Physics 141.
Lecture 19.

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Physics 141.
Lecture 19.

- Course Information.
- Quiz.
- Topics to be discussed today:
  - Angular momentum.
  - Conservation of angular momentum at the macroscopic and the microscopic level.
  - Precession.

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Course Information

- Homework set # 8 is due on Friday November 8.
- The collisions of Lab # 5 will take place on Monday 11/11 in Spurrier Gym.
  - If you did not pickup your 12-pack, please pick up one today.
  - Make sure you empty your cans between now and 11/11 and bring your 12 empty cans to Spurrier Gym.
- Exam # 3 will take place on Tuesday 11/19 between 8 am and 9.20 am.
- Exam # 3 will cover the material of Chapters 8, 9, 10, and 11.

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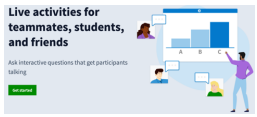
Quiz lecture 19.
PollEv.com/frankwolfs050

- The quiz today will have four questions.
- I will collect your answers electronically using the Poll Everywhere system.
- You have 60 seconds to answer each question.



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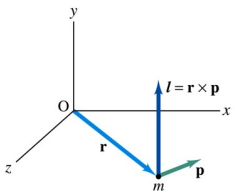
Angular momentum. Definition.

- The angular momentum is defined as the vector product between the position vector and the linear momentum.
- Note:
  - Compare this definition with the definition of the torque.
  - Angular momentum is a vector.
  - The unit of angular momentum is kg m^2/s.
  - The angular momentum depends on both the magnitude and the direction of the position and linear momentum vectors.
  - Under certain circumstances the angular momentum of a system is conserved!



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Conservation of angular momentum.

- Consider the change in the angular momentum of a particle:

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r}\times\vec{p}) = m\left(\vec{r}\times\frac{d\vec{v}}{dt} + \frac{d\vec{r}}{dt}\times\vec{v}\right) = m(\vec{r}\times\vec{a} + \vec{v}\times\vec{v}) =$$

$$= \vec{r}\times m\vec{a} = \vec{r}\times\sum\vec{F} = \sum\vec{\tau}$$

- When the net torque is equal to 0 Nm:

$$\sum\vec{\tau} = 0 = \frac{d\vec{L}}{dt} \Rightarrow \vec{L} = \text{constant}$$

- When we take the sum of all torques, the torques due to the internal forces cancel and the sum is equal to torque due to all external forces.

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Conservation of angular momentum. Planetary motion.

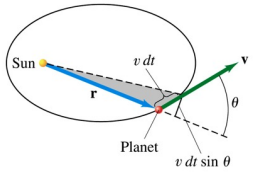
- Consider planetary motion:

$$\vec{L} = \vec{r}\times\vec{p} = mrv\sin(\theta)\hat{z} = \\ = m\frac{rvdt\sin(\theta)}{dt}\hat{z} = 2m\frac{dA}{dt}\hat{z}$$

- The gravitational force is an internal force. In the absence of external forces, the angular momentum is conserved. We conclude that

$$\frac{dA}{dt}=\text{constant}$$

- This is of course Kepler's Law.



$$\frac{dA}{dt} = \text{constant}$$

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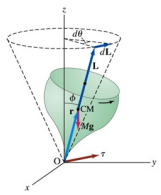
Conservation of angular momentum.

- The connection between the angular momentum $\vec{L}$ and the torque $\vec{\tau}$

$$\sum\vec{\tau} = \frac{d\vec{L}}{dt}$$

is only true if $\vec{L}$ and $\vec{\tau}$ are calculated with respect to the same reference point (which is at rest in an inertial reference frame).

- The relation is also true if $\vec{L}$ and $\vec{\tau}$ are calculated with respect to the center of mass of the object (note: the center of mass can accelerate).



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Conservation of angular momentum. A demonstration.

