

Physics 141.  
Review Exam # 3.



**Fuel efficient aviation.**  
(see <http://www.treehugger.com/files/2009/02/fuel-efficient-aviation.php>)

Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester

1

---

---

---

---

---

---

---

---

Before we start out review .....

- Do not forget the deadlines associated with Lab # 5.
- Due today:
  - Complete the discussion and comparison of the results with your colliding partner and submit final results (velocities and errors).
  - If you do not complete the video analysis and the comparison of the results, and if you do not submit the final results of your analysis, your grade on lab report # 5 will be reduced.

Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester

2

---

---

---

---

---

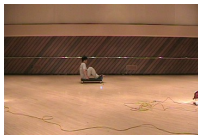
---

---

---

Analysis of experiment # 5.  
Updated Timeline.

- ✓ 11/14: collisions in the May room
- ✓ 11/20: analysis files available.
  - <https://www.pas.rochester.edu/~tdimino/phy141/lab05/>
- ✓ 11/20: each student has determined his/her best estimate of the velocities before and after the collisions (analysis during regular lab periods).
- 11/21: complete discussion and comparison of results with colliding partners and submit final results (velocities and errors).
- 11/25: results will be compiled, linear momenta and kinetic energies will be determined, and results will be distributed.
- 12/4: office hours by lab TA/TIs to help with analysis and conclusions.
- 12/6: students submit lab report # 5.



Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester

3

---

---

---

---

---

---

---

---

### Physics 141 Exam # 3.

- Exam # 3 will take place on Tuesday 11/28 between 8 am and 9.20 am in Hoyt.
- Exam # 3 will cover the material of Chapters 8, 9, 10, and 11.
- General remarks:
  - You must answer question 11 and 12 in one blue booklet.
  - You must answer question 13 in the second blue booklet.
  - **If you answer all analytical questions in one blue booklet, question 13 will not be graded.**
  - You must enter your student ID in the appropriate place on the scantron form. **No ID on the form, no grade for the multiple-choice questions.**
  - If you arrive after 8.45 am, you will not be allowed to take the exam.
  - You cannot leave the exam before 8.45 am.

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

4

---

---

---

---

---

---

---

---

### Extra office hours on Monday 11/27

- There will be extra office hours on 11/27 to answer any last-minute questions about exam # 3.
  - Frank Wolfs: 11 am – 1 pm, using Zoom:  
(<https://rochester.zoom.us/j/91686690889?pwd=MEkvUXUvR0VZc0g0aIN5K1MvRFBEZz09>).
  - Guilherme Fiusa: 3 pm - 5 pm, B&L 304.
  - Aidan Bachmann: 10 am - 11 am, POA.
  - Nathan Skerrett: 5.30 pm – 6.30 pm, POA.
- There will be no recitations and office hours on Tuesday, Wednesday, Thursday, and Friday during the week of the exam.

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

5

---

---

---

---

---

---

---

---

### Surviving Phy 141 Exams.

- Time your work:
  - Exam has 10 MC + 3 analytical questions.
  - Work 15 minutes on the MC questions.
  - Work 15 minutes on each of the analytical questions (45 minutes total).
  - You now have 30 minutes left to finish those questions you did not finish in the first 15 minutes.
- Write neatly – you cannot earn credit if we cannot read what you wrote!
- Write enough so that we can see your line of thought – you cannot earn credit for what you are thinking!

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

6

---

---

---

---

---

---

---

---

### Surviving Phy 141 Exams.

- Every problem should start with a diagram, showing all forces (direction and approximate magnitude) and dimensions. All forces and dimensions should be labeled with the variables that will be used in your solution.
- Indicate what variables are known and what variables are unknown.
- Indicate which variable needs to be determined.
- Indicate the principle(s) that you use to solve the problem.
- If you make any approximations, indicate them.
- Check your units!

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

7

---

---

---

---

---

---

---

---

### Review exam # 3. Chapter 8.

- In Chapter 8 quantization of energy is discussed.
- Quantization of energy leads to well defined emission and absorption patterns in atomic spectra.
- Quantization of energy is observed both at the nuclear and at the atomic level.
- Sections not included: none (sorry).

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

8

---

---

---

---

---

---

---

---

### Review exam # 3. Chapter 8.

- Terminology introduced :
  - Energy quantization.
  - Emission and absorption spectra.
  - Temperature dependence of emission and absorption spectra.
  - Vibrational and rotational spectra.

