Physics 141.
Lecture 23.

The first and the best airline in the world.

December 5^{th}.
An important day in the Netherlands.

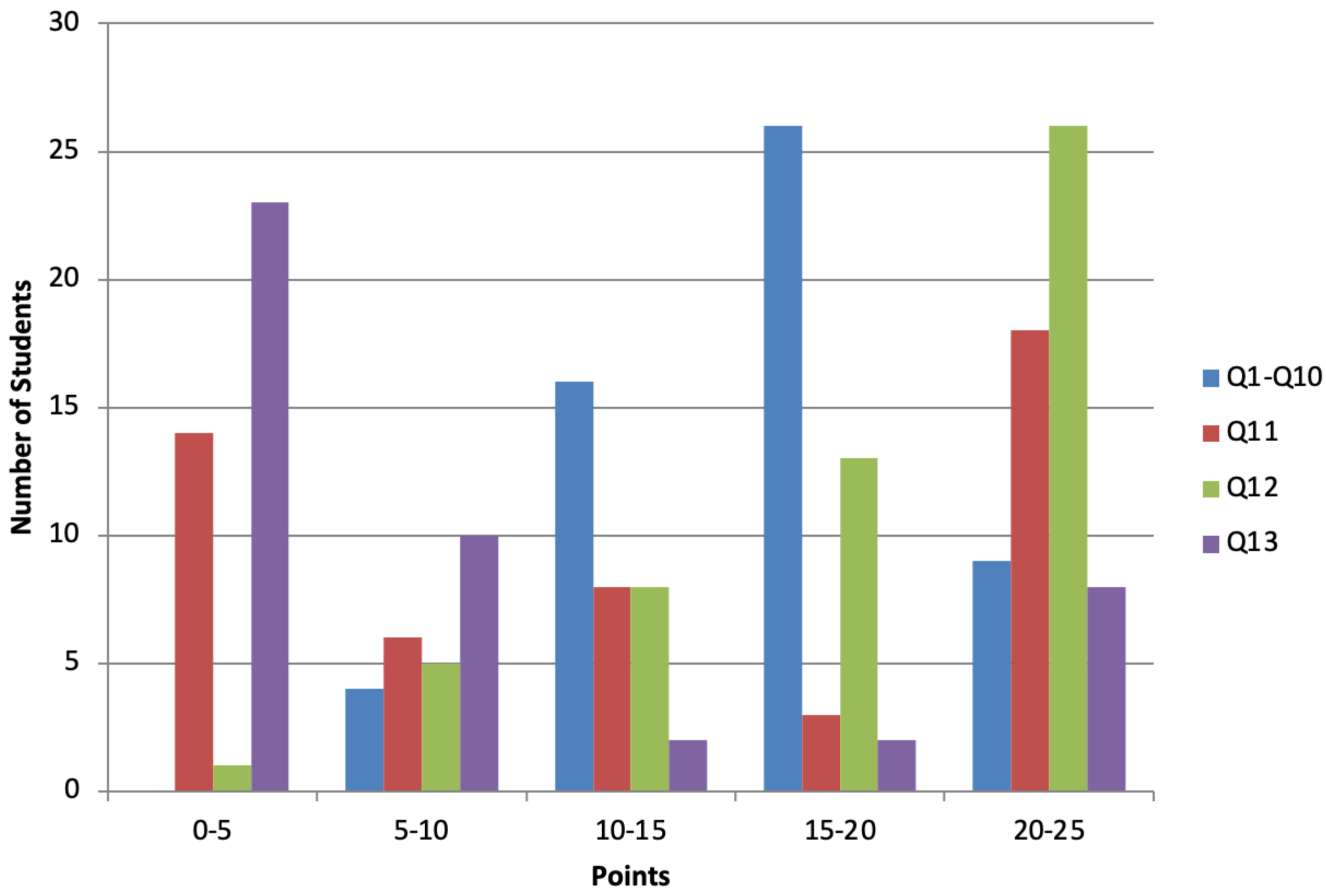
Physics 141.
Lecture 23.

- Course information:
  - Laboratory # 5 – lab report is due on Wednesday 12/6 at noon.
  - Homework set # 10 is due on Friday 12/8 at noon.
  - Results Exam # 3.
- Quiz
- Finish the discussion of Chapter 12:
  - The energy distribution of an ideal gas.
  - How do we confirm the energy distribution?
- Start the discussion of Supplement S1, Gases and Heat Engines:
  - The ideal gas law.

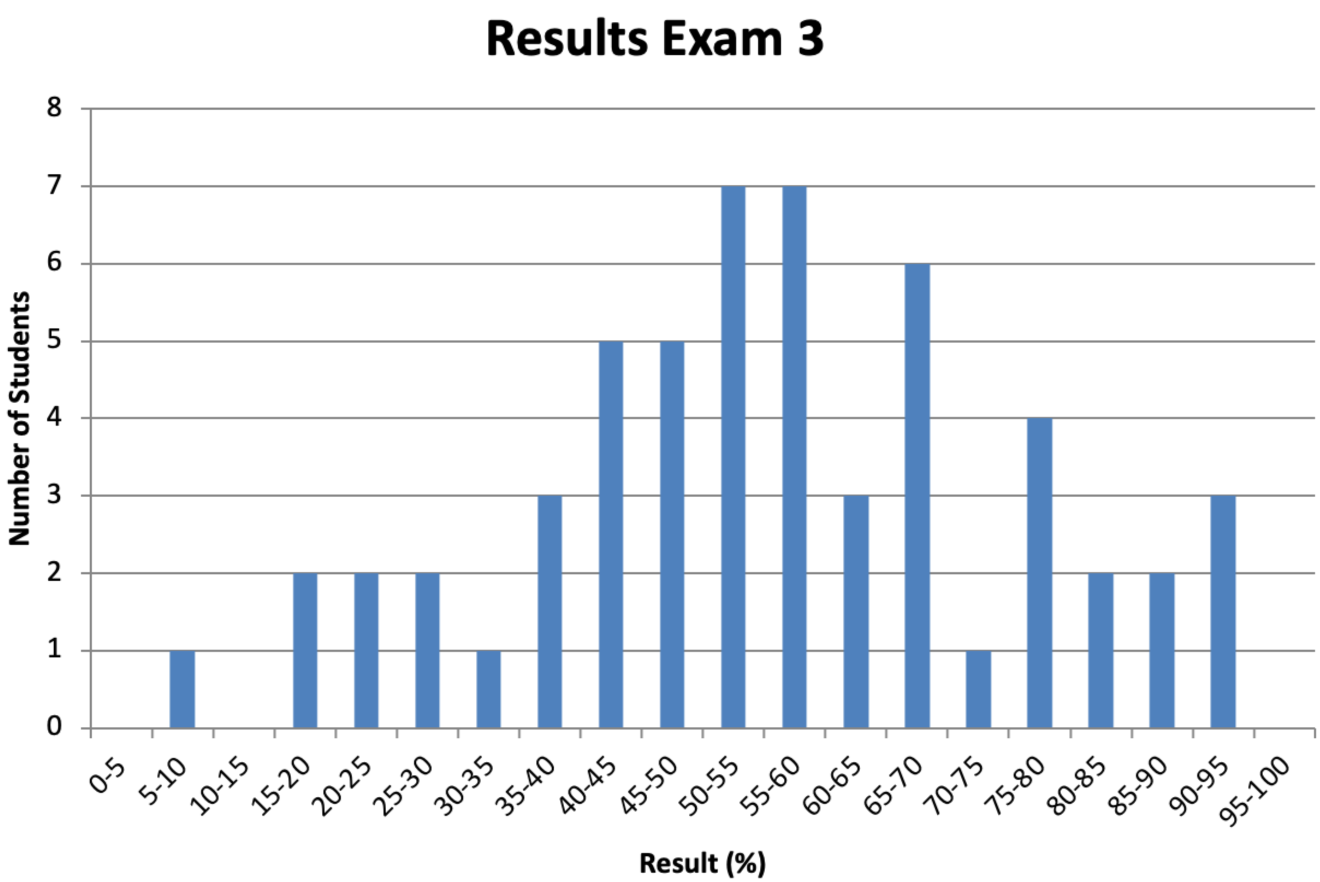
Analysis of experiment # 5.
Updated Timeline.