- Ignoring the mass of the bicycle wheel, the external torque will be close to zero if we use the center of the disk as our reference point.
- Since the external torque is zero, angular momentum thus should be conserved.
- I can change the orientation of the wheel by applying internal forces. In which direction will I need to spin to conserve angular momentum?

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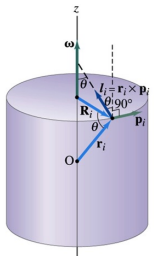
Angular momentum of rotating rigid objects.

- Consider a rigid object rotating around the z axis.
- The magnitude of the angular momentum of a part of a small section of the object is equal to

$$|\vec{l}_i| = r_i p_i \hat{l}_i$$

- Due to the symmetry of the object, we expect that the angular momentum of the object will be directed on the z axis. Thus, we only need to consider the z component of this angular momentum.



Note the direction of $\vec{l}_i$!!!! (perpendicular to $\vec{r}_i$ and $\vec{p}_i$)

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Angular momentum of rotating rigid objects.

- The z component of the angular momentum is

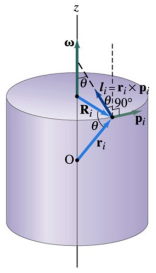
$$l_{i,z} = r_i p_i \cos(\theta) = R_i p_i = m_i R_i v_i$$

- The total angular momentum of the rotating object is the sum of the angular momenta of the individual components:

$$L_z = \sum_i l_{i,z} = \sum_i m_i R_i^2 \omega = I\omega$$

- The total angular momentum is thus equal to

$$\vec{L} = I\omega\hat{z}$$



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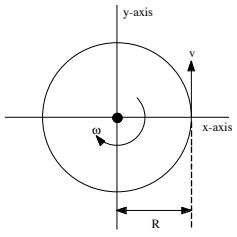
Conservation of angular momentum. Sample problem.

- A cockroach with mass m runs counterclockwise around the rim of a lazy Susan (a circular dish mounted on a vertical axle) of radius R and rotational inertia I with frictionless bearings. The cockroach's speed (with respect to the earth) is v, whereas the lazy Susan turns clockwise with angular speed ω_0. The cockroach finds a bread crumb on the rim and, of course, stops. (a) What is the angular speed of the lazy Susan after the cockroach stops? (b) Is mechanical energy conserved ?



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Conservation of angular momentum. Sample problem.

- The initial angular momentum of the cockroach is

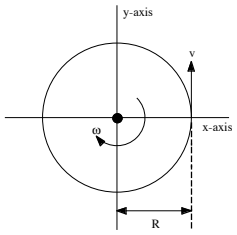
$$\vec{L}_c = \vec{r}_c \times \vec{p}_c = Rmv\hat{z}$$

- The initial angular momentum of the lazy Susan is

$$\vec{L}_d = -I\omega_0\hat{z}$$

- The total initial angular momentum is thus equal to

$$\vec{L} = \vec{L}_c + \vec{L}_d = (Rmv - I\omega_0)\hat{z}$$



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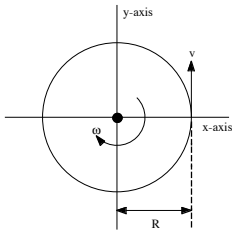
Conservation of angular momentum. Sample problem.

- When the cockroach stops, it will move in the same way as the rim of the lazy Susan. The forces that bring the cockroach to a halt are internal forces, and angular momentum is thus conserved.
- The moment of inertia of the lazy Susan + cockroach is equal to

$$I_f = I + mR^2$$

- The final angular velocity of the system is thus equal to

$$\omega = \frac{L_f}{I_f} = \frac{Rmv - I\omega_0}{I + mR^2}$$



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Conservation of angular momentum. Sample problem.

