Physics 141.
Lecture 19.

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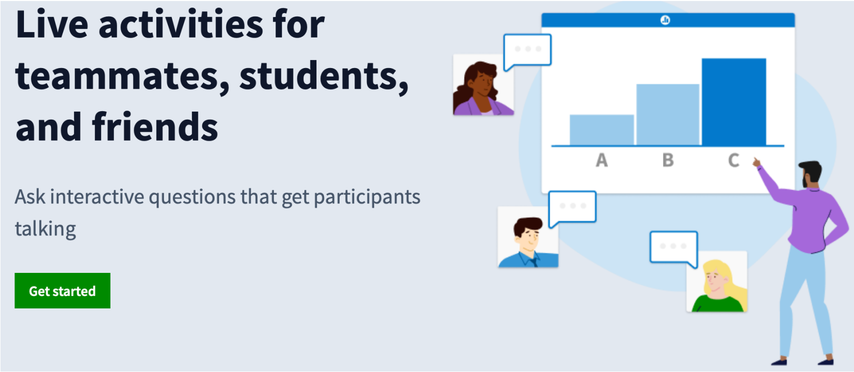
- Course Information.
- Quiz.
- Topics to be discussed today:
  - Angular momentum.
  - Conservation of angular momentum at the macroscopic and the microscopic level.
  - Precession.

Course Information

- Homework set # 8 is due on Friday November 8.
- The collisions of Lab # 5 will take place on Monday 11/11 in Spurrier Gym.
  - If you did not pickup your 12-pack, please pick up one today.
  - Make sure you empty your cans between now and 11/11 and bring your 12 empty cans to Spurrier Gym.
- Exam # 3 will take place on Tuesday 11/19 between 8 am and 9.20 am.
- Exam # 3 will cover the material of Chapters 8, 9, 10, and 11.

Quiz lecture 19.
PollEv.com/frankwolfs050

- The quiz today will have four questions.
- I will collect your answers electronically using the Poll Everywhere system.
- You have 60 seconds to answer each question.

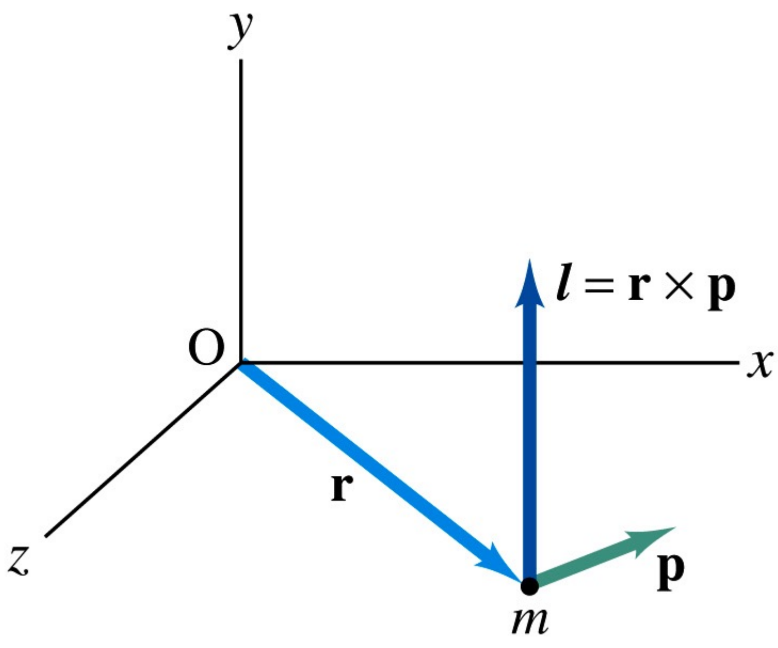


Live activities for teammates, students, and friends

Ask interactive questions that get participants talking

Angular momentum.
Definition.