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

9

---

---

---

---

---

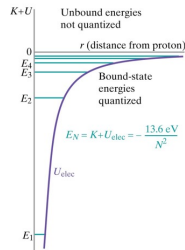
---

---

---

## Review chapter 8. Energy quantization in the hydrogen atom.

- Experiments examining the details of atomic structure showed that the electrons can only occupy certain specific energy levels.
- The measurements show that the total energy of the electron,  $K + U$ , is equal to  $-13.6/N^2$  eV where  $N$  is an integer ( $N = 1, 2, \dots$ ).
- The quantization of the energy levels is a result of quantum-mechanical effects (to be discussed in more detail in Physics 143). Also see chapter 11.



Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester

10

---

---

---

---

---

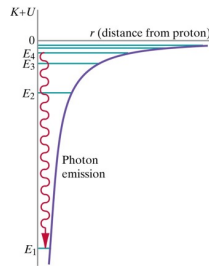
---

---

---

## Review chapter 8. Energy quantization: emission patterns.

- Light is emitted when an excited atom makes a transition to a lower energy level.
- Since the energy level of atoms are quantized, the light emitted will have discrete wavelengths (energies).
- The energy levels serve as a signature (finger print) for the atom, and the emission pattern can be used to identify the atom.



Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester

11

---

---

---

---

---

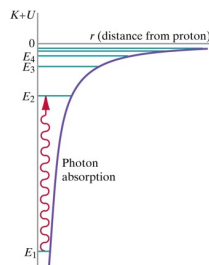
---

---

---

## Review chapter 8. Energy quantization: absorption patterns.

- When an atom is in its ground state, it can only absorb photons of specific frequencies.
- Only photons with an energy that exactly match possible transitions between energy levels in the atom are absorbed by the atom.
- The absorption spectrum can also be used as a signature of the atoms.



Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester

12

---

---

---

---

---

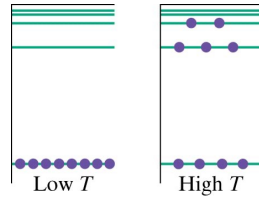
---

---

---

## Review chapter 8. Temperature dependence of absorption.

- The absorption pattern depends on the population of states in the sample.
- Atoms with excited states populated are able to absorb lower-energy light (longer wavelength) since the energy spacing between atomic levels decreases with increasing energy.
- Since the population patterns depends on temperature, we expect to see a temperature dependence of the absorption spectrum.



Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

13

---

---

---

---

---

---

---

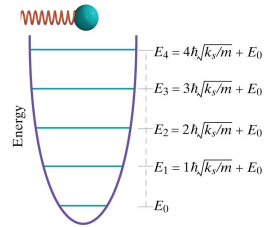
---

---

---

## Review chapter 8. The vibrational model.

- The effect of the quantum treatment of the "spring" (the vibrational model) is that the energy of the system is quantized.
- It turns out that in many cases a potential well consistent with the spring force is consistent with the observed atomic properties.
- The energy levels in this well are quantized and the spacing between the levels is  $(h/2\pi)\sqrt{k_s/m}$ .
- One important consequence of this quantization is that atoms can only transfer energy to each other in discrete amounts.



Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

14

---

---

---

---

---

---

---

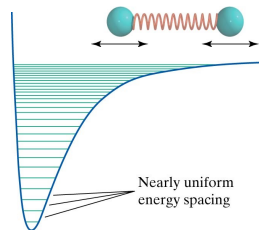
---

---

---

## Review chapter 8. The vibrational model.

- When we measure the vibrational energy levels for a two-atomic molecule we find that at low energies the vibrational model works fine (**nearly uniform energy spacing**).
- At higher energies, the potential well starts to deviate from a harmonic oscillator well, and the vibrational energy levels are no longer uniformly spaced.



Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

15

---

---

---

---

---

---

---

---

---

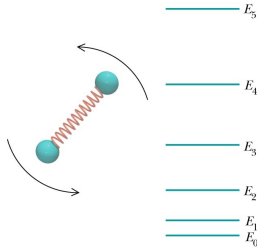
---

### Review chapter 8. The rotational model.

- Molecules can also carry rotational energy.
- The rotational energy of a molecule is also quantized, but the spacing between levels increases with increasing excitation energy.
- The rotational energy is found to be equal to

$$E_l = \frac{1}{2I} l(l+1)h^2$$

where  $l$  is an integer (0, 1, ... )  
and  $I$  is the moment of inertia  
(depends on mass and shape).



Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester

16

---

---

---

---

---

---

---

---

---

---

### Review chapter 8. Different systems, different energies.

State	Energy level spacing (eV)
Hadronic	100,000,000
Nuclear	1,000,000
Atomic	1
Molecular (vibrational)	0.01
Molecular (rotational)	0.0001

Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester

17

---

---

---

---

---

---

---

---

---

---

### Problem 8.P14.

**••P14** Hydrogen atoms: **(a)** What is the minimum kinetic energy in electron volts that an electron must have to be able to ionize a hydrogen atom that is in its ground state (that is, remove the electron from being bound to the proton)? **(b)** If electrons of energy 12.8 eV are incident on a gas of hydrogen atoms in their ground state, what are the energies of the photons that can be emitted by the excited gas? **(c)** If instead of electrons, photons of all energies between 0 and 12.8 eV are incident on a gas of hydrogen atoms in the ground state, what are the energies at which the photons are absorbed?

Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester

18

---

---

---

---

---

---

---

---

---

---

Review chapter 8.

---

End of Chapter 8.

Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester

19

---

---

---

---

---

---

---

---

Review exam # 3.  
Chapter 9.

---

- In this Chapter, the motion of multi-particle systems is discussed.
- It is found to be useful to describe the motion of a multi-particle systems in terms of the motion of its center of mass.
- The potential and the kinetic energy of a multi-particle system is discussed.
- Excluded sections: 9.5 and 9.6 (but you are expected to understand the outcome of the derivations discussed in these sections).

Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester

20

---

---

---

---

---

---

---

---

Review exam # 3.  
Chapter 9.

---

- Terminology introduced :
  - The center of mass.
  - The momentum principle for multi-particle systems.
  - The kinetic energy for multi-particle systems:
    - Translational kinetic energy.
    - Rotational kinetic energy.
    - Vibrational kinetic energy.

Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester

21

---

---

---

---

---

---

---

---

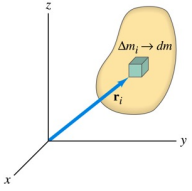
### Review chapter 9. The position of the center of mass.

---

- The **position of the center of mass** is defined as

$$\vec{r}_{cm} = \frac{\sum_i \vec{r}_i m_i}{\sum_i m_i}$$

- If we are not dealing with discrete point masses we need to replace the sum with an integral.

$$\vec{r}_{cm} = \frac{1}{M} \int_V \vec{r} dm$$


Frank L. H. Wolfs      Department of Physics and Astronomy, University of Rochester

---

---

---

---

---

---

---

---

22

### Review chapter 9. Motion of the center of mass.

---

To examine the **motion of the center of mass** we start with its position and then determine its velocity and acceleration:

$$M\vec{r}_{cm} = \sum_i m_i \vec{r}_i$$

$$M\vec{v}_{cm} = \sum_i m_i \vec{v}_i$$

$$M\vec{a}_{cm} = \sum_i m_i \vec{a}_i$$

Note: we have assumed that the system can be treated non-relativistically.

Frank L. H. Wolfs      Department of Physics and Astronomy, University of Rochester

---

---

---

---

---

---

---

---

23

### Review chapter 9. Motion of the center of mass.

---

- The expression for  $M\vec{a}_{cm}$  can be rewritten in terms of the forces on the individual components:

$$M\vec{a}_{cm} = \frac{d}{dt}(M\vec{v}_{cm}) = \frac{d\vec{P}_{cm}}{dt} = \sum_i \vec{F}_i = \vec{F}_{net, ext}$$

- We conclude that **the motion of the center of mass is only determined by the external forces**. Forces exerted by one part of the system on other parts of the system are called internal forces. According to Newton's third law, the sum of all internal forces cancel out (for each interaction there are two forces acting on two parts: they are equal in magnitude but pointing in an opposite direction and cancel if we take the vector sum of all internal forces).

Frank L. H. Wolfs      Department of Physics and Astronomy, University of Rochester

---

---

---

---

---

---

---

---

24



### Review chapter 9. Motion of the center of mass.

- Now consider the special case where there are no external forces acting on the system:

$$\frac{d\vec{P}_{tot}}{dt} = 0$$

- This equations tells us that the total linear momentum of the system is constant.
- In the case of an extended object, we find the total linear momentum by adding the linear momenta of all of its components:

$$\vec{P}_{tot} = M\vec{v}_{cm} = \sum_i m_i \vec{v}_i = \sum_i \vec{p}_i$$

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

25

---

---

---

---

---

---

---

---

### Review chapter 9. Energy of a multi-particle system.

- In order to determine the (mechanical) energy of a multiple-particle system we need to determine both its kinetic energy and its potential energy:

- The kinetic energy of the system will be the sum of the kinetic energy of the center-of-mass and the kinetic energy of the motion of the particles with respect to the center of mass.
- The potential energy of the system may or may not depend on the position of the center of mass:
  - The gravitational potential energy can be expressed in terms of the position of the center of mass.
  - The electrostatic potential energy depends on the position of charges, not on the position of mass, and does not depend on the position of the center of mass.