- The initial kinetic energy of the system is equal to

$$K_i = \frac{1}{2}mv^2 + \frac{1}{2}I\omega_0^2 = \frac{1}{2}\frac{Imv^2 + I^2\omega_0^2 + m^2R^2v^2 + mR^2I\omega_0^2}{I + mR^2}$$

Cockroach **Lazy Susan**

- The final kinetic energy of the system is equal to

$$K_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}(I + mR^2)\left(\frac{Rmv - I\omega_0}{I + mR^2}\right)^2 = \frac{1}{2}\frac{(Rmv - I\omega_0)^2}{I + mR^2}$$

- The change in the kinetic energy is thus equal to

$$\Delta K = \frac{1}{2}\frac{ml(-2Rv\omega_0 - v^2 - R^2\omega_0^2)}{I + mR^2} = -\frac{1}{2}\frac{ml}{I + mR^2}(v + R\omega_0)^2$$

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3 Minute 55 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 3 minute 55 second intermission.
- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.
  - Solve a WeBWorK problem.

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Conservation and angular momentum at the atomic and nuclear level.

- Particles at the atomic and nuclear level have two different forms of angular momentum:
  - Translational angular momentum: the angular momentum associated with the "orbital" motion of the particles. This angular momentum is also called the **orbital angular momentum**.
  - Rotational angular momentum: the angular momentum associated with the rotation of the particles around their symmetry axis. This angular momentum is called the **spin of the particle**.
- The angular momentum at the atomic and nuclear level is quantized; its projection along the x, y, or z axis is an integer or half-integer multiple of $\hbar = h/2\pi$.

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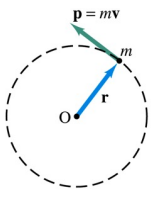
Implications of quantization of angular momentum.

- Consider the "classical" picture of the motion of electrons in atoms.
- If the angular momentum is a integer multiple of $\hbar$, the orbit must be such that

$$rp = N\hbar$$

- In order to carry out circular motion, the force on the electron must be equal to

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{mv^2}{r} = \frac{p^2}{mr}$$



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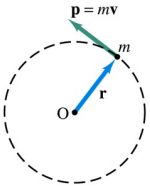
Implications of quantization of angular momentum.

- Eliminating p from the force equation shows us that

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}=\frac{p^2}{mr}=\frac{\left(\frac{N\hbar}{r}\right)^2}{mr}=\frac{N^2\hbar^2}{mr^3}$$

- This equation can be used to determine the radius r:

$$r=4\pi\varepsilon_0\frac{N^2\hbar^2}{me^2}$$



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Implications of quantization of angular momentum.

- The quantization of r results in a quantization of both the potential and the kinetic energy of the electron:

$$U=-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}=-\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{me^4}{N^2\hbar^2}$$

$$K=\frac{1}{2}\frac{p^2}{m}=\frac{1}{2m}\frac{N^2\hbar^2}{r^2}=\frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{me^4}{N^2\hbar^2}$$

- The total energy of the electron is thus equal to

$$E=K+U=-\frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{me^4}{N^2\hbar^2}=-\frac{13.6}{N^2}\text{eV}$$

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Implications of quantization of angular momentum.

- The energy levels of an electron in the hydrogen atom exactly match the levels predicted using this simple model, and the quantization of the energy levels is thus a direct consequence of the quantization of angular momentum.
- In addition to the orbital angular momentum of the electrons in the atom, they also poses spin. The projection of the spin of the electron on a particular axis will be either $+\frac{1}{2}\hbar$ or $-\frac{1}{2}\hbar$. It will **never** be zero. The electron is said to be a spin $\frac{1}{2}$ particle.
- Many other particles, such as muons, neutrinos, and quarks, are spin $\frac{1}{2}$ particles.

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Implications of quantization of angular momentum.

- Since quarks are the building blocks of hadrons, we also expect that hadrons have a well-defined spin.
  - Hadrons that contain three quarks can either be spin $\frac{1}{2}$ or spin $\frac{3}{2}$.
  - Hadrons that contain two quarks can either be spin 0 or spin 1.
- The total spin of a particle limits how particles can be distributed across the various energy levels of the system.
  - If the spin is a half integer, the particle is called a **Fermion**, and it must obey the Pauli exclusion principle (two fermions can not be in the exact same quantum state).
  - If the spin is an integer, the particle is called a **Boson**, and it is not subject to the Pauli exclusion principle (there is not limit to the number of Bosons that can be in the exact same quantum state).