- ✓11/14: collisions in the May room
- ✓ 11/20: analysis files available.
  - https://www.pas.rochester.edu/~tdimino/phy141/lab05/
- ✓ 11/20: each student has determined his/her best estimate of the velocities before and after the collisions (analysis during regular lab periods).
- ✓ 11/21: complete discussion and comparison of results with colliding partners and submit final results (velocities and errors).
- ✓ 11/25: results will be compiled, linear momenta and kinetic energies will be determined, and results will be distributed.
- ✓ 12/4: office hours by lab TA/TIs to help with analysis and conclusions.
- 12/6: students submit lab report # 5.

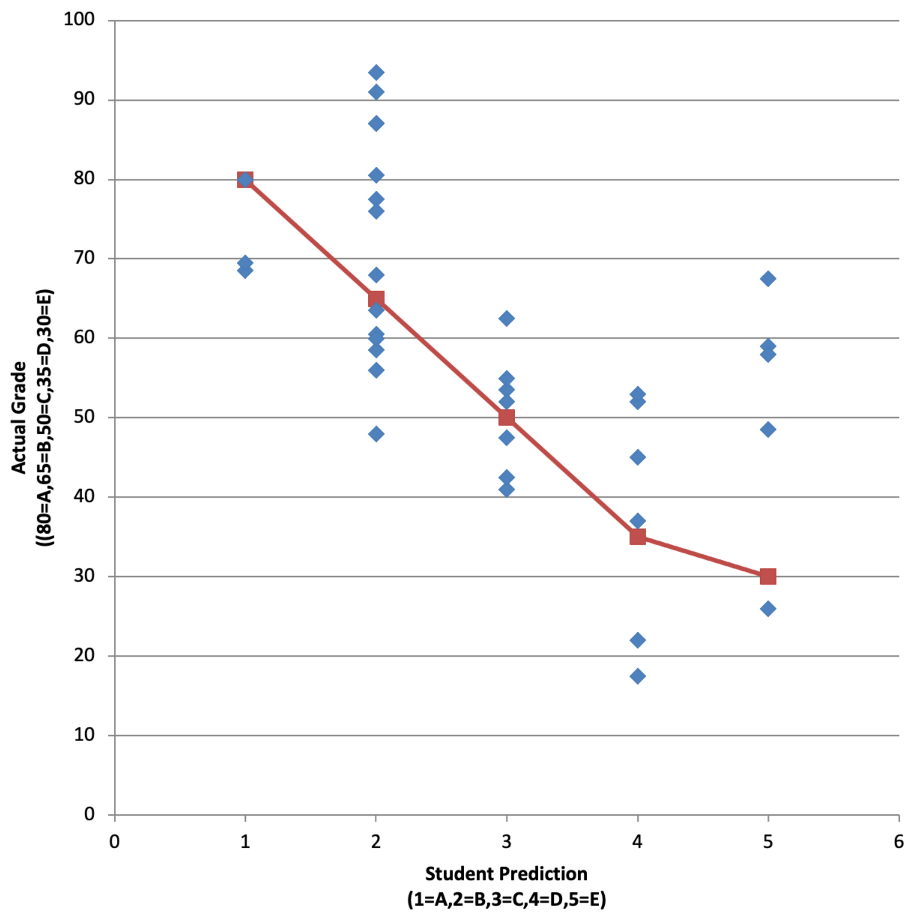
Results Exam # 3.



Results Exam # 3.

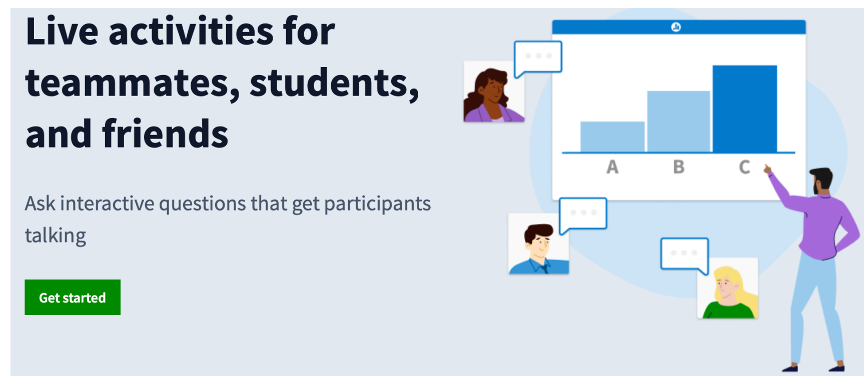


Results Exam # 3.



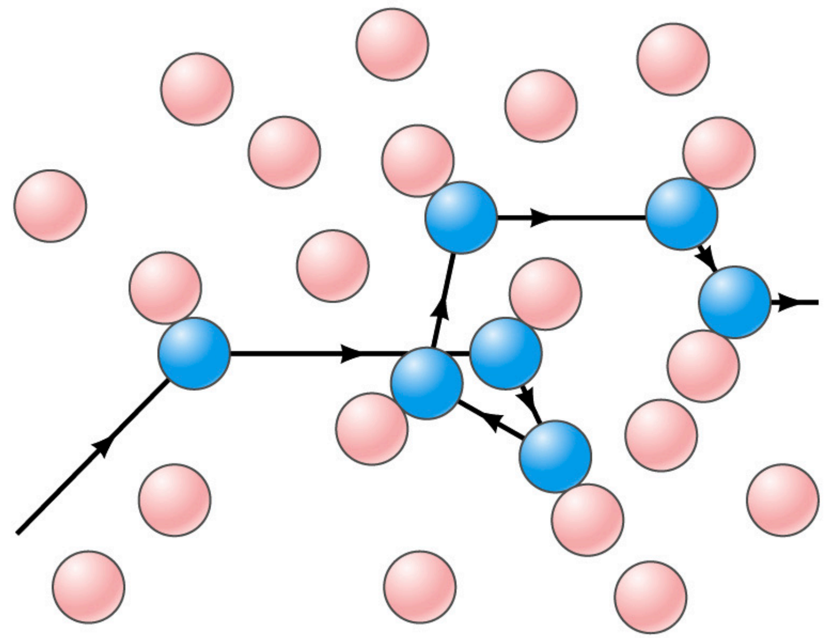
Quiz lecture 23.
PollEv.com/frankwolfs050

- The quiz today will have three questions.
- I will collect your answers electronically using the Poll Everywhere system.
- You have 30 seconds to answer each question.