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

26

---

---

---

---

---

---

---

---

### Review chapter 9. Kinetic energy of a multi-particle system.

- The kinetic energy of a multiple-particle system will have two components:

$$K = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} \sum_i m_i v_i^2 = K_{translational} + K_{relative}$$

- The **translational component**: the kinetic energy associated with the motion of the center of mass.
- The **relative component**: the kinetic energy associated with the motion of the particles with respect to the center of mass. This type of motion can be vibrational, rotational, a combination of these two, etc.

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

27

---

---

---

---

---

---

---

---

## Review chapter 9. Potential energy of a multi-particle system.

- Consider a multi-particle system located close to the surface of the earth.
- The gravitational potential energy of this system is equal to

$$U = \sum m_i g y_i = g \left\{ \sum m_i y_i \right\} = g M y_{cm}$$

- The gravitational potential energy thus depends on the vertical position of the center of mass of the system.

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

28

---

---

---

---

---

---

---

---

## Problem 9.P43.

••P43 A string is wrapped around a uniform disk of mass  $M$  and radius  $R$ . Attached to the disk are four low-mass rods of radius  $b$ , each with a small mass  $m$  at the end (Figure 9.63).

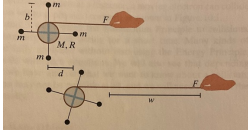


Figure 9.63

The apparatus is initially at rest on a nearly frictionless surface. Then you pull the string with a constant force  $F$ . At the instant when the center of the disk has moved a distance  $d$ , an additional length  $w$  of string has unwound off the disk. (a) At this instant, what is the speed of the center of the apparatus? Explain your approach. (b) At this instant, what is the angular speed of the apparatus? Explain your approach.

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

29

---

---

---

---

---

---

---

---

## Review chapter 9.

End of Chapter 9.

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

30

---

---

---

---

---

---

---

---

### Review exam # 3. Chapter 10.

- In this Chapter the momentum principle is used to establish a connection between the collision force and the change in the linear momentum of the collision partners.
- If the collision force is the only force acting on the collision partners, then the total linear momentum (the sum of the momenta of the partners) will be conserved.
- Excluded sections: none (sorry).

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

31

---

---

---

---

---

---

---

---

### Review exam # 3. Chapter 10.

- Terminology introduced:
  - The collision force.
  - Conservation of linear momentum.
  - Elastic and inelastic collisions.
  - Motion of the center of mass.
  - Scattering experiments (Rutherford scattering).

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

32

---

---

---

---

---

---

---

---

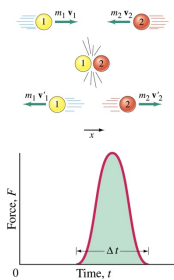
### Review chapter 10. The collision force.

- During a collision, a strong force is exerted on the colliding objects for a short period of time.
- The result of the **collision force** is a change in the linear momentum of the colliding objects:

$$\vec{p}_f - \vec{p}_i = \int_{\vec{p}_i}^{\vec{p}_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t) dt$$

- The integral of the force is called the collision impulse  $J$ :

$$\vec{J}_i = \int_{\vec{p}_i}^{\vec{p}_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t) dt$$



Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

33

---

---

---

---

---

---

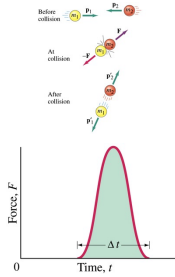
---

---

## Review chapter 10. Elastic and inelastic collisions.

• If we consider both colliding object, then the collision force becomes an internal force and the total linear momentum of the system must be conserved if there are no external forces acting on the system.

- Collisions are usually divided into two groups:
  - **Elastic collisions:** kinetic energy is conserved.
  - **Inelastic collisions:** kinetic energy is NOT conserved.



Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester

34

---

---

---

---

---

---

---

---

## Review chapter 10. Elastic collisions in one dimension.

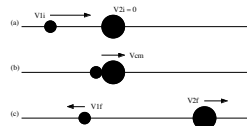
• The final state of elastic collisions in one dimension is completely defined if we know the initial conditions.

