Equation Sheet Physics 141

The momentum principle:

$$d\vec{\mathbf{p}} = \vec{\mathbf{F}}_{nat}dt$$

$$\vec{\mathbf{p}} = \frac{m\vec{\mathbf{v}}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\vec{\mathbf{p}}_{new} = \vec{\mathbf{p}}_{old} + \vec{\mathbf{F}}_{net} \Delta t$$

$$\vec{\mathbf{r}}_{new} = \vec{\mathbf{r}}_{old} + \frac{1}{\sqrt{1 + \left(\frac{p}{mc}\right)^2}} \left(\frac{\vec{\mathbf{p}}}{m}\right) \Delta t$$

Equations of motion in 1D for constant acceleration and low velocities ( $v \ll c$ ):

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v(t) = \frac{dx(t)}{dt} = v_0 + at$$

$$a(t) = \frac{dv(t)}{dt} = a = \text{constant}$$

Requirement for uniform circular motion:

$$F_r = \frac{mv^2}{r}$$

Rotational motion:

$$d = \theta r$$

$$v = \omega r$$
  $\omega = \frac{d\theta}{dt}$ 

$$a = \alpha r$$
  $\alpha = \frac{d\omega}{dt}$ 

Gravitational force:

$$\vec{\mathbf{F}} = G \frac{m_1 m_2}{r^2} \hat{\mathbf{r}}$$

 $\vec{\mathbf{F}} = m\vec{\mathbf{g}}$  (close to the surface of the Earth)

Electrostatic force:

$$\vec{\mathbf{F}} = -\frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \,\hat{\mathbf{r}}$$

Harmonic motion:

$$F = -kx$$

$$x(t) = x_{\text{max}} \cos(\omega t + \phi)$$
 where  $\omega = \sqrt{\frac{k}{m}}$ 

$$T = \frac{2\pi}{\omega}$$

Damped harmonic motion:

$$x(t) = x_m e^{-\frac{bt}{2m}} e^{it\sqrt{\frac{k}{m}}}$$

Driven harmonic motion:

$$x(t) = \frac{F_0}{\omega_0^2 - \omega^2} \cos(\omega t + \phi)$$

Stress and strain:

$$\frac{F}{A} = Y \frac{\Delta L}{L}$$

$$Y = \frac{k_s}{d}$$

Work done by a force:

$$W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}}$$
 constant force

$$= \int_{0}^{\vec{r}_{2}} \vec{F} \cdot d\vec{r} \quad \text{variable force}$$

Power:

$$P = \frac{dW}{dt} = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$$

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Work-energy theorem:

$$\Delta E_{system} = W$$

$$E_{system} = (E_1 + E_2 + E_3 + .....) + U$$

Relativistic energy relations:

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} = mc^2 + K$$

$$E^2 - \left(pc\right)^2 = \left(mc^2\right)^2$$

$$K = \frac{mc^{2}}{\sqrt{1 - \frac{v^{2}}{c^{2}}}} - mc^{2} \xrightarrow[v \ll c]{} \frac{1}{2} mv^{2}$$

Potential energy:

$$\Delta U = -W_{\text{internal}} = -\int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$$

$$\vec{\mathbf{F}} = -\vec{\nabla}U = \begin{pmatrix} -\frac{\partial U}{\partial x} \\ -\frac{\partial U}{\partial y} \\ -\frac{\partial U}{\partial z} \end{pmatrix}$$

$$U_{gravity} = -G \frac{m_1 m_2}{r}$$

$$U_{gravity} = mgh$$

$$U_{\rm electric} = \frac{1}{4\pi\varepsilon_0} \frac{q_1q_2}{r}$$

$$U_{spring} = \frac{1}{2}kx^2$$

Heat capacity:

$$\Delta E_{thermal} = mC\Delta T$$

Friction forces:

$$f_{s} \leq \mu_{s} N$$

$$f_k = \mu_k N$$

Drag force (air):

$$\vec{\mathbf{F}}_{air} = -\frac{1}{2} C \rho A v^2 \hat{\mathbf{v}}$$

Energy levels for the Hydrogen atom:

$$E_N = \frac{-13.6}{N^2}$$
 eV,  $N = 1, 2, 3, ...$ 

Vibrational energy levels:

$$E_N = E_0 + N\hbar\omega_0 = E_0 + N\hbar\sqrt{\frac{k_s}{m}}, N = 0, 1, 2, ...$$

Energy and wavelength of light:

$$E_{photon} = \frac{hc}{\lambda_{light}}$$

Center of mass:

$$\vec{\mathbf{r}}_{cm} = \frac{\sum_{i} m_{i} \vec{\mathbf{r}}_{i}}{\sum_{i} m_{i}} = \frac{1}{M} \sum_{i} m_{i} \vec{\mathbf{r}}_{i}$$

$$\vec{\mathbf{r}}_{cm} = \frac{\int \vec{\mathbf{r}} dm}{\int dm} = \frac{1}{M} \int \vec{\mathbf{r}} dm$$

Motion of the center of mass:

$$M\vec{a}_{cm} = \vec{F}_{net.ext}$$

Gravitational potential energy of a multi-particle system:

$$U = Mgy_{cr}$$

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Kinetic energy of a multi-particle system:

$$K = K_{trans} + K_{rel} = \frac{1}{2} M v_{cm}^{2} + K_{rel}$$

Impulse of a force:

$$\vec{\mathbf{J}} = \int \vec{\mathbf{F}} dt$$

Momentum and impulse:

$$\vec{\mathbf{J}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i$$

Conservation of linear momentum:

$$\Delta \vec{\mathbf{P}}_{system} + \Delta \vec{\mathbf{P}}_{surroundings} = 0$$

Elastic collision in one dimension (mass 2 at rest before the collision):

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_{_1}}{m_{_1} + m_{_2}} v_{_{1i}}$$

Completely inelastic collision in one dimension (mass 2 at rest before the collision):

$$v_f = \frac{m_1}{m_1 + m_2} v_i$$