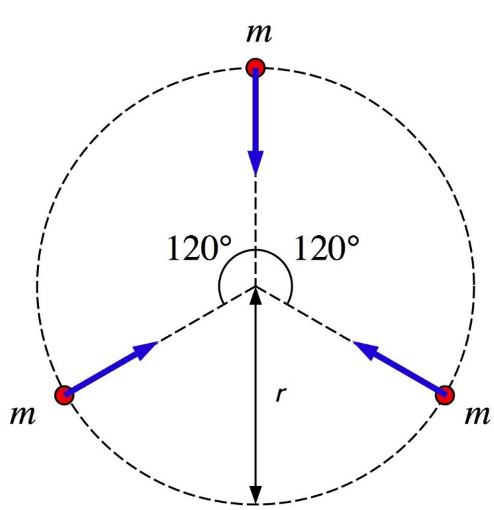

Figure 9: Net force on each of the three masses.

b) What is the magnitude of the net force acting on each body?

The distance between the masses is $r\sqrt{3}$. The magnitude of the gravitational force between each mass pair is thus equal to

$$F_{pair} = G\frac{m^2}{3r^2} \tag{11}$$

The radial component of this force is equal to

$$F_{radial} = F_{pair}\cos\left(\frac{\pi}{6}\right) = G\frac{m^2}{3r^2}\frac{1}{2}\sqrt{3} = G\frac{m^2}{2\sqrt{3}r^2} \tag{12}$$

The magnitude of the net force acting on each body is thus equal to

$$F_{net} = 2F_{radial} = G\frac{m^2}{\sqrt{3}r^2} \tag{13}$$

c) What is the orbital period of each body?

Since the bodies are in a stable orbit, the net force acting on them must be directed towards the center of the orbit, and have a magnitude equal to mv^2/r. Using the result from part b) we must thus require that

$$F_{net} = G\frac{m^2}{\sqrt{3}r^2} = \frac{mv^2}{r} \tag{14}$$

or

$$v^2 = G\frac{m}{\sqrt{3}r} \tag{15}$$

The speed v is equal to $2\pi r/T$ where T is the orbital period. The requirement for a stable circular orbit now becomes

$$\frac{4\pi^2 r^2}{T^2} = G\frac{m}{\sqrt{3}r} \tag{16}$$

or

$$T = \sqrt{4\sqrt{3}\frac{\pi^2 r^3}{Gm}} \tag{17}$$

Problem 13 (25 points) **Answer in booklet 1**

Consider a two-dimensional elastic collision involving particles of equal mass m in which one of the particles is initially at rest, as shown in Fig. 10.

Figure 10: The collision system before the collision.

After the collision, the angle between the directions of these two particles is A, as shown in Fig. 11.

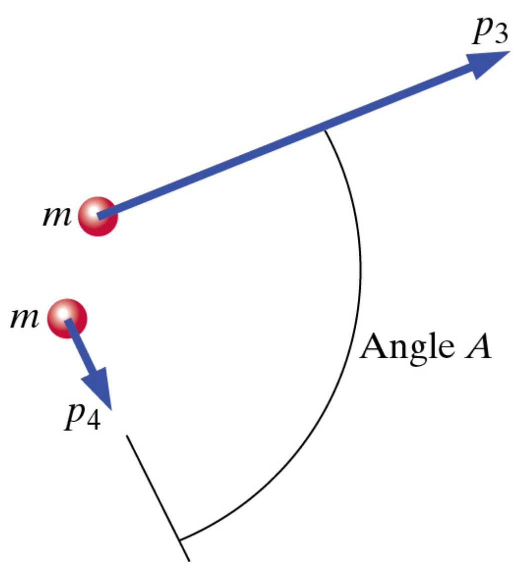

Figure 11: The collision system after the collision.

a) Use vector notation to write down the relation between the initial linear momentum of the incident particle, $\vec{p}_1$, and the linear momenta of the two particles after the collision, $\vec{p}_3$ and $\vec{p}_4$.