• The final velocity of mass  $m_1$  is:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

• The final velocity of mass  $m_2$  is:

$$v_{2f} = \frac{m_1}{m_2} (v_{1i} - v_{1f}) = \frac{2m_1}{m_1 + m_2} v_{1i}$$



Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester

35

---

---

---

---

---

---

---

---

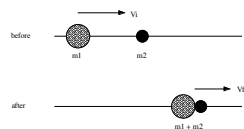
## Review chapter 10. Inelastic collisions in one dimension.

• In inelastic collisions, kinetic energy is not conserved.

• A special type of inelastic collisions are the completely inelastic collisions, where the two objects stick together after the collision.

• Conservation of linear momentum in a completely inelastic collision requires that

$$m_1 v_i = (m_1 + m_2) v_f$$



Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester

36

---

---

---

---

---

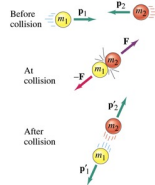
---

---

---

## Review chapter 10. Collisions in two or three dimensions.

- Collisions in two or three dimensions are approached in the same way as collisions in one dimension.
- The x, y, and z components of the linear momentum must be conserved if there are no external forces acting on the system.
- The collisions can be elastic or inelastic.



Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

37

---

---

---

---

---

---

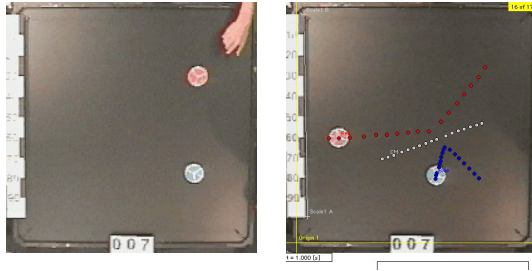
---

---

---

---

## Review chapter 10. Motion of the center of mass.



Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

38

---

---

---

---

---

---

---

---

---

---

## Problem 10.P.30

**10.P.37** At the PEP II facility at the Stanford Linear Accelerator Center (SLAC) in California and at the KEKB facility in Japan, electrons with momentum 9.03 GeV/c were made to collide head-on with positrons whose momentum is 3.10 GeV/c ( $1 \text{ GeV} = 10^9 \text{ eV}$ ; see Figure 10.42. That is,  $pc$  for the electron is 9.03 GeV and  $pc$  for the positron is 3.10 GeV. The values of  $pc$  and the corresponding energies are so large with respect to the electron or positron rest energy ( $0.5 \text{ MeV} = 0.0005 \text{ GeV}$ ) that for the purposes of this analysis you may, if you wish, safely consider the electron and positron to be massless.

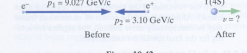


Figure 10.42

- (a) The electron-positron collision produces in an intermediate state a particle called the  $\Upsilon(4S)$  ("Upsilon 4S"), in the reaction  $e^- + e^+ \rightarrow \Upsilon(4S)$ . Show that the rest energy of the  $\Upsilon(4S)$  is 10.58 GeV.
- (b) What is the speed of the  $\Upsilon(4S)$  produced in the collision?

(c) The  $\Upsilon(4S)$  decays almost immediately into two "B" mesons:  $\Upsilon(4S) \rightarrow B^+ + B^-$ . The  $B^+$  is the antiparticle of the  $B^0$  and has the same rest energy:  $Mc^2 = 5.28 \text{ GeV}$  as the  $B^0$ . Consider the case in which both "B" mesons are emitted at the same angle  $\theta$  to the direction of the moving  $\Upsilon(4S)$ , as shown in Figure 10.43. Calculate this angle  $\theta$ .

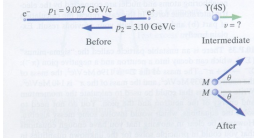


Figure 10.43

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

39

---

---

---

---

---

---

---

---

---

---

### Review Midterm Exam # 3. Chapter 11.

- The focus of this Chapter is rotational motion and angular momentum.
- Rotational motion and angular momentum is described in terms of angular variables, such as angular position, velocity, and acceleration, and torque.
- There is a great deal of symmetry between the way we use linear variables and the way we use of angular variables.
- We discussed the requirements of conservation of angular momentum (no external torques)
- We also discussed the concept and consequences of the quantization of angular momentum.
- Sections excluded: 11.12 (page 453 – 455), and 11.13.

Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester

40

---

---

---

---

---

---

---

---

### Review Midterm Exam # 3. Chapter 11.

- Terminology introduced:
  - Angular position, velocity, and acceleration.
  - Rotation axis.
  - Moment of inertia.
  - Rolling motion.
  - Torque.
  - Angular momentum.

Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester

41

---

---

---

---

---

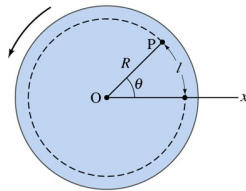
---

---

---

### Review Chapter 11. Rotational variables.

- The variables that are used to describe rotational motion are:
  - Angular position  $\theta$
  - Angular velocity  $\omega = d\theta/dt$
  - Angular acceleration  $\alpha = d\omega/dt$
- The rotational variables are related to the linear variables:
  - Linear position  $l = R\theta$
  - Linear velocity  $v = R\omega$
  - Linear acceleration  $a = R\alpha$



Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester

42

---

---

---

---

---

---

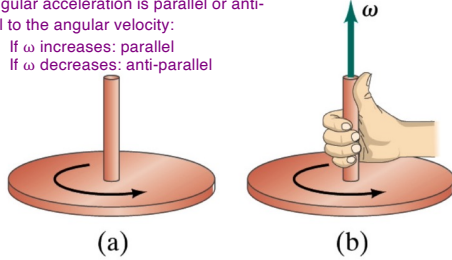
---

---

## Review Chapter 11. Rotational variables.

Angular velocity and acceleration are vectors! They have a magnitude and a direction. The direction of  $\omega$  is found using the right-hand rule. The angular acceleration is parallel or anti-parallel to the angular velocity:

- If  $\omega$  increases: parallel
- If  $\omega$  decreases: anti-parallel



Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester

43

---

---

---

---

---

---

---

---

## Review Chapter 11. The moment of inertia.

- The kinetic energy of a rotation body is equal to

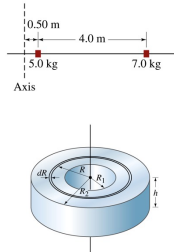
$$K = \frac{1}{2} I \omega^2$$

where  $I$  is the moment of inertia which is defined (for discrete mass distributions) as

$$I = \sum_i m_i r_i^2$$

- For continuous mass distributions

$$I \text{ is defined as } I = \int r^2 dm$$



Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester

44

---

---

---

---

---

---

---

---

## Review Chapter 11. Moments of inertia.

- As part of the exam you will receive a table of moments of inertia for various objects (see Figure on the right).
- There will be no analytical questions that require you to calculate the moment of inertia for non-uniform objects (like you had on WeBWork).
- But .... You need to know who to determine the moment of inertia using the parallel-axis theorem.

Moments of inertia of various objects of uniform composition.		
(1) Thin hoop of radius $R_0$	Through center	$\frac{1}{2} MR_0^2$
(2) Thin hoop of radius $R_0$ and width $w$	Through center	$\frac{1}{2} MR_0^2 + \frac{1}{12} Mw^2$
(3) Solid cylinder of radius $R_0$	Through center	$\frac{1}{2} MR_0^2$
(4) Hollow cylinder of inner radius $R_1$ and outer radius $R_2$	Through center	$\frac{1}{2} M(R_1^2 + R_2^2)$
(5) Uniform sphere of radius $R_0$	Through center	$\frac{2}{5} MR_0^2$
(6) Long uniform rod of length $L$	Through center	$\frac{1}{12} ML^2$
(7) Long uniform rod of length $L$	Through end	$\frac{1}{3} ML^2$
(8) Rectangular thin plate of length $L$ and width $w$	Through center	$\frac{1}{12} M(L^2 + w^2)$

Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester

45

---

---

---

---

---

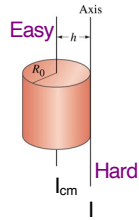
---

---

---

## Review Chapter 11. Parallel-axis theorem.

- Calculating the moment of inertia with respect to a symmetry axis of the object is in general easy.
- It is much harder to calculate the moment of inertia with respect to an axis that is not a symmetry axis.
- However, we can make a hard problem easier by using the parallel-axis theorem:



$$I = I_{cm} + Mh^2$$

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

46

---

---

---

---

---

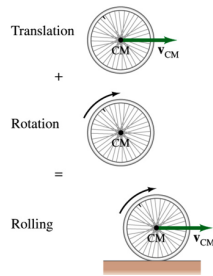
---

---

---

## Review Chapter 11. Rolling motion.

- Rolling motion is a combination of translational and rotational motion.
- The kinetic energy of rolling motion has thus two contributions:
  - Translational kinetic energy =  $(1/2) M v_{CM}^2$ .
  - Rotational kinetic energy =  $(1/2) I_{cm} \omega^2$ .
- We assume that the wheel does not slip:  $\omega = v / R$ .



Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

47

---

---

---

---

---

---

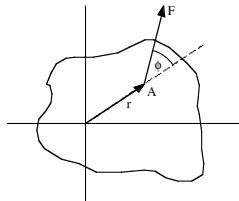
---

---

## Review Chapter 11. Torque.

- The torque  $\tau$  of the force  $F$  is proportional to the angular acceleration of the rigid body:
 
$$\tau = I\alpha$$
- This equation looks similar to Newton's second law for linear motion:
 
$$F = ma$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$



- Note:
 

<u>linear motion</u>	<u>rotational motion</u>
mass $m$	moment $I$
force $F$	torque $\tau$

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

48

---

---

---

---

---

---

---

---

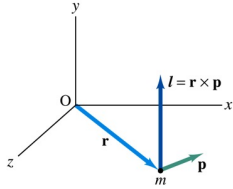


## Review Chapter 11. Angular momentum.

- The angular momentum is defined as the vector product between the position vector and the linear momentum.

Note:

- Compare this definition with the definition of the torque.
- Angular momentum is a vector.
- The unit of angular momentum is  $\text{kg m}^2/\text{s}$ .
- The angular momentum depends on both the magnitude and the direction of the position and linear momentum vectors.
- Under certain circumstances the angular momentum of a system is conserved!



Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

49

---

---

---

---

---

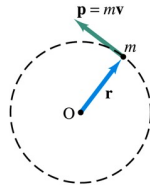
---

---

---

## Review Chapter 11. Angular momentum and circular motion.

- Consider an object carrying out circular motion.
- For this type of motion, the position vector will be perpendicular to the momentum vector.
- The magnitude of the angular momentum is equal to the product of the magnitude of the radius  $r$  and the linear momentum  $p$ :



$$L = mvr = mr^2(v/r) = I\omega$$

- Note: compare this with  $p = mv$ !

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

50

---

---

---

---

---

---

---

---

## Review Chapter 11. Conservation of angular momentum.

- Consider the change in the angular momentum of a particle:

$$\begin{aligned} \frac{d\vec{L}}{dt} &= \frac{d}{dt}(\vec{r} \times \vec{p}) = m \left( \vec{r} \times \frac{d\vec{v}}{dt} + \frac{d\vec{r}}{dt} \times \vec{v} \right) = m(\vec{r} \times \vec{a} + \vec{v} \times \vec{v}) = \\ &= \vec{r} \times m\vec{a} = \vec{r} \times \sum \vec{F} = \sum \vec{\tau} \end{aligned}$$

- When the net torque is equal to 0 Nm:

$$\sum \vec{\tau} = 0 = \frac{d\vec{L}}{dt} \Rightarrow \vec{L} = \text{constant}$$

- When we take the sum of all torques, the torques due to the internal forces cancel and the sum is equal to torque due to all external forces.

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

51

---

---

---

---

---

---

---

---

## Review Chapter 11. Quantization of angular momentum.

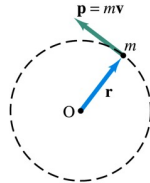
- Consider the "classical" picture of the motion of electrons in atoms.
- The angular momentum is a integer multiple of  $h/2\pi$ , the orbit must be such that

$$rp = N \frac{h}{2\pi} = N\hbar$$

- This leads to a quantization of the orbital radius and energy:

$$r = 4\pi\epsilon_0 \frac{N^2 \hbar^2}{me^2}$$

$$E = K + U = -\frac{1}{2} \left( \frac{1}{4\pi\epsilon_0} \right)^2 \frac{me^4}{N^2 \hbar^2} = -\frac{13.6}{N^2} \text{ eV}$$



Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

52

---

---

---

---

---

---

---

---

## Review Chapter 11. Quantization of angular momentum.

- The energy levels of an electron in the Hydrogen atom exactly match the levels predicted using this simple model, and the quantization of the energy levels is a direct consequence of the quantization of angular momentum.
- In addition to the orbital angular momentum of the electrons in the atom, they also possess spin. The projection of the spin of the electron on a particular axis will be either  $+(1/2)\hbar/2\pi$  or  $-(1/2)\hbar/2\pi$ . It will **never** be zero. The electron is said to have a spin 1/2 particle.
- Many other particles, such as muons, neutrinos, and quarks, are spin 1/2 particles.