Probing molecular speed. The mean-free path.

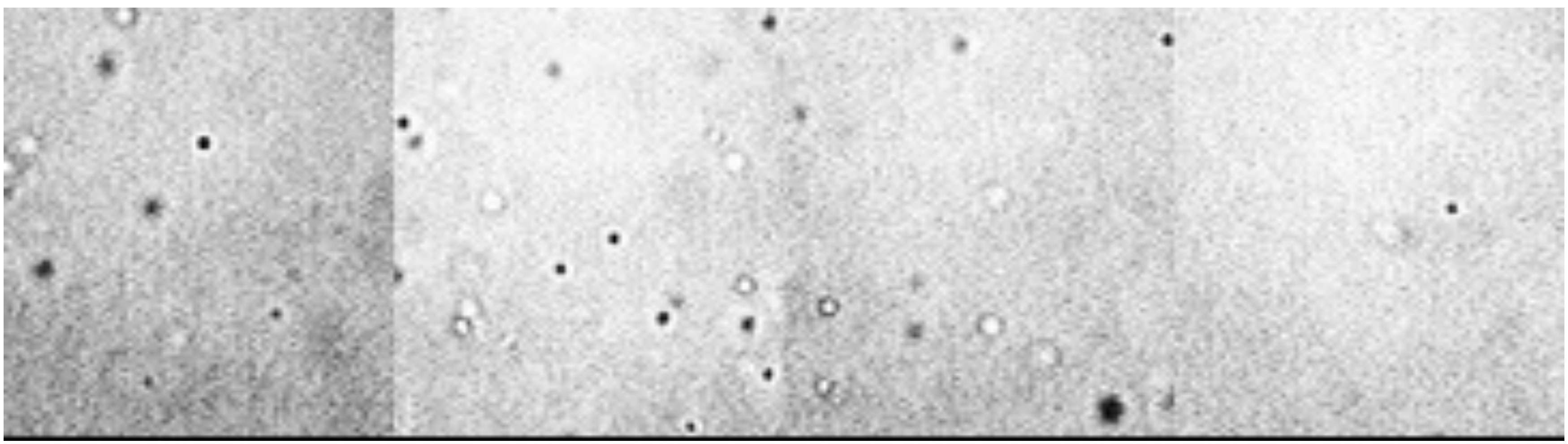
- The RMS velocities of individual gas molecules are large. For example, for hydrogen at room temperature, the RMS velocity is 1920 m/s.
- Despite the large RMS velocity, the average diffusion velocity is much smaller and is largely determined by the mean-free path of the molecules.
- We expect that the mean-free path is inversely proportional to the cross-sectional area of the molecules and inversely proportional to the density.



Typical values of the mean-free path are between 10^{-8} and 10^{-7} m

Probing molecular speeds in liquids.

0.5 μm particles in water, 50/50 glycerol-water, 75/25 glycerol-water, glycerol



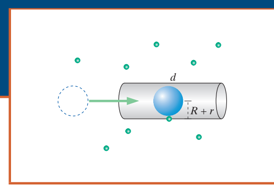
http://www.physics.emory.edu/~weeks/squishy/BrownianMotionLab.html

Chapter 13.
Aka Supplement S1.

SUPPLEMENT

S1

Gases and Heat Engines



OBJECTIVES

After studying this supplement, you should be able to

- Determine the flow rate of a gas through a hole, given microscopic information about the gas
- Determine the change in temperature and pressure of a gas undergoing compression or expansion
- Determine the efficiency of a heat engine

S1.1 GASES, SOLIDS, AND LIQUIDS

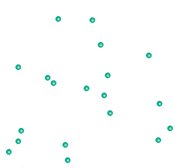


Figure S1.1 Molecules in a gas.

In contrast to a solid, a gas has no fixed structure. The gas molecules are not bound to each other but move around very freely, which is why a gas does not have a well-defined shape of its own; it fills whatever container you put it in (Figure S1.1). Think, for example, of the constantly shifting shape of a cloud, or the deformability of a balloon, in contrast with the rigidity of a block of aluminum. On average, gas molecules are sufficiently far apart that most of the time they hardly interact with each other. This low level of interaction is what makes it feasible to model a gas in some detail, using relatively simple concepts.

In a gas the molecular motion must be sufficiently violent that molecules can't stay stuck together. At high enough temperatures, any molecules that do manage to bind to each other temporarily soon get knocked apart again by high-speed collisions with other molecules. However, at a low enough temperature, molecules move sufficiently slowly that collisions are no longer violent enough to break intermolecular bonds. Rather, more and more molecules stick to each other in a growing mass as the gas turns into a liquid or, at still lower temperatures, a solid.

Liquids Are More Complex

Figure S1.2 Molecules in a liquid.

A liquid is intermediate between a solid and a gas. The molecules in a liquid are sufficiently attracted to each other that the liquid doesn't fly apart like a gas (Figure S1.2), yet the attraction is not strong enough to keep each molecule near a fixed equilibrium position as in a solid. The molecules in a liquid can slide past each other, giving liquids their special property of fluid flow (unlike solids) with fixed volume (unlike gases).

The analysis of liquids in terms of atomic, microscopic models is quite difficult compared with gases, where the molecules only rarely come in contact with other atoms, or compared with solids, where the atoms never move very far away from their equilibrium positions. For this reason, in

this introductory textbook with its emphasis on atomic-level description and analysis we concentrate mostly on understanding gases and solids.

An active field of research, called "molecular dynamics," models liquids, gases, and solids by computational modeling using the Momentum Principle and appropriate forces between the molecules. The difference between this work and the computational models you have made is that sophisticated programming techniques are required to deal with very large numbers of molecules, sometimes as many as a million, in a reasonable amount of computer time.

S1.2 GAS LEAKS THROUGH A HOLE

In Chapter 12 we used statistical mechanics to determine the average speed of a gas molecule. Here we'll see some interesting phenomena in which the average speed plays a role. We will model the gas molecules as little balls that don't attract each other and interact only in brief elastic collisions.

