Physics 141.
Review Exam # 1.

It's all physics!
Surviving Phy 141 Exams.

- **Time your work:**
  - Exam has 10 MC + 3 analytical questions.
  - Work 15 minutes on the MC questions.
  - Work 15 minutes on each of the analytical questions (45 minutes total).
  - You now have 30 minutes left to finish those questions you did not finish in the first 15 minutes.

- **Write neatly – you cannot earn credit if we cannot read what you wrote!**

- **Write enough so that we can see your line of thought – you cannot earn credit for what you are thinking!**
Surviving Phy 141 Exams.

- Every problem should start with a diagram, showing all forces (direction and approximate magnitude) and dimensions. All forces and dimensions should be labeled with the variables that will be used in your solution.

- Indicate what variables are known and what variables are unknown.

- Indicate which variable needs to be determined.

- Indicate the principle(s) that you use to solve the problem.

- If you make any approximations, indicate them.

- Check your units!
Review Midterm Exam # 1.
Chapter 1.

• The focus of this Chapter is an introduction to the matter around us and their interactions.

• The parameters used to describe motion are introduced.

• The linear momentum of a particle is defined and the effect of relativistic velocities is described.

• We discussed how to explore the properties of interactions by looking at changes in the linear momentum of the particles being examined.

• Sections excluded: none (sorry).
Review Midterm Exam # 1.
Chapter 1.

• Terminology introduced:
  • Vectors and their use to describe motion (position, velocity, acceleration).
  • Linear momentum (relativistically correct).
  • Techniques to study the properties of interactions.
Review Chapter 1.
Linear momentum.

- The linear momentum of a particle is defined as:

\[ \vec{p} = \frac{m \vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}} \]

where \( v \) is the velocity and \( m \) is the rest-mass of the particle.

- At low velocities, \( v \ll c \), the definition of the linear momentum of the particle approaches the definition you should have seen in your high-school physics course:

\[ \vec{p} = \frac{m \vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}} \approx m \vec{v} \]
Predicting motion.

• If we know the interaction acting on our particle and the time over which this interaction is acting, we can determine the change in the linear momentum of our particle:

$$\vec{p}_{\text{new}} = \vec{p}_{\text{old}} + \Delta \vec{p}$$

• The new position of our particle can be found if we know its velocity:

For Low velocity:

$$\vec{r}_{\text{new}} = \vec{r}_{\text{old}} + \vec{v} \Delta t$$

For High velocity:

$$\vec{r}_{\text{new}} = \vec{r}_{\text{old}} + \frac{1}{\sqrt{1 + \left( \frac{p_{\text{old}}}{mc} \right)^2}} \vec{p}_{\text{old}} \Delta t$$

$$\vec{r}_{\text{new}} = \vec{r}_{\text{old}} + \frac{\vec{p}_{\text{old}}}{m} \Delta t$$
Review Chapter 1.
Probing interactions.

• **Effect of an interaction:**
  
  change in the magnitude of the linear momentum

  and/or

  change in the direction of the linear momentum.

Interaction required!
Review Midterm Exam # 1.
Chapter 2.

• The focus of this Chapter is the connection between the interactions between a system and its surroundings and the linear momentum of the system.

• We introduced the momentum principle, which relates the change in the momentum of the system to the force and the time during which this force is acting.

• We showed how the momentum principle can be used to study the time evolution of a system. We explored how to use this principle both in the relativistic limit and in the low-velocity limit.

• Sections excluded: none (sorry).
Review Midterm Exam # 1.
Chapter 2.

- Terminology introduced:
  - The momentum principle.
  - The net force.
  - The impulse of a force.
  - Equations of motion associated with constant forces.
  - Conservation of linear momentum.
Review Chapter 2.
The momentum principle.

• The change in the linear momentum of an object is proportional to the strength of the interaction and to the duration of the interaction. This principle is known as the **momentum principle**:

\[ \Delta \vec{p} = \vec{F}_{\text{net}} \Delta t \]

• This equation allows us to calculate the time-dependence of the linear momentum if we know the initial value and the time/position dependence of the interaction.
Review Chapter 2.
Quantifying the extent of an interaction.

• If we do not know the interaction, but we measure the change in the linear momentum we can determine extent of the interaction:

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$$

• In the non-relativistic limit this relation becomes

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \approx m \frac{d\vec{v}}{dt} = m\ddot{a}$$

• If the net force acting on a system is zero, the change in its linear momentum will be zero, and linear momentum will be conserved.
Review Chapter 2. Linear motion in one dimension.