- The angular momentum is defined as the vector product between the position vector and the linear momentum.
- Note:
  - Compare this definition with the definition of the torque.
  - Angular momentum is a vector.
  - The unit of angular momentum is kg m^2/s.
  - The angular momentum depends on both the magnitude and the direction of the position and linear momentum vectors.
  - Under certain circumstances the angular momentum of a system is conserved!



Conservation of angular momentum.

- Consider the change in the angular momentum of a particle:

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r}\times\vec{p}) = m\left(\vec{r}\times\frac{d\vec{v}}{dt} + \frac{d\vec{r}}{dt}\times\vec{v}\right) = m(\vec{r}\times\vec{a} + \vec{v}\times\vec{v}) =$$

$$= \vec{r}\times m\vec{a} = \vec{r}\times\sum\vec{F} = \sum\vec{\tau}$$

- When the net torque is equal to 0 Nm:

$$\sum\vec{\tau} = 0 = \frac{d\vec{L}}{dt} \Longrightarrow \vec{L} = \text{constant}$$

- When we take the sum of all torques, the torques due to the internal forces cancel and the sum is equal to torque due to all external forces.

Conservation of angular momentum. Planetary motion.

- Consider planetary motion:

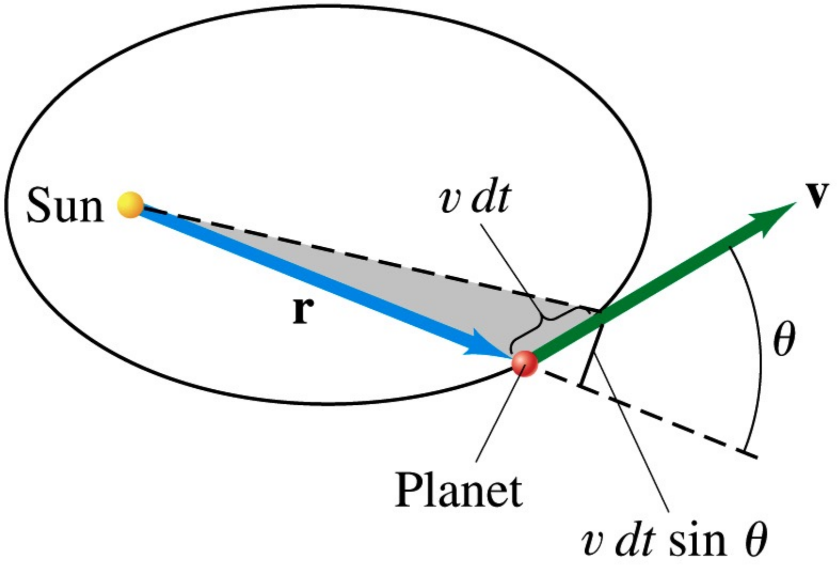
$$\vec{L} = \vec{r} \times \vec{p} = mrv\sin(\theta)\hat{z} =$$

$$= m\frac{rvdt\sin(\theta)}{dt}\hat{z} = 2m\frac{dA}{dt}\hat{z}$$

- The gravitational force is an internal force. In the absence of external forces, the angular momentum is conserved. We conclude that

$$\frac{dA}{dt}\text{=constant}$$

- This is of course Kepler's Law.



$$\frac{dA}{dt} = \text{constant}$$

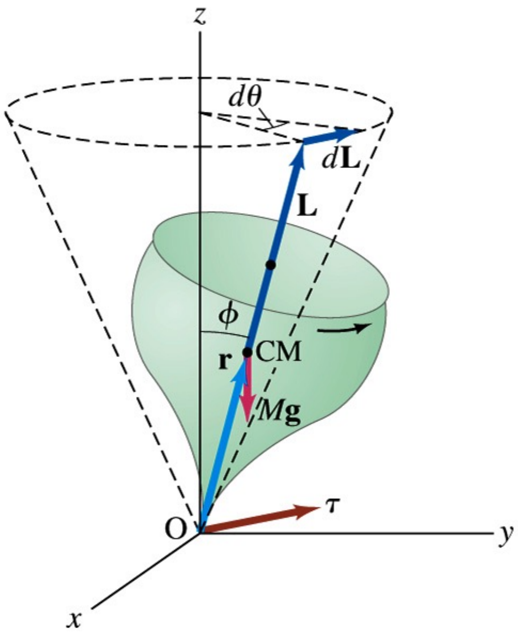
Conservation of angular momentum.