We will frequently use the symbol $n = N/V$ to stand for the number of gas molecules per unit volume, which we will express in SI units as number per cubic meter:

NUMBER DENSITY: NUMBER PER CUBIC METER

$$\text{Definition: } n \equiv \frac{N}{V}$$

N is the number of gas molecules in the volume V. The units of n are molecules per cubic meter.

Warning: You may be familiar from chemistry with the ideal gas law written in the form $PV = nRT$, where n is the number of moles. Here n means something else—the number of molecules per cubic meter, N/V.

EXAMPLE ***n* at Standard Temperature and Pressure**

Standard Temperature and Pressure (or STP) is defined for a gas to be $0\ °C = 273$ K and the average air pressure at sea level. Under STP conditions the ideal gas law can be used to show that one mole of a gas will occupy a volume of 22.4 liters $= 22.4 \times 10^3\ \text{cm}^3 = 22.4 \times 10^{-3}\ \text{m}^3$. What is the number density n of a gas at STP?

Solution One mole consists of 6.02×10^{23} molecules, so

$$n = \frac{6.02 \times 10^{23}\ \text{molecules}}{22.4 \times 10^{-3}\ \text{m}^3} = 2.68 \times 10^{25}\ \text{molecules/m}^3$$

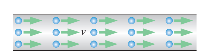


Figure S1.3 Side view of molecules all traveling to the right with speed v inside a tube. There are n molecules per cubic meter inside the tube.

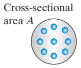


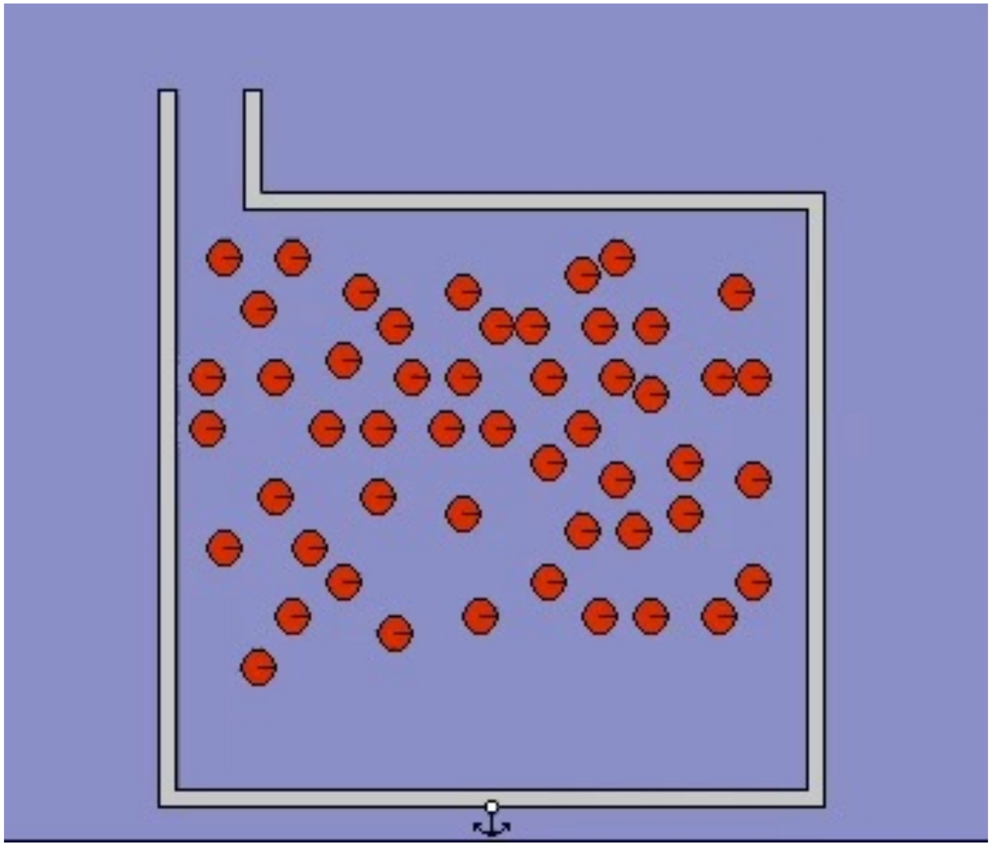
Figure S1.4 End view of molecules all traveling the same direction in a tube of cross-sectional area A.

One-Directional Gas

We will calculate the leakage rate of a gas through a small hole in a container filled with the gas. First we'll consider a simplified one-directional example, in order to understand the basic issues before stating the results for a real three-dimensional gas.

The chain of reasoning that we follow is basically geometric. Consider a situation in which many gas molecules are all traveling to the right inside a tube. For the moment, temporarily assume that they all have the same speed v (Figure S1.3). The cross-sectional area of the tube is A (Figure S1.4).

Supplement S1.
The kinetic theory of gases.



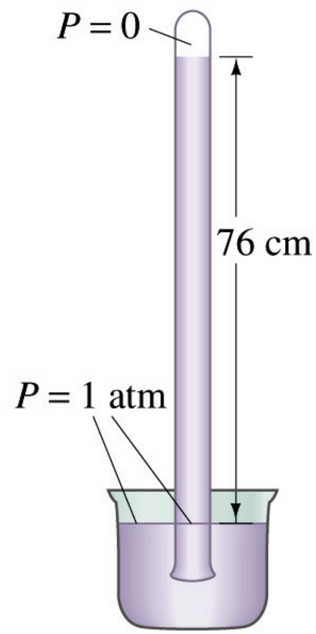
http://eml.ou.edu/Physics/module/thermal/ketcher/Idg4.avi

The kinetic theory of gases. Thermodynamic variables.

- The kinetic theory of gases provides a framework to connect the microscopic properties of the molecules in a gas (such as their rms velocity) to the macroscopic properties of the gas (such as volume, temperature, and pressure).
- The volume of a gas is defined by the size of the enclosure of the gas. During a change in the state of a gas, the volume may or may not remain constant (this depends on the procedure followed).
- The temperature of a gas has been defined in terms of the entropy of the system (see discussion in Chapter 12).
- We will now briefly discuss pressure.

Thermodynamic variables. Pressure.