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

53

---

---

---

---

---

---

---

---

## Review Chapter 11. Quantization of angular momentum.

- Since quarks are the building blocks of hadrons, we also expect that hadrons have a well defined spin.
  - Hadrons that contain three quarks can either be spin 1/2 or spin 3/2.
  - Hadrons that contain two quarks can either be spin 0 or spin 1.
- The total spin of a particle limits how particles can be distributed across the various energy levels of the system.
  - If the spin is a half integer, the particle is called a **Fermion**, and it must obey the Pauli exclusion principle (two fermions can not be in the exact same quantum state).
  - If the spin is an integer, the particle is called a **Boson**, and it is not subject to the Pauli exclusion principle (there is not limit to the number of Bosons that can be in the exact same quantum state).
- The spin of macroscopic objects will also be quantized, but the difference between different spin states is so small that it is impossible to observed effects of this quantization.

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

54

---

---

---

---

---

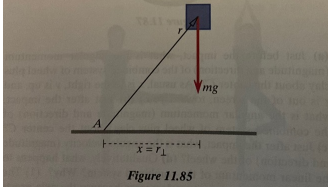
---

---

---

### Example Problem: Problem 11.P33.

••P33 Let's compare the Momentum Principle and the Angular Momentum Principle in a simple situation. Consider a mass  $m$  falling near the Earth (Figure 11.85). Neglecting air resistance, the Momentum Principle gives  $dp_y/dt = -mg$ , yielding  $dv_y/dt = -g$  (nonrelativistic). Choose a location  $A$  off to the side, on the ground. Apply the Angular Momentum Principle to find an algebraic expression for the rate of change of angular momentum of the mass about location  $A$ .



Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

55

---

---

---

---

---

---

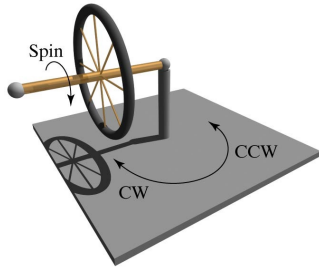
---

---

---

---

### Example Problem: Problem 11.P84.



Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

56

---

---

---

---

---

---

---

---

---

---

### Review Midterm Exam # 3. Equilibrium.

- This topics focuses on the conditions for equilibrium.
- The conditions for equilibrium are:
  - First condition: net force = 0 N
  - Second condition: net torque = 0 Nm
- Both conditions must be satisfied for the object to be equilibrium.
- Static equilibrium:
  - The conditions for equilibrium are met.
  - $P = 0 \text{ kg m/s}$
  - $L = 0 \text{ kg m}^2/\text{s}$

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

57

---

---

---

---

---

---

---

---

---

---

## Review Midterm Exam # 3. Equilibrium.

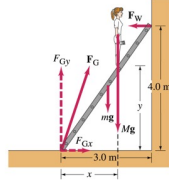
- Equilibrium in 3D:

With respect to every reference point!

$$\begin{aligned} \sum F_x &= 0 & \sum \tau_x &= 0 \\ \sum F_y &= 0 & \text{and } \sum \tau_y &= 0 \\ \sum F_z &= 0 & \sum \tau_z &= 0 \end{aligned}$$

- Equilibrium in 2D:

$$\begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum \tau_z &= 0 \end{aligned}$$



Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

58

---

---

---

---

---

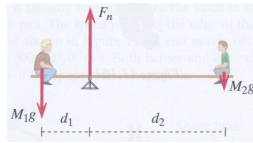
---

---

---

## Example Problem Problem 11.P56

- Calculate the torque due to the gravitational force of each person.
- Can you determine the normal force  $F_N$ ?
- What is the net torque with respect to the pivot point?
  - Rotate clockwise?
  - Rotate counter clockwise?
  - No rotate?
- When will the seesaw
  - Rotate clockwise?
  - Rotate counter clockwise?
  - No rotate?



Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

59

---

---

---

---

---

---

---

---

## Study tips.

- Review the homework assignments related to this material and look at the solutions that are posted on the WEB.
- Review the end-of-chapter problems, especially those for which you have received the solutions. However, make sure you read **all** other problems and determine if you know what approach to take to solve them.
- Use the practice exams to determine how well prepared you are for the exam. Note: problems for the material covered on Exam # 3 are:
  - Exam 2: Problems 5, 6, 7, 8, 9, 10, 11, and 12
  - Exam 3: Problems 3, 5, 9, and 13.

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester

60

---

---

---

---

---

---

---

---

Good luck preparing for exam # 3.

Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester

---

---

---

---

---

---

---