

The momentum principle:

$$d\vec{p} = \vec{F}_{net} dt$$

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\vec{p}_{new} = \vec{p}_{old} + \vec{F}_{net} \Delta t$$

$$\vec{r}_{new} = \vec{r}_{old} + \frac{1}{\sqrt{1 + \left(\frac{p}{mc}\right)^2}} \left(\frac{\vec{p}}{m}\right) \Delta t$$

Equations of motion in 1D for constant acceleration and low velocities ($v \ll c$):

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v(t) = \frac{dx(t)}{dt} = v_0 + at$$

$$a(t) = \frac{dv(t)}{dt} = a = \text{constant}$$

Requirement for uniform circular motion:

$$F_r = \frac{mv^2}{r}$$

Rotational motion:

$$d = \theta r$$

$$v = \omega r \quad \omega = \frac{d\theta}{dt}$$

$$a = \alpha r \quad \alpha = \frac{d\omega}{dt}$$

Gravitational force:

$$\vec{F} = G \frac{m_1 m_2}{r^2} \hat{r}$$

$$\vec{F} = m\vec{g} \quad (\text{close to the surface of the Earth})$$

Electrostatic force:

$$\vec{F} = -\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$$

Harmonic motion:

$$F = -kx$$

$$x(t) = x_{\max} \cos(\omega t + \phi) \quad \text{where } \omega = \sqrt{\frac{k}{m}}$$

$$T = \frac{2\pi}{\omega}$$

Damped harmonic motion:

$$x(t) = x_m e^{-\frac{bt}{2m}} e^{it\sqrt{\frac{k}{m}}}$$

Driven harmonic motion:

$$x(t) = \frac{F_0}{\omega_0^2 - \omega^2} \cos(\omega t + \phi)$$

Stress and strain:

$$\frac{F}{A} = Y \frac{\Delta L}{L}$$

$$Y = \frac{k_s}{d}$$

Work done by a force:

$$W = \vec{F} \cdot \vec{d} \quad \text{constant force}$$

$$= \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} \quad \text{variable force}$$

Power:

$$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

Work-energy theorem:

$$\Delta E_{\text{system}} = W$$

$$E_{\text{system}} = (E_1 + E_2 + E_3 + \dots) + U$$

Relativistic energy relations:

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} = mc^2 + K$$

$$E^2 - (pc)^2 = (mc^2)^2$$

$$K = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - mc^2 \xrightarrow{v \ll c} \frac{1}{2}mv^2$$

Potential energy:

$$\Delta U = -W_{\text{internal}} = -\int \vec{F} \cdot d\vec{r}$$

$$\vec{F} = -\vec{\nabla}U = \begin{pmatrix} -\frac{\partial U}{\partial x} \\ -\frac{\partial U}{\partial y} \\ -\frac{\partial U}{\partial z} \end{pmatrix}$$

$$U_{\text{gravity}} = -G \frac{m_1 m_2}{r}$$

$$U_{\text{gravity}} = mgh$$

$$U_{\text{electric}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

$$U_{\text{spring}} = \frac{1}{2}kx^2$$

Heat capacity:

$$\Delta E_{\text{thermal}} = mC\Delta T$$

Friction forces:

$$f_s \leq \mu_s N$$

$$f_k = \mu_k N$$

Drag force (air):

$$\vec{F}_{\text{air}} = -\frac{1}{2}C\rho A v^2 \hat{v}$$

Energy levels for the Hydrogen atom:

$$E_N = \frac{-13.6}{N^2} \text{ eV}, N = 1, 2, 3, \dots$$

Vibrational energy levels:

$$E_N = E_0 + N\hbar\omega_0 = E_0 + N\hbar\sqrt{\frac{k_s}{m}}, N = 0, 1, 2, \dots$$

Energy and wavelength of light:

$$E_{\text{photon}} = \frac{hc}{\lambda_{\text{light}}}$$

Center of mass:

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i} = \frac{1}{M} \sum_i m_i \vec{r}_i$$

$$\vec{r}_{\text{cm}} = \frac{\int \vec{r} dm}{\int dm} = \frac{1}{M} \int \vec{r} dm$$