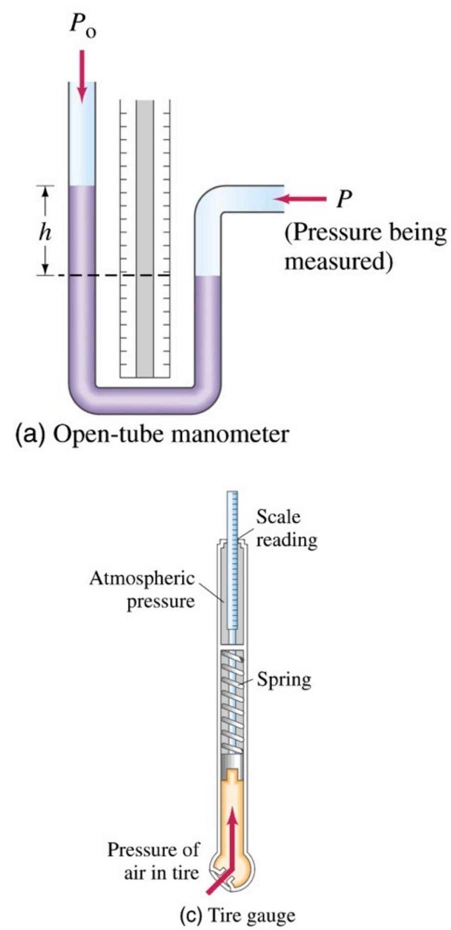
- Pressure is an important thermodynamic variable.
- **Pressure is defined as the force per unit area**.
- **The SI unit is pressure is the Pascal: 1 Pa = 1 N/m^2**. Another common unit is the atm (atmospheric pressure) which is the pressure exerted by the atmosphere on us (1 atm = 1.013 x 10^5 N/m^2).
- A pressure of 1 atm will push a mercury column up by 76 cm.



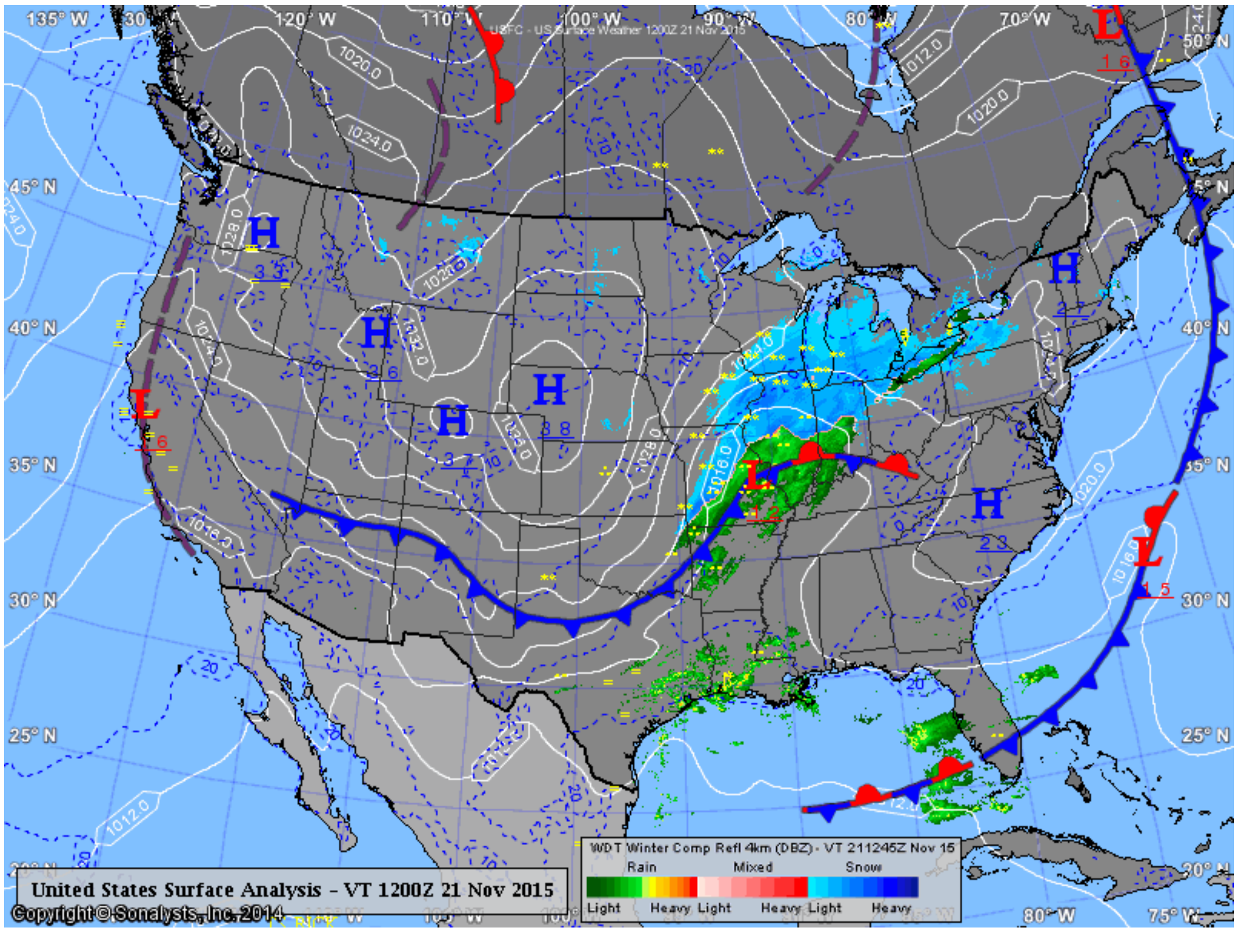
Note: if we would use water, the column would be about 10 m high.

Thermodynamic variables. Pressure.

- Many devices that measure pressure, actually measure the pressure difference between the pressure of interest and the atmospheric pressure.
- Atmospheric pressure changes with altitude. The higher you go, the less air is pressing on your head! Airplanes use the atmospheric pressure to measure altitude.
- But keep into consideration that the atmospheric pressure at a fixed location and altitude is not constant!



Thermodynamic variables.
Pressure.



The kinetic theory of gases. Thermodynamic variables.

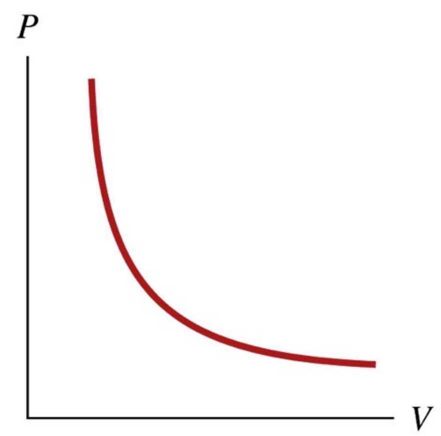
- The **volume** of a gas is defined by the size of the enclosure of the gas. During a change in the state of a gas, the volume may or may not remain constant (this depends on the procedure followed).

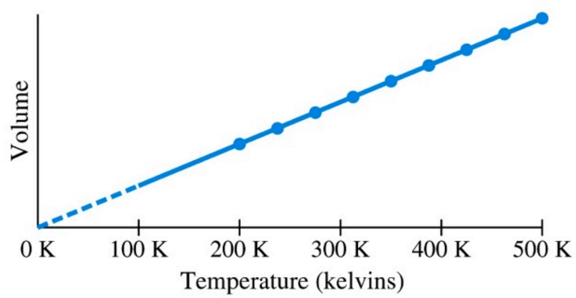
- The **temperature** of a gas has been defined in terms of the entropy of the system (see discussion in Chapter 12).