Parameters define initial conditions!

\[ x(t) = \int_{t_0}^{t} v(t') \, dt' \quad x(t) = x_0 + v_0 t + \frac{1}{2} a t^2 \]

\[ v(t) = \frac{dx}{dt} = \int_{t_0}^{t} a(t') \, dt' \quad v(t) = v_0 + at \]

\[ a(t) = \frac{dv}{dt} = \frac{d^2 x}{dt^2} \quad a(t) = a = \text{constant} \]

The same for different observers!
Review Chapter 2.
Motion in three dimensions: constant $a$.

$$ \mathbf{r}(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} \qquad \mathbf{v}(t) = \begin{pmatrix} v_x(t) \\ v_y(t) \\ v_z(t) \end{pmatrix} \qquad \mathbf{a}(t) = \begin{pmatrix} a_x(t) \\ a_y(t) \\ a_z(t) \end{pmatrix} $$

where

$$ x(t) = x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \quad y(t) = y_0 + v_{0y} t + \frac{1}{2} a_y t^2 \quad z(t) = z_0 + v_{0z} t + \frac{1}{2} a_z t^2 $$

$$ v_x(t) = v_{0x} + a_x t \quad v_y(t) = v_{0y} + a_y t \quad v_z(t) = v_{0z} + a_z t $$

$$ a_x(t) = a_x = \text{constant} \quad a_y(t) = a_y = \text{constant} \quad a_z(t) = a_z = \text{constant} $$

Note: A non-zero acceleration in one direction only affects motion in that direction.
Review Chapter 2.
A special case: projectile motion in 2D.

\[ x(t) = x_0 + v_{0x} t \]
\[ y(t) = y_0 + v_{0y} t - \frac{1}{2} gt^2 \]
\[ v_x(t) = v_{0x} = \text{constant} \]
\[ v_y(t) = v_{0y} - gt \]
\[ a_x(t) = 0 \]
\[ a_y(t) = -g = \text{constant} \]

Note: The non-zero gravitational acceleration only affects motion in the vertical direction; not in the horizontal direction.
Review Chapter 2.
Understanding graphs.

• Make sure you understand what information you can obtained from graphs showing velocity or position as function of time.
  • The slope of the position vs time graph is the velocity.
  • The slope of the velocity vs time graph is the acceleration.
  • The sign of position and velocity is important.
2.P.59 A small dense ball with mass 1.5 kg is thrown with initial velocity (5, 8, 0) m/s at time $t = 0$ at a location we choose to call the origin ((0, 0, 0) m). Air resistance is negligible.

(a) When the ball reaches its maximum height, what is its velocity (a vector)? It may help to make a simple diagram.

(b) When the ball reaches its maximum height, what is $t$? You know how $v_y$ depends on $t$, and you know the initial and final velocities.

(c) Between the launch at $t = 0$ and the time when the ball reaches its maximum height, what is the average velocity (a vector)? You know how to determine average velocity when velocity changes at a constant rate.

(d) When the ball reaches its maximum height, what is its location (a vector)? You know how average velocity and displacement are related.
Review Chapter 2.
Sample problem.

(e) At a later time the ball’s height $y$ has returned to zero, which means that the average value of $v_y$ from $t = 0$ to this time is zero. At this instant, what is the time $t$?

(f) At the time calculated in part (e), when the ball’s height $y$ returns to zero, what is $x$? (This is called the “range” of the trajectory.)

(g) At the time calculated in part (e), when the ball’s height $y$ returns to zero, what is $v_y$?

(h) What was the angle to the $x$ axis of the initial velocity?

(i) What was the angle to the $x$ axis of the velocity at the time calculated in part (e), when the ball’s height $y$ returned to zero?
Review Midterm Exam # 1.
Chapter 3.

• The focus of this Chapter is the study of motion induced by an external forces.

• The primary force on which the Chapter focuses is the gravitational force. Other forces, such as the electric force and the spring force, are briefly described.

• The four fundamental interactions and their relative strengths are introduced in this Chapter.

• The different types of motion discussed in this Chapter include orbital motion and chaos.

• Sections excluded: none (sorry).
Review Midterm Exam # 1.
Chapter 3.

• Terminology introduced:
  • Newton's laws of motion.
  • The four fundamental interactions.
  • The gravitational force law.
  • The Shell theorem.
  • Mass and weight.
  • The principle of superposition.
  • Orbital motion.
Review Chapter 3.
The four fundamental interactions.

For PHY 141: Know the order of the strength of these forces.

http://particleadventure.org/particleadventure/frameless/chart.html
The gravitational force is given by the following relation:

\[ \mathbf{F}_{\text{grav}} = G \frac{m_1 m_2}{r_{12}^2} \hat{r} \]

The constant \( G \) is the gravitational constant which is measured to be \( 6.67 \times 10^{-11} \) N m\(^2\)/kg\(^2\).