The initial linear momentum of the system is the linear momentum of the incident particle: $\vec{p}_1$. The final linear momentum of the system is the vector sum of $\vec{p}_3$ and $\vec{p}_4$. Conservation of linear momentum thus requires that

$$\vec{p}_1 = \vec{p}_3 + \vec{p}_4 \tag{18}$$

b) Use the scalar product to obtain an expression for the magnitude of $\vec{p}_1$ in terms of the magnitudes of $\vec{p}_3$ and $\vec{p}_4$ and the angle A.

Using the properties of the scalar product we can calculate the magnitude of $\vec{p}_1$:

$$|\vec{p}_1|^2 = \vec{p}_1 \bullet \vec{p}_1 = (\vec{p}_3 + \vec{p}_4) \bullet (\vec{p}_3 + \vec{p}_4) = |\vec{p}_3|^2 + |\vec{p}_4|^2 + 2\vec{p}_3 \bullet \vec{p}_4 \tag{19}$$

The vector product between $\vec{p}_3$ and $\vec{p}_4$ can be expressed in terms of the magnitudes of $\vec{p}_3$ and $\vec{p}_4$ and the angle A:

$$|\vec{p}_1|^2 = |\vec{p}_3|^2 + |\vec{p}_4|^2 + 2|\vec{p}_3||\vec{p}_4|\cos(A) \tag{20}$$

c) Use the relation you derived in part b) to obtain an expression that relates the kinetic energy of the incident particle, K_1, to the kinetic energies of the outgoing particles, K_3 and K_4.

The incident kinetic energy K_1 is equal to

$$K_1 = \frac{|\vec{p}_1|^2}{2m} \tag{21}$$

Using Eq. 20, we can rewrite this equation as

$$K_1 = \frac{\vec{p}_3|^2 + |\vec{p}_4|^2 + 2|\vec{p}_3||\vec{p}_4|\cos(A)}{2m} = K_3 + K_4 + 2\sqrt{K_3}\sqrt{K_4}\cos(A) \tag{22}$$

d) Since the collision is elastic, what can you conclude about the angle A?

Since the collision is elastic, $K_1 = K_3 + K_4$. Using this relation between kinetic energies and Eq. 22 we can conclude that

$$K_1 = K_3 + K_4 + 2\sqrt{K_3}\sqrt{K_4}\cos(A) = K_1 + 2\sqrt{K_3}\sqrt{K_4}\cos(A) \tag{23}$$

This requires that

$$2\sqrt{K_3}\sqrt{K_4}\cos(A) = 0 \tag{24}$$

or

$$\cos(A) = 0 \tag{25}$$

since K_3 and K_4 are not equal to 0. The angle A must thus be equal to 90 degrees.

Problem 14 (25 points) **Answer in booklet 1**

A small block of mass m slides along a friction-less loop-the-loop track, as shown in Fig. 12. The block is released from rest at point P.