- The **pressure** of a gas is defined as the force per unit area. The SI unit is pressure is the Pascal: 1 Pa = 1 N/m^2. Another common unit is the atm (atmospheric pressure) which is the pressure exerted by the atmosphere on us (1 atm = 1.013 x 10^5 N/m^2).

The equation of state of a gas.

- In order to specify the state of a gas, we need to measure its temperature, its volume, and its pressure. The relation between these variables and the mass of the gas is called **the equation of state**.
- The equation of state of a gas was initially obtained on the basis of observations.
  - Boyle's Law (1627 - 1691):
    ***pV* = constant** for gases maintained at constant temperature.
  - Charle's Law (1746 - 1823):
    ***V/T* = constant** for gases maintained at constant pressure.
  - Gay-Lussac's Law (1778 - 1850):
    ***p/T* = constant** for gases maintained at constant volume



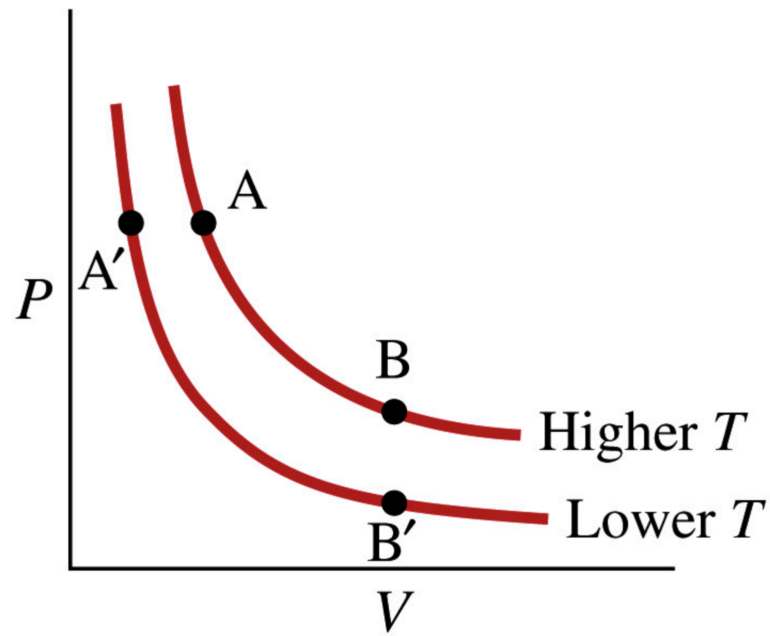


(b)

The equation of state of a gas.

- Combining the various gas laws we can obtain a single more general relation between pressure, temperature, and volume: $pV = \text{constant } T$.

- Another observation that needs to be included is the dependence on the amount of gas: if pressure and temperature are kept constant, the volume is proportional to the mass m: $pV = \text{constant } mT$.



The equation of state of a gas.

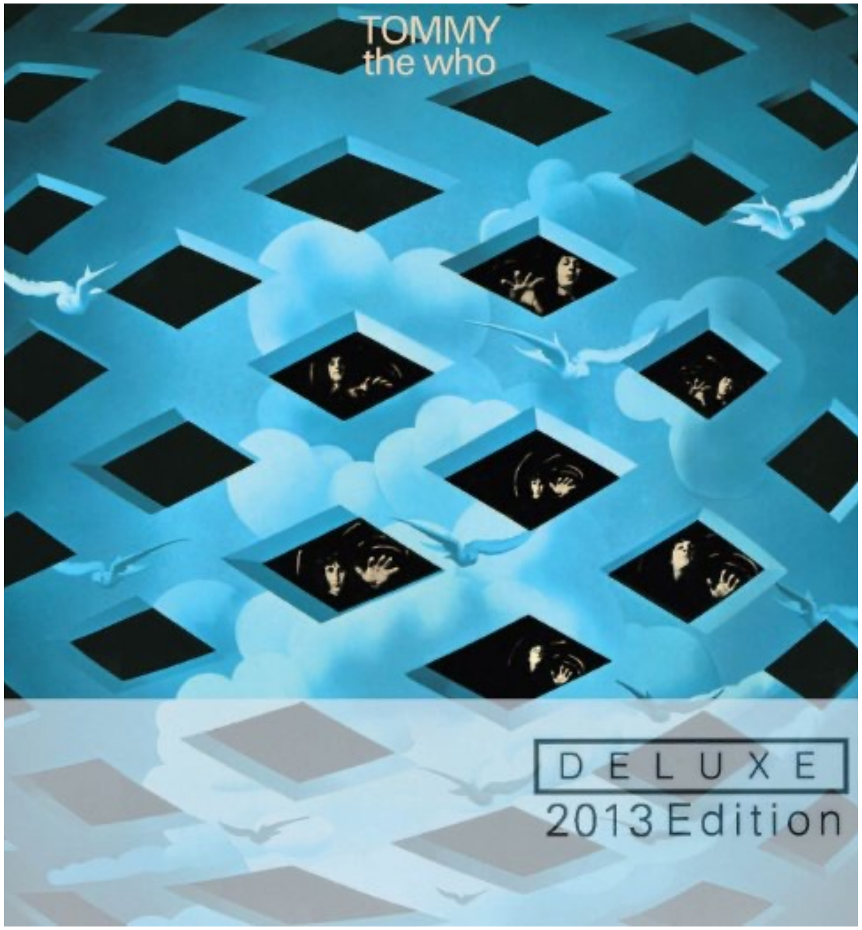
- The equation of state of a gas can be written as

$$pV = NkT$$

where
  - p = pressure (in Pa).
  - V = volume (in m^3).
  - N = number of molecules of gas (1 mole = 6.02×10^{23} molecules or atoms). Note the number of molecules in a mole is also known as Avogadro's number N_A.
  - T = temperature (in K).

- Note: the equation of state is the equation of state of an ideal gas. Gases at very high pressure and/or close to the freezing point show deviations from the ideal gas law.

3 Minute 34 Second Intermission.



- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 3 minute 34 second intermission.
- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.
  - Solve a WeBWorK problem.

The equation of state of a gas. Example problem.

- A cylinder contains oxygen at 20°C and a pressure of 15 atm at a volume of 12 l. The temperature is raised to 35°C , and the volume is reduced to 8.5 l. What is the final pressure of the gas?

- Since the amount of gas does not change, we can rewrite the ideal gas law in the following way: pV/T = constant. Since we know the initial state, we can determine the missing information about the final state:

$$p_{\mathrm{i}}V_{\mathrm{i}}/T_{\mathrm{i}} = p_{\mathrm{f}}V_{\mathrm{f}}/T_{\mathrm{f}}$$

The equation of state of a gas. Example problem.