Note: the gravitational force does not depend on the momentum of the particles.
Review Chapter 3.
The shell theorem.

• Consider a shell of material of mass $m_1$ and radius $R$.

• In the region outside the shell, the gravitational force on a point mass $m_2$ will be identical to what it would have been if all the mass of the shell was located at its center.

• In the region inside the shell, the gravitational force on a point mass $m_2$ is equal to $0$ N.

\[
\vec{F}_{\text{grav}} = G \frac{m_1 m_2}{r_{12}^2} \hat{r}
\]
Review Chapter 3. 
The superposition principle.

If several forces are acting on our object, we can use the **Superposition Principle** to determine the net force acting on our object:

The net force on an object is the vector sum of the individual force acting on it by other object. Each individual interaction is unaffected by the presence of other interacting objects.
Review Chapter 3.
Quantifying the extent of an interaction.

• **If our system contains multiple particles:**

  - We need to consider multiple forces acting on these particles (internal and external forces).
  
  - The results of these forces will be changes in the linear momenta of these particles.
  
  - **The total linear momentum of this system of particles will be the vector sum of the linear momentum associated with each particle.**

  - The net force is the vector sum of the force acting on each particle. Due to Newton’s third law, the internal forces cancel, and the net force is equal to the vector sum of the external forces.

  - If the sum of all the external force is equal to zero, then the net force on the system will be zero and the change in the linear momentum will be zero. In that case, **linear momentum will be conserved.**
In order for an object of mass $m$ to be in a circular orbit of radius $r$, a net force must be acting on it with a magnitude of $mv^2/r$, directed towards the center of the earth.

The only force that acts in this direction is the gravitational force.

The orbital velocity must satisfy the following relation:

$$v^2 = \frac{GM_{\text{earth}}}{r}$$

This relation can be rewritten in terms of the orbital period:

$$r^3 = \frac{GM_{\text{earth}}T^2}{4\pi^2}$$
Review Chapter 3.
Orbital motion.

• **Remarks:**

  • Orbital motion requires that the net force on each mass if directed towards the center of the orbit.

  • No mass needs to be located at the center of the orbit.

  • Calculating the net force requires vector addition of the individual forces.
3.P.40 A steel ball of mass $m$ falls from a height $h$ onto a scale calibrated in newtons. The ball rebounds repeatedly to nearly the same height $h$. The scale is sluggish in its response to the intermittent hits and displays an average force $F_{\text{avg}}$, such that $F_{\text{avg}} T = F \Delta t$, where $F \Delta t$ is the brief impulse that the ball imparts to the scale on every hit, and $T$ is the time between hits.

Calculate this average force in terms of $m$, $h$, and physical constants. Compare your result with what the scale reads if the ball merely rests on the scale. Explain your analysis carefully (but briefly).
Review Midterm Exam # 1.
Chapter 4.

• The focus of this Chapter is the atomic nature of matter and the forces that act on our system and influences its motion.

• We discussed a model of a solid in terms of a lattice of atoms that are interconnected by springs. Many dynamic properties of a solid can be understood in terms of this simple model.

• We discussed various forces and types of motion:
  • Simple-harmonic motion are discussed and the force requirements for this type of motion.

• Sections excluded: none (sorry).
Review Midterm Exam # 1.
Chapter 4.

• Terminology introduced:
  • The atomic model of a solid.
  • Tension.
  • Stress and strain.
  • Harmonic motion.
Review Midterm Exam # 1.
Chapter 4.

- We can visualize a solid as a collection of atoms of mass $m$, interconnected by springs.
- The atoms are not at rest in a solid, but continuously vibrate around an equilibrium position.
- The temperature of the solid is a measure of the kinetic energy associated with the motion of the atoms.
- This simple model can explain many important properties of matter, but many others can only be explained in terms of quantum mechanics.
Review Chapters 4.
The atomic model of a wire.

- A commonly used atomic model of a wire is a model in which the atoms are connected via springs of spring constant $k$.
- The inter-atomic separation will increase when we move up the wire.
- The assumption that the tension in the wire is constant is thus a poor approximation.
- When we connect a mass to the wire, there still will be a dependence of the spring force on position, but this dependence will be much smaller than it was before.
Review Chapters 4.
Stress and strain.

- When we apply a force to an object that is kept fixed at one end, its dimensions can change.

- If the force is below a maximum value, the change in dimension is proportional to the applied force. This is called Hooke's law:

  \[ F = k \Delta L \]

- This force region is called the elastic region.
Review Chapters 4.
Stress and strain.