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The building blocks of matter: combining quarks.

Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$
Baryons are fermionic hadrons.
There are about 120 types of baryons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c²	Spin
p	proton	uud	1	0.938	1/2
$\bar{p}$	anti-proton	$\bar{u}\bar{u}\bar{d}$	−1	0.938	1/2
n	neutron	udd	0	0.940	1/2
Λ	lambda	uds	0	1.116	1/2
Ω^-	omega	sss	−1	1.672	3/2

Mesons $q\bar{q}$
Mesons are bosonic hadrons.
There are about 140 types of mesons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c²	Spin
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	−1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.770	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2 .980	0

http://particleadventure.org/particleadventure/frameless/chart.html

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The building blocks of matter: grouped according to spin.

FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c²	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	−1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	−1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	−1

Quarks spin = 1/2

Flavor	Approx. Mass GeV/c²	Electric charge
u up	0.003	2/3
d down	0.006	−1/3
c charm	1.3	2/3
s strange	0.1	−1/3
t top	175	2/3
b bottom	4.3	−1/3

BOSONS force carriers spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c²	Electric charge
γ photon	0	0
W^-	80.4	−1
W^+	80.4	+1
Z^0	91.187	0

Strong (color) spin = 1

Name	Mass GeV/c²	Electric charge
g gluon	0	0

http://particleadventure.org/particleadventure/frameless/chart.html

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Implications of quantization of angular momentum.

- A final remark (before talking about precession):

  The spin of macroscopic objects will also be quantized, but the difference between different spin states is so small that it is impossible to observed effects of this quantization.

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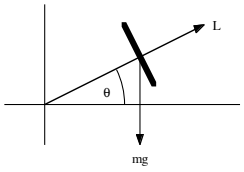
Precession.

- Consider a rotating rigid object spinning around its symmetry axis.
- The object carries a certain angular momentum L.
- Consider what will happen when the object is balanced on the tip of its axis (which makes an angle θ with the horizontal plane).
- The gravitational force, which is an external force, will generate a toques with respect of the tip of the axis.



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Precession.

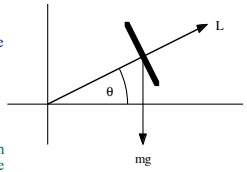
- The external torque is equal to

$$|\vec{\tau}| = |\vec{r}\times\vec{F}| = |\vec{r}||\vec{F}|\sin\left(\frac{\pi}{2}+\theta\right) = Mgr\cos(\theta)$$

- The external torque causes a change in the angular momentum:

$$d\vec{L} = \vec{\tau}dt$$

- Thus:
  - The change in the angular momentum points in the same direction as the direction of the torque.
  - The torque will thus change the direction of $\vec{L}$ but not its magnitude.



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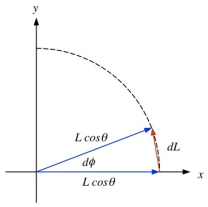
Precession.

- The effect of the torque can be visualized by looking at the motion of the projection of the angular momentum in the *xy* plane.
- The angle of rotation of the projection of the angular momentum vector when the angular momentum changes by $d\vec{L}$ is equal to



$$d\phi = \frac{dL}{L\cos(\theta)} = \frac{Mgr\cos(\theta)\,dt}{L\cos(\theta)} = \frac{Mgrdt}{L}$$

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Precession.

- Since the projection of the angular momentum during the time interval dt rotates by an angle $d\phi$, we can calculate the rate of precession:

$$\Omega = \frac{d\phi}{dt} = \frac{Mgr}{L} = \frac{Mgr}{I\omega}$$

- We conclude the following:
  - The rate of precessions does not depend on the angle θ.
  - The rate of precession decreases when the angular momentum increases.

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Up next: equilibrium.

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