- The final pressure of the gas is equal to

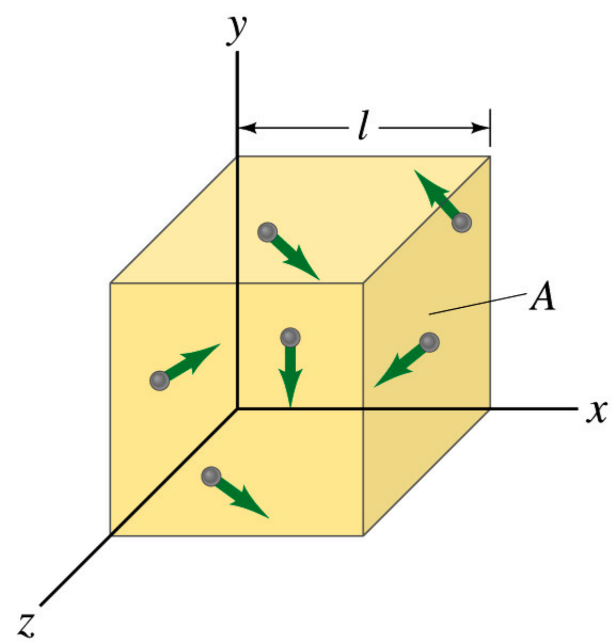
$$p_f = p_i\,(V_i/V_f)\;/\;(T_i/T_f)$$

- Note:
  - This relation will preserve the units of pressure.
  - The units of volume cancel, and we can keep the volume in units of liters. Note: for whatever we unit we choose, zero volume in SI units, correspond to zero volume in all other units.
  - The units of temperature must be in Kelvin. The temperature ratio $T_i/T_f = (273.15 + 20)/(273.15 + 35) = 0.951$ when T is expressed in Kelvin. The ratio would be 0.571 when T is expressed in Celsius.

- When we use the correct units, we find that $p_f = 22$ atm.

The molecular point of view of a gas.

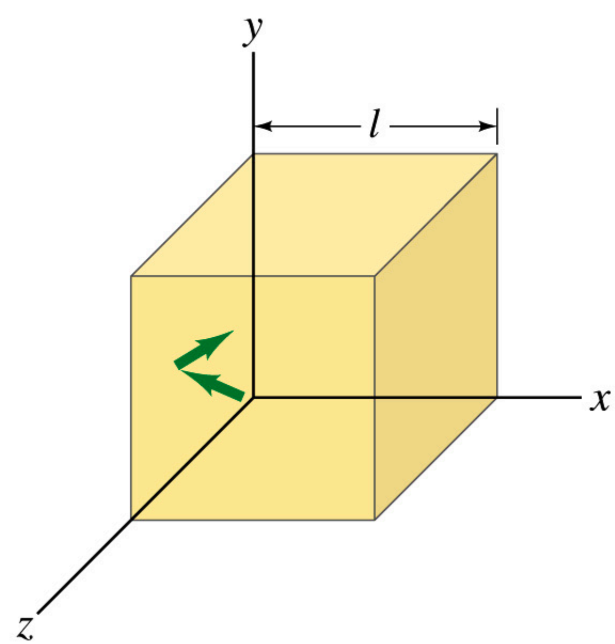
- Consider a gas contained in a container.
- The molecules in the gas will continuously collide with the walls of the vessel.
- Each time a molecule collides with the wall, it will carry out an elastic collision.
- Since the linear momentum of the molecule is changed, the linear momentum of the wall will change too.
- Since force is equal to the change in linear momentum per unit time, the gas will exert a force on the walls.



The molecular point of view of a gas.

- Consider the collision of a single molecule with the left wall.
- In this collision, the linear momentum of the molecule changes by mv_x - $(-mv_x) = 2mv_x$.
- The same molecule will collide with this wall again after a time $2l/v_x$.
- The force that this single molecule exerts on the left wall is thus equal to

$$\Delta p/\Delta t = (2mv_x)/(2l/v_x) = mv_x^2/l$$



The molecular point of view of a gas.

- The force that this single molecule exerts on the left wall is thus equal to

$$F_{\text{left}} = mv_x^2/l$$

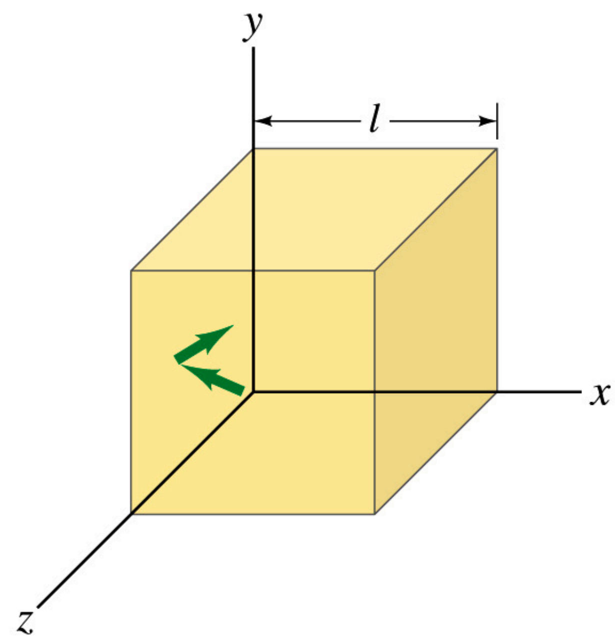
- If the pressure exerted on the left wall by this molecule is equal to

$$p_{\text{left}} = F_{\text{left}}/A = mv_x^2/(lA)$$

  where A is the area of the left wall.

- The volume of the gas is equal to lA and we can thus rewrite the pressure on the left wall:

$$p_{\text{left}} = mv_x^2/V$$



The molecular point of view of a gas.

- The pressure that many molecules exerts on the left wall is equal to

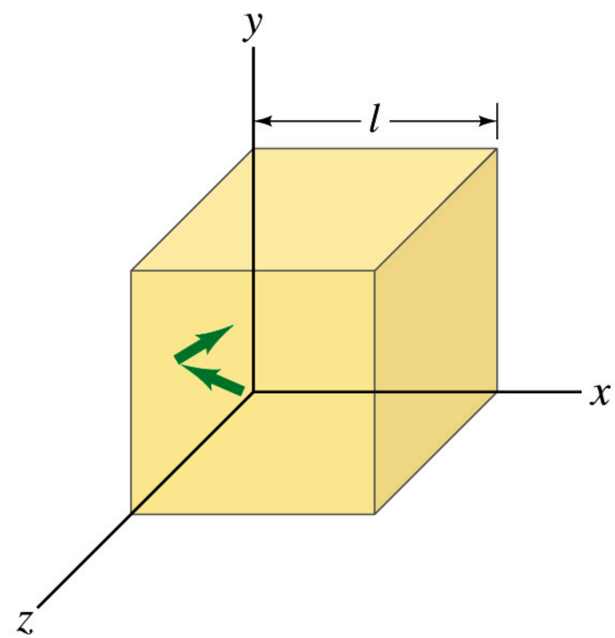
$$p_{\text{left}} = m(v_{1x}^2+v_{2x}^2+v_{3x}^2+\ldots)/V$$

- This equation can be rewritten in terms of the average of the square of the x component of the molecular velocity and the number of molecules (N):

$$p_{\text{left}} = mN(v_x^2)_{\text{average}}/V$$

- Assuming that there is no preferential direction, the average square of the x, y, and z components of the molecular velocity will be the same:

$$(v_x^2)_{\text{average}} = (v_y^2)_{\text{average}} = (v_z^2)_{\text{average}}$$



The molecular point of view of a gas.