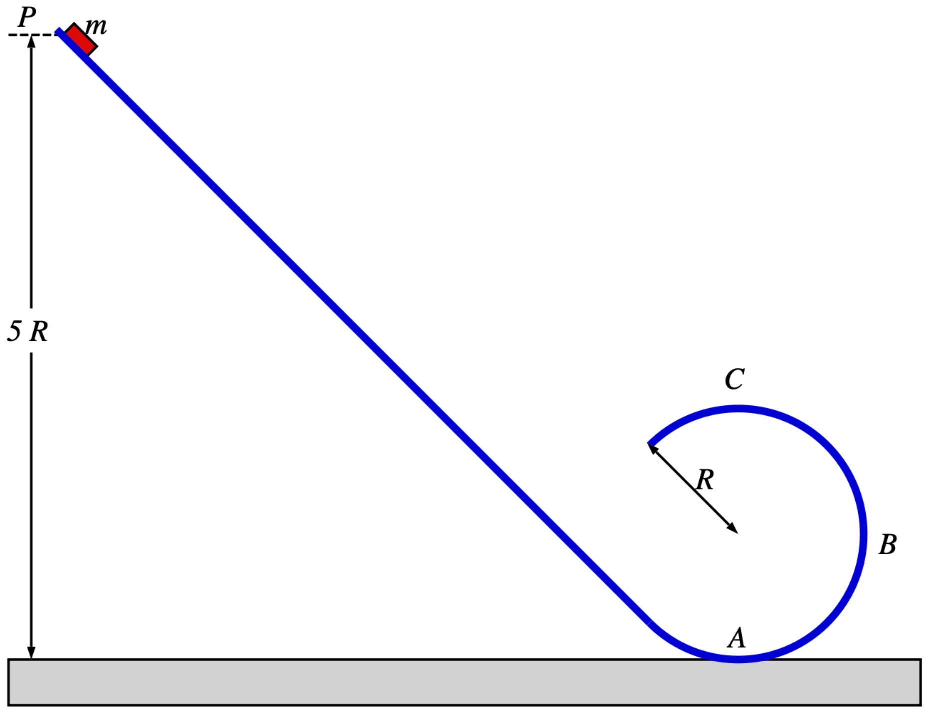

Figure 12: Motion on the loop-the-loop.

a) Draw a force diagram of all forces acting on the block at point A.

There are two forces acting on mass m at point A:

(a) The normal force N exerted by the surface of the loop-the-loop on the block

(b) The gravitational force

The directions of these two forces are indicated in the Fig. 13.

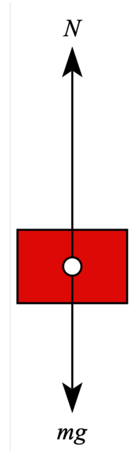

Figure 13: Forces on the block at point A.

b) What is the speed of the block at point A?

The velocity of the block at point A can be found by applying conservation of energy. At the moment that the block is released, its mechanical energy is entirely due to gravitational potential energy. The total energy of the block at this instant is therefore given by

$$E_P = U_P + K_P = mg(5R) = 5mgR \tag{26}$$

Here we have assumed that the vertical position at point A (the bottom of the loop-the-loop) corresponds to $U = 0$ J.

At point A, the mechanical energy of the block is entirely due to kinetic energy

$$E_A = U_A + K_A = \frac{1}{2}m{v_A}^2 \tag{27}$$

Since total mechanical energy is conserved, we conclude that

$$\frac{1}{2}m{v_A}^2 = 5mgR \tag{28}$$

The velocity of the block at point A is therefore equal to

$$v_A = \sqrt{10gR} \tag{29}$$

c) What is the net force acting on the block at point B (specify direction and magnitude)?

The block is carrying out circular motion. In order for the block to carry out this circular motion, a radial force must be present. At point B, the radial force can only be provided by the normal force N. In addition to the normal force, a gravitational force mg acts along the vertical direction. These two forces are drawn in the force diagram shown in Fig.14.

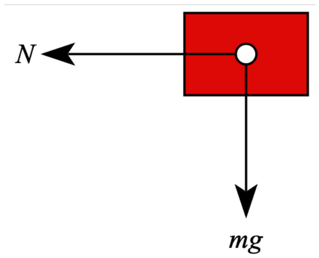

Figure 14: Forces on the block at point B.

The velocity of the block at point B can be found by applying conservation of energy:

$$E_B = U_B + K_B = mgR + \frac{1}{2}m{v_B}^2 = E_P = 5mgR \tag{30}$$

The velocity at B is thus equal to

$$v_B = \sqrt{8gR} \tag{31}$$

The normal force N can now be determined:

$$N_B = \frac{m{v_B}^2}{R} = 8mg \tag{32}$$

The magnitude of the net force acting on the block at point B is equal to

$$|F_B| = \sqrt{(mg)^2 + N^2} = \sqrt{65}mg \tag{33}$$

d) At what height above the bottom of the loop should the block be released so that it is on the verge of losing contact with the track at the top of the loop (point C)?