- The elongation $\Delta L$ in the elastic region can be specified as follows:

$$\Delta L = \frac{1}{Y} \frac{F}{A} L$$

where

- $L =$ original length
- $A =$ cross sectional area
- $Y =$ Young’s modulus

- **Stress** is defined as the force per unit area ($= F/A$).
- **Strain** is defined as the fractional change in length ($\Delta L/L$).

Note: the ratio of stress to strain is equal to the Young’s Modulus.
Review Chapters 4.
Simple harmonic motion (SHM).

The key to the understanding of the atomic model of matter is the understanding of the spring-like interaction between the atoms.

\[ x(t) = A \cos(\omega t + \phi) \]
Review Chapters 4.
SHM: what forces are required?

• Consider we observe simple harmonic motion.
• The observation of the equation of motion can be used to determine the nature of the force that generates this type of motion.
• In order to do this, we need to determine the acceleration of the object carrying out the harmonic motion:

\[
x(t) = A \cos(\omega t + \phi)\\
\]

\[
v(t) = \frac{dx}{dt} = \frac{d}{dt} \left( A \cos(\omega t + \phi) \right) = -\omega A \sin(\omega t + \phi)\\
\]

\[
a(t) = \frac{dv}{dt} = \frac{d}{dt} \left( -\omega A \sin(\omega t + \phi) \right) = -\omega^2 A \cos(\omega t + \phi) = -\omega^2 x(t)\\
\]
Review Chapters 4.
What forces are required for SHM?

• Using Newton’s second law we can determine the force responsible for the harmonic motion:

\[ F = ma = -m\omega^2x \]

• We conclude:

*Simple harmonic motion is the motion executed by a particle of mass \( m \), subject to a force \( F \) that is proportional to the displacement of the particle, but opposite in sign.*

• Any force that satisfies this criterion can produce simple harmonic motion. If more than one force is present, you need to examine the net force, and make sure that the net force is proportional to the displacements, but opposite in sign.
Review Chapters 4.
SHM: examples.

- **The simple pendulum:**
  - The pendulum will carry out SHM with an angular frequency \( \omega = \sqrt{g/L} \).
  - The period of the pendulum is thus \( 2\pi/\omega = 2\pi\sqrt{L/g} \) which is independent of the mass of the pendulum.

- **The torsion pendulum:**
  - The pendulum will carry out SHM with an angular frequency \( \omega = \sqrt{K/I} \).
  - By measuring the period of the pendulum, we can determine the torsion constant \( K \) of the wire.
Review Chapters 4.  
Damped Harmonic Motion.

• If we add a damping force (such as the drag force) to the equation of motion we obtain:

\[
\frac{d^2 x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \frac{k}{m} x = 0
\]

• The solution of this equation is:

\[
x(t) \approx x_m e^{-\frac{bt}{2m}} e^{it\sqrt{\frac{k}{m}}}
\]

Damping Term  SHM Term

Not included on Exam # 1!
Review Chapters 4.
Driven Harmonic Motion.

- When we apply a driving force to our system, the equation of motion becomes
  \[
  \frac{d^2 x}{dt^2} = -\omega_0^2 x + F_0 \sin(\omega t)
  \]
- The steady-state solution of this equation of motion is
  \[
  x(t) = A \cos(\omega t + \phi)
  \]
- The amplitude \(A\) depends on the natural and the driving frequencies:
  \[
  A = \frac{F_0}{\left(\omega_0^2 - \omega^2\right)}
  \]

Note:
- \(\omega\) = driving frequency
- \(\omega_0\) = natural frequency

Not included on Exam #1!
One word of caution.
Never draw a centrifugal force!
Another word of caution: make sure you know how to decompose your force(s).

- In many problems you need to decompose an external force into components that are parallel and perpendicular to a surface.

- Know how to do this!
  - Check that your decomposition makes sense.
  - The component of the gravitational force parallel to the inclined plane is \( mg \sin \theta \). Does this make sense? What do you expect to get if \( \theta = 0 \)?
• **When looking at complex systems:**

  • Determine all external forces.

  • How do the components of the system move? If the components of the system carry out the same motion then the acceleration of the system is the ratio of the total external force divided by the total mass.

  • Now look at each component separately. Since we know its acceleration, we can determine the net force acting on it. The net force is the sum of the external and internal forces acting on this component. If some of these forces are not known, their properties can be determined in this way.
4.P.45 (a) In outer space, a rod is pushed to the right by a constant force $F$ (Figure 4.53). Describe the pattern of interatomic distances along the rod. Include a specific comparison of the situation at locations A, B, and C. Explain briefly in terms of fundamental principles.

$F$

$A$ $B$ $C$

Figure 4.53

*Hint:* Consider the motion of an individual atom inside the rod, and various locations along the rod.
The end!

Good luck preparing for Exam # 1.