- The connection between the angular momentum $\vec{L}$ and the torque $\vec{\tau}$

$$\sum \vec{\tau} = \frac{d\vec{L}}{dt}$$

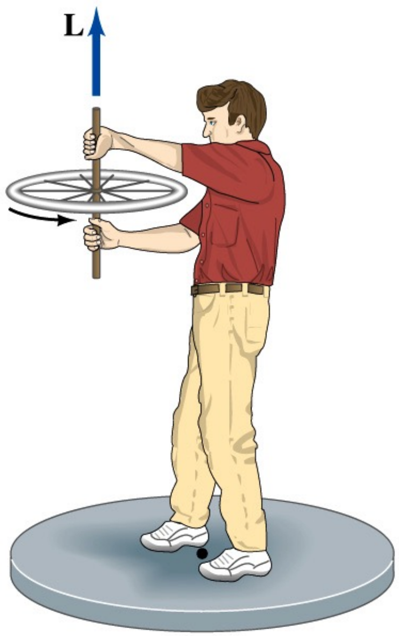
is only true if $\vec{L}$ and $\vec{\tau}$ are calculated with respect to the same reference point (which is at rest in an inertial reference frame).

- The relation is also true if $\vec{L}$ and $\vec{\tau}$ are calculated with respect to the center of mass of the object (note: the center of mass can accelerate).



Conservation of angular momentum. A demonstration.

- Ignoring the mass of the bicycle wheel, the external torque will be close to zero if we use the center of the disk as our reference point.
- Since the external torque is zero, angular momentum thus should be conserved.
- I can change the orientation of the wheel by applying internal forces. In which direction will I need to spin to conserve angular momentum?

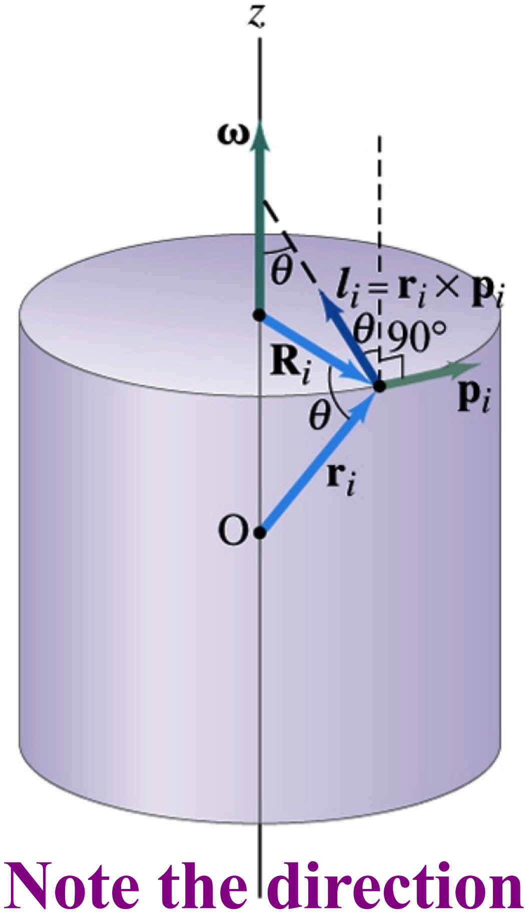


Angular momentum of rotating rigid objects.

- Consider a rigid object rotating around the *z* axis.
- The magnitude of the angular momentum of a part of a small section of the object is equal to

$$\left|\vec{l}_i\right| = r_i p_i \hat{l}_i$$

- Due to the symmetry of the object, we expect that the angular momentum of the object will be directed on the *z* axis. Thus, we only need to consider the *z* component of this angular momentum.