The block will be on the verge of losing contact with the track at point C if the normal force equals zero. At this point, the radial force acting on the block is equal to the gravitational force mg. The velocity of the block at point C has to satisfy the following relation

$$\frac{m{v_C}^2}{R} = mg \tag{34}$$

and v_C is given by

$$v_C = \sqrt{gR} \tag{35}$$

The total energy of the block at point C can now be obtained

$$E_C = U_C + K_C = mg\,(2R) + \frac{1}{2}m{v_C}^2 = 2mgR + \frac{1}{2}mgR = \frac{5}{2}mgR \tag{36}$$

This total energy can be provided if the block is released from rest from a height h equal to

$$h = \frac{5}{2}R \tag{37}$$

e) What would your answer to part d) be if instead of a block sliding down the track you would use a sphere of radius r and mass m?

If instead of a block, we are using a sphere of radius r, we will have to take into consideration that the kinetic energy now has two components: a translational component and a rotational component:

$$K_C = \frac{1}{2}m{v_C}^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}m{v_C}^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\frac{{v_C}^2}{r^2} = \frac{7}{10}m{v_C}^2 \tag{38}$$

The total energy at C is thus equal to

$$E_C = U_C + K_C = mg\,(2R) + \frac{7}{10}m{v_C}^2 = 2mgR + \frac{7}{10}mgR = \frac{27}{10}mgR \tag{39}$$

This total energy can be provided if the block is released from rest from a height h equal to

$$h = \frac{27}{10}R \tag{40}$$

Problem 15 (25 points) **Answer in booklet 2**

Consider the classical model of a hydrogen atom in which an electron of mass m is in a circular orbit of radius r around a proton of mass M. In this model, we will assume that the proton remains at rest at the center of the circular orbit. Since the electron and the proton have charges with opposite signs, the electric force between them is attractive.

a) The electric force between the electron and the proton is attractive and is much larger than the gravitational force acting between them. The electric force is directed in the same direction as the gravitational force, and has a magnitude equal to

$$|F_{el}| = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

where e is the magnitude of the charge of the electron. What is the velocity of the electron in an orbit of radius r, assuming uniform circular motion? You can ignore relativistic effects and the rest mass of the electron.

In order to achieve uniform circular motion, the magnitude of the radial force on the electron must be equal to mv^2/r. This requires that

$$|F_{el}| = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{mv^2}{r} \tag{41}$$

or

$$v = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{mr}} \tag{42}$$

b) The electric potential energy of the electron is equal to

$$U_{el} = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

What is the total energy of the electron in an orbit of radius r? You can ignore relativistic effects and the rest mass of the electron.

The total energy of the electron is the sum of its kinetic energy and its potential energy:

$$E = K_{el} + U_{el} = \frac{1}{2}mv^2 - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} = -\frac{1}{8\pi\varepsilon_0}\frac{e^2}{r} \tag{43}$$

c) What is the magnitude of the angular momentum of the electron in an orbit of radius r?

For uniform circular motion, the linear momentum is perpendicular to the radial position vector. The magnitude of the angular momentum is thus equal to

$$\left|\vec{L}\right| = r\,(mv) = mr\sqrt{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{mr}} = \sqrt{\frac{1}{4\pi\varepsilon_0}mre^2} \tag{44}$$

d) Quantum mechanics tells us that the orbital angular momentum of the electron is quantized and is an integer multiple of $h/2\pi$:

$$\left|\vec{L}\right| = N\hbar = N\frac{h}{2\pi}$$

where N is 1, 2, 3, ... The quantization of angular momentum leads to a quantization of the energy of the electron. Based on this model, what are the energies of the photons that can be emitted by the hydrogen atom when it decays from an excited state with $N = 3$ to its ground state ($N = 1$)?