Motion of the center of mass:

$$M\vec{a}_{\text{cm}} = \vec{F}_{\text{net,ext}}$$

Gravitational potential energy of a multi-particle system:

$$U = Mgy_{\text{cm}}$$

Kinetic energy of a multi-particle system:

$$K = K_{trans} + K_{rel} = \frac{1}{2} M v_{cm}^2 + K_{rel}$$

Impulse of a force:

$$\vec{J} = \int \vec{F} dt$$

Momentum and impulse:

$$\vec{J} = \vec{p}_f - \vec{p}_i$$

Conservation of linear momentum:

$$\Delta \vec{P}_{system} + \Delta \vec{P}_{surroundings} = 0$$

Elastic collision in one dimension (mass 2 at rest before the collision):

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

Completely inelastic collision in one dimension (mass 2 at rest before the collision):

$$v_f = \frac{m_1}{m_1 + m_2} v_i$$

Moment of inertia:

$$I = \sum_i m_i r_i^2 \quad \text{Discrete mass distribution}$$

$$I = \int_{Volume} r^2 dm \quad \text{Continuous mass distribution}$$

Kinetic energy of a rotating rigid object:

$$K = \frac{1}{2} I \omega^2$$

Torque:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Newton's "second" law for rotational motion:

$$\vec{\tau} = I \vec{\alpha}$$

Angular momentum of a single particle:

$$\vec{L} = \vec{r} \times \vec{p}$$

Angular momentum of a rotating rigid object:

$$\vec{L} = I \vec{\omega}$$

The angular momentum principle:

$$\frac{d\vec{L}}{dt} = (\vec{r} \times \vec{F})_{\text{net, ext}} = \vec{\tau}_{\text{net, ext}}$$

Number of micro states:

$$\Omega = \frac{(q + N - 1)!}{q!(N - 1)!}$$

Definition of entropy S :

$$S = k \ln \Omega$$

Definition of temperature T :

$$\frac{1}{T} = \frac{dS}{dE_{\text{int}}}$$

The Boltzmann distribution:

$$P(\Delta E) \propto e^{-\Delta E/kT}$$

The Maxwell-Boltzmann velocity distribution:

$$P(v) = 4\pi \left(\frac{M}{2\pi kT} \right)^{\frac{3}{2}} v^2 e^{-\frac{1}{2} M v^2 / (kT)}$$

Root-mean-square speed:

$$v_{rms} = \sqrt{v^2} = \sqrt{\frac{3kT}{M}}$$

Average translational kinetic energy of an ideal gas:

$$\bar{K}_{trans} = \frac{3}{2} kT$$

Mean-free path d :

$$\frac{N}{V} \left[\pi (R+r)^2 d \right] \approx 1$$

Number of gas molecules hitting an area A per second:

$$\text{molecules/s} = \frac{1}{4} \frac{N}{V} A \bar{v}$$

Ideal gas law:

$$pV = NkT$$

Work done by a gas:

$$W = \int_{V_1}^{V_2} p dV$$

First law of thermodynamics:

$$\Delta E_{\text{system}} = W + Q$$

Isothermal compression:

$$W = NkT \ln \left(\frac{V_1}{V_2} \right)$$

Adiabatic compressions:

$$pV^\gamma = pV^{C_p/C_V} = \text{constant}$$

Heat capacity C defined:

$$\Delta Q = C \Delta T$$

Heat capacities per molecule for ideal gasses:

$$C_V = \frac{3}{2} k \quad \text{monatomic gas}$$

$$C_V \geq \frac{3}{2} k \quad \text{other gases}$$

$$C_p = C_V + k$$

Rate of thermal energy transfer:

$$\frac{dQ}{dt} = \sigma A \frac{T_H - T_L}{L}$$

Efficiency of a heat engine:

$$\text{efficiency} = \frac{|W|}{Q_H} = 1 - \frac{T_L}{T_H}$$

Quality factor of a heat pump:

$$K = \frac{|Q_C|}{|W|}$$