- The force on the left wall can be rewritten in terms of the average squared velocity

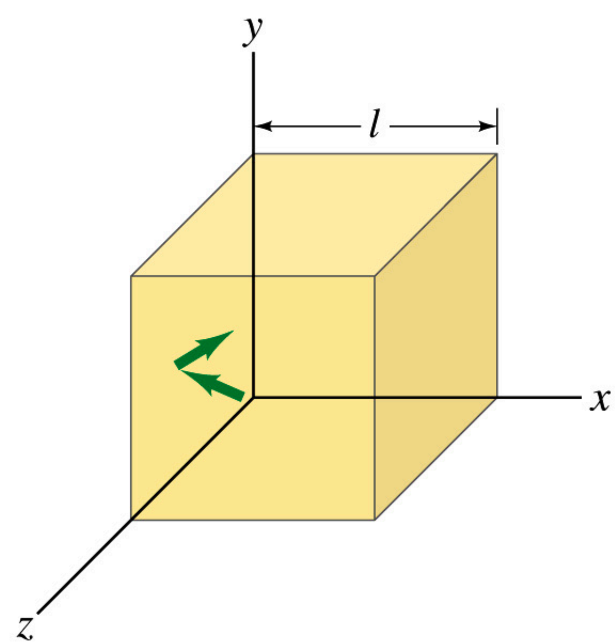
$$p_{\text{left}} = mN(v^2)_{\text{average}}/3V$$

- Assuming there is no preferential direction of motion of the molecules, the pressure on all walls will be the same and we thus conclude:

$$pV = mN(v^2)_{\text{average}}/3$$

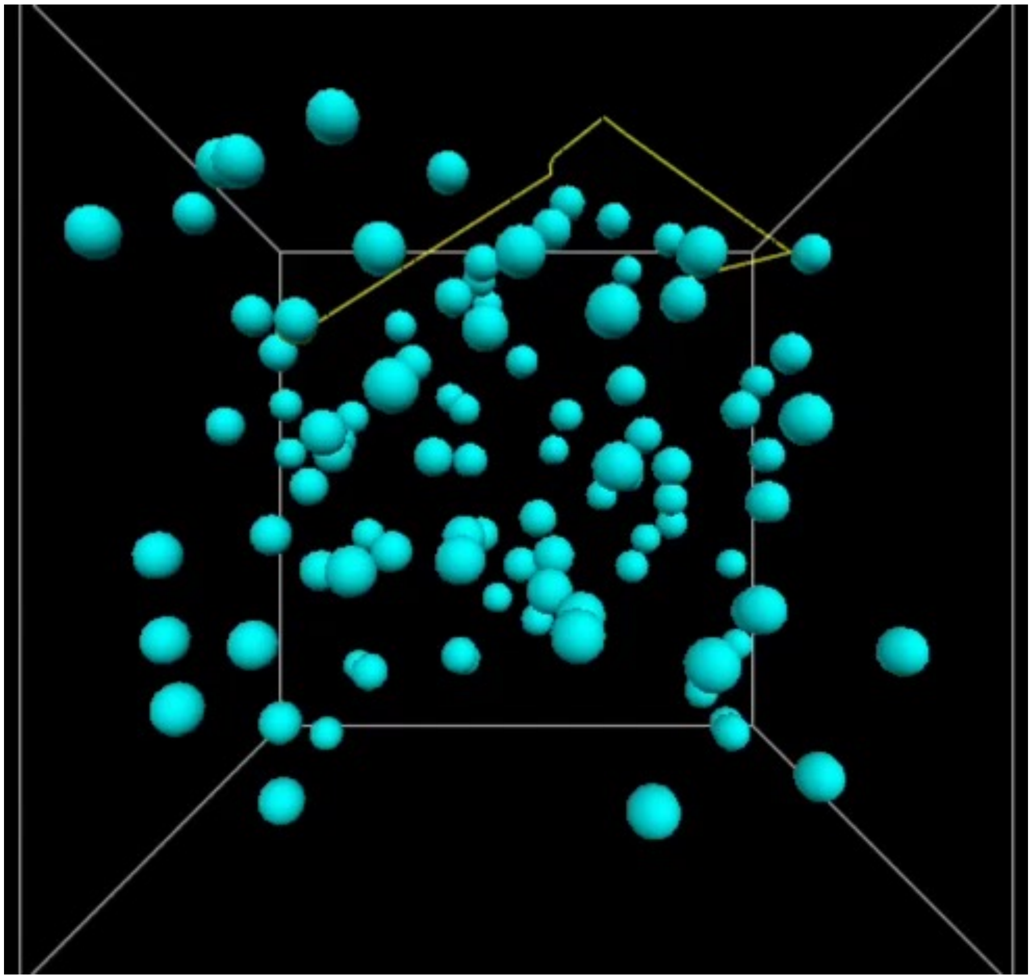
- Compare this to the ideal gas law:

$$pV = NkT$$

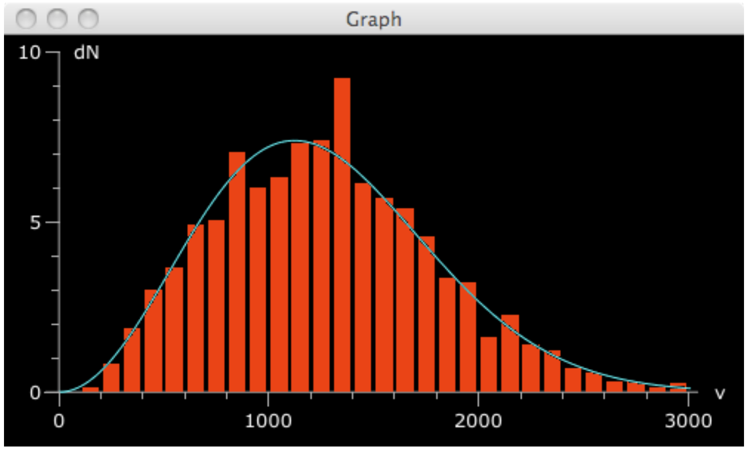


$$K_{\text{average}} = (1/2)m(v^2)_{\text{average}} = (3/2)\,kT$$

Simulating an ideal gas.



- Ideal gas simulations:
  - Assume elastic collisions between the gas molecules.
  - Assume elastic collisions between the gas molecules and the walls.
- Results agree very well with measured values.



Done for today!