The quantization of angular momentum requires that

$$\sqrt{\frac{1}{4\pi\varepsilon_0}mre^2} = N\hbar \tag{45}$$

or

$$r = 4\pi\varepsilon_0\frac{N^2\hbar^2}{me^2} \tag{46}$$

The energy of the electron is thus equal to

$$E_N = -\frac{1}{8\pi\varepsilon_0}\frac{e^2}{r} = -\frac{1}{8\pi\varepsilon_0}\frac{e^2}{\left(4\pi\varepsilon_0\frac{N^2\hbar^2}{me^2}\right)} = -\frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{me^4}{N^2\hbar^2} \tag{47}$$

The energy levels of the lowest three states of the atom are:

$$E_1 = -\frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{me^4}{\hbar^2} \tag{48}$$

$$E_2 = -\frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{1}{4}\frac{me^4}{\hbar^2} \tag{49}$$

$$E_3 = -\frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{1}{9}\frac{me^4}{\hbar^2} \tag{50}$$

The possible transition energies are thus

$$E_{3\to 1} = E_3 - E_1 = -\frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{me^4}{\hbar^2}\left\{\frac{1}{9} - 1\right\} = \frac{8}{9}\left(\frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{me^4}{\hbar^2}\right) \tag{51}$$

$$E_{3\to 2} = E_3 - E_2 = -\frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{me^4}{\hbar^2}\left\{\frac{1}{9} - \frac{1}{4}\right\} = \frac{5}{36}\left(\frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{me^4}{\hbar^2}\right) \tag{52}$$

$$E_{2\to 1} = E_2 - E_1 = -\frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{me^4}{\hbar^2}\left\{\frac{1}{4} - 1\right\} = \frac{3}{4}\left(\frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{me^4}{\hbar^2}\right) \tag{53}$$

Problem 16 (25 points) **Answer in booklet 3**

A block with mass m is pushed along a horizontal floor by a force P that makes an angle A with the horizontal, as shown in Fig. 15a. The coefficients of kinetic and static friction between the block and the floor are μ_k and μ_s, respectively.

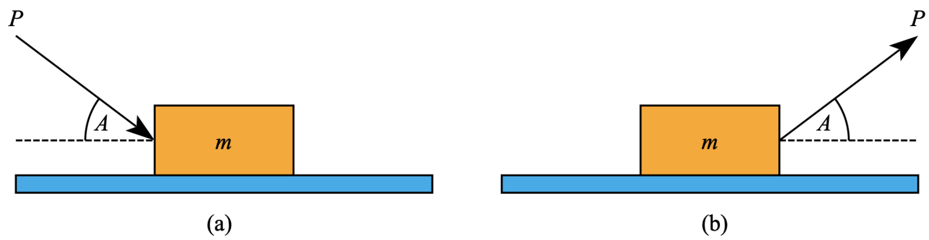

Figure 15: Pushing and pulling a block.

a) Calculate the maximum force that can be applied without moving the block.

Figure 16 shows all forces acting on the block. These forces include the gravitational force W, the normal force N, the static friction force f and the applied force P. Since the block remains at rest, the net force acting on it must be zero. This implies that

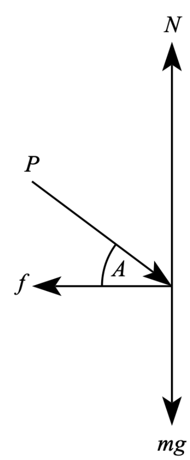

Figure 16: Force acting on the block shown in Fig. 15a.

$$\sum F_x = P\cos(A) - f = 0 \tag{54}$$

and

$$\sum F_y = N - mg - P\sin(A) = 0 \tag{55}$$

Equation 55 can be used to obtain the normal force N:

$$N = mg + P\sin(A) \tag{56}$$

The static friction force f is related to the normal force in the following manner

$$f \leq \mu_s N = \mu_s (mg + P\sin(A)) \tag{57}$$

Since the static friction force f has a maximum value, the applied force P also has a maximum value. If we apply a force larger than this maximum value, the static friction force is not strong enough to balance the applied force, and the block will start to move. The applied force P and the static friction force f are related in the following manner (see also Eq. 54):

$$P = \frac{f}{\cos(A)} \tag{58}$$

The maximum applied force P can now be easily obtained from the known maximum static friction force shown in Eq. 57

$$P_{\max} = \frac{\mu_s (mg + P_{\max}\sin(A))}{\cos(A)} \tag{59}$$

which can be solved for P_{max}:

$$P_{\max} = \frac{\mu_s mg}{\cos(A) - \mu_s \sin(A)} \tag{60}$$

b) If the applied force P is larger than the maximum force calculated in a), the block will start to move. Calculate the acceleration of the block.

We now assume that the applied force P is larger than the maximum force calculated in a). If this is the case, the block will start to move, and the friction force is now the kinetic friction force. Following the same procedure as in a) we can calculate the kinetic friction force f:

$$f = \mu_k (mg + P\sin(A)) \tag{61}$$

The net force in the horizontal direction is given by

$$\sum F_x = P\cos(A) - f = P\cos\theta - \mu_k (mg + P\sin(A)) \tag{62}$$

The acceleration of the block is given by

$$a = \frac{\sum F_x}{m} = \frac{P\cos(A) - \mu_k (mg + P\sin(A))}{m} \tag{63}$$

c) As a result of the applied force P, the block moves over a distance d. What is the work done on the block by force P?

The work W done by the applied force P is equal to the scalar product between the force vector and the displacement vector:

$$W = \vec{P} \bullet \vec{d} = Pd\cos(A) \tag{64}$$

Instead of pushing the block with a force P, we can pull the block with the same force P, as shown in Fig. 15b.

d) Calculate the maximum force that can be applied without moving the block. Is this force larger or smaller than the maximum force calculated in part a)?

All the forces acting on the block are shown in Fig. fig:solutionsProblem16d. Comparing the situation shown in part a) with the current situation we observe that the only difference is the direction of the applied force P. This difference is however very important since it changes the value of the normal force, and therefore the magnitude of the static friction force.

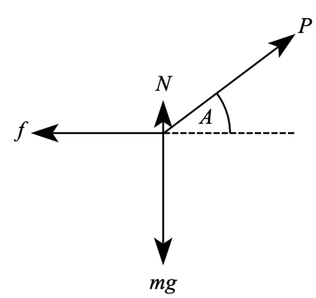

Figure 17: Force acting on the block shown in Fig. 15b.

The derivation of the maximum force in a) can also be used to determine the maximum force in this situation of we replace sin(θ) by -sin(θ). We conclude that

$$P_{\max} = \frac{\mu_s mg}{\cos(A) + \mu_s \sin(A)} \tag{65}$$

The maximum force in this situation is less than the maximum force obtained in part a).

e) If the applied force P is larger than the maximum force calculated in part d), the block will start to move. Calculate the acceleration of the block.

We now assume that the applied force P is larger than the maximum force calculated in a). If this is the case, the block will start to move, and the friction force is now the kinetic friction force. Following the same procedure as in d) we can calculate the kinetic friction force f:

$$f = \mu_k (mg - P\sin(A)) \tag{66}$$

The net force in the horizontal direction is given by

$$\sum F_x = P\cos(A) - f = P\cos(A) - \mu_k (mg - P\sin(A)) \tag{67}$$

The acceleration of the block is given by

$$a = \frac{\sum F_x}{m} = \frac{P\cos(A) - \mu_k (mg - P\sin(A))}{m} \tag{68}$$

f) As a result of the applied force P, the block moves over a distance d. What is the work done on the block by force P?

The work W done by the applied force P is equal to the scalar product between the force vector and the displacement vector:

$$W = \vec{P} \bullet \vec{d} = Pd\cos(A) \tag{69}$$

g) In which situation (block being pulled or pushed) will the block have the highest kinetic energy when it has been displaced over a distance d? Your answer needs to be well motivated.

In the two cases, the work done by the applied force P on the block is the same, but the work done by the friction force is different. Equations 61 and 66 show that the magnitude of the work done by the friction force is less when we pull the block compared to when we push the block. The total work done on the block by all forces is thus larger when we pull the block than when we push the block. Consequently, the change in kinetic energy (and thus in velocity) is larger when we pull the block then when we push the block.

Problem 17 (25 points) **Answer in booklet 3**

A rod of length L and negligible mass is attached to a uniform disk of mass M and radius R, as shown in Fig. 18.

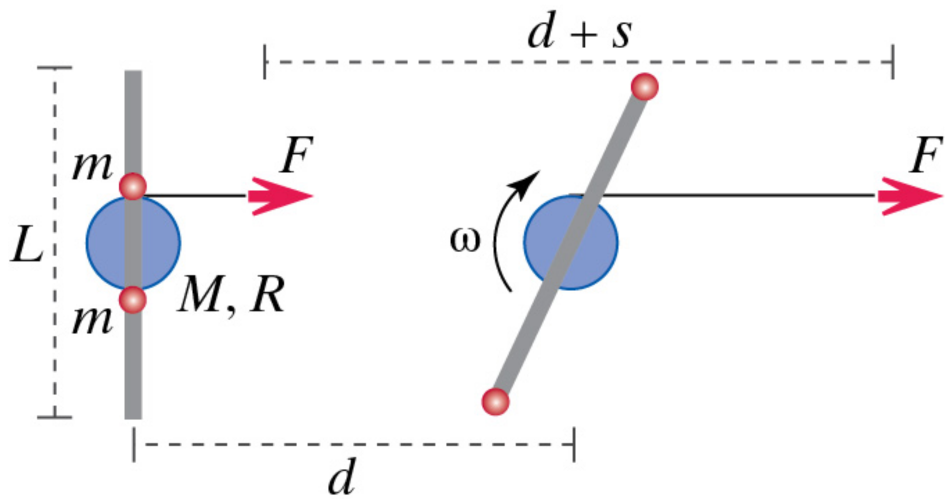

Figure 18: System studied in Problem 17.

A string is wrapped around the disk, and you pull on the string with a constant force F. Two small balls, each of mass m, slide along the rod with negligible friction. The apparatus start from rest, and when the center of the disk has moved a distance d, a length of string s has come off the disk, and the balls have collided with the end of the rod and stuck there. The apparatus slides on a nearly frictionless table.

a) At this instant, what is the speed v of the center of the disk?

The work done by the force F on the center of mass is equal to

$$W_{cm} = Fd \tag{70}$$

The work done on the center of mass is equal to the change in its kinetic energy. Since the center of mass is initially at rest, the work done on the center of mass is equal to the final kinetic energy of the center of mass:

$$K_{\text{cm,final}} = \frac{1}{2}(M + 2m)v_{\text{cm,final}}^2 = Fd \tag{71}$$

The final velocity of the center of mass is thus equal to

$$v_{\text{cm,final}} = \sqrt{\frac{2Fd}{M + 2m}} \tag{72}$$

b) At this instant, the angular speed of the disk is ω. What is the rotational energy of the system at this instant?

The moment of inertia I of the system at this instant is equal to

$$I = \frac{1}{2}MR^2 + 2m\left(\frac{1}{2}L\right)^2 = \frac{1}{2}MR^2 + \frac{1}{2}mL^2 = \frac{1}{2}\left(MR^2 + mL^2\right) \tag{73}$$

The rotation energy of the system is equal to

$$K_{\text{rotational}} = \frac{1}{2}I\omega^2 = \frac{1}{4}\left(MR^2 + mL^2\right)\omega^2 \tag{74}$$

c) What is the change in the internal energy of the system at this instant?

The total work done on the system by force F is

$$W_{\text{total}} = F(d+s) \tag{75}$$

This work changes the kinetic energy of the center of mass, the rotational energy of the system, and its internal energy. The change in the internal energy of the system is thus equal to

$$E_{\textbf{internal}} = W - E_{\textbf{cm}} - E_{\textbf{rotational}} = F(d+s) - Fd - \frac{1}{4}\left(MR^2 + mL^2\right)\omega^2 \tag{76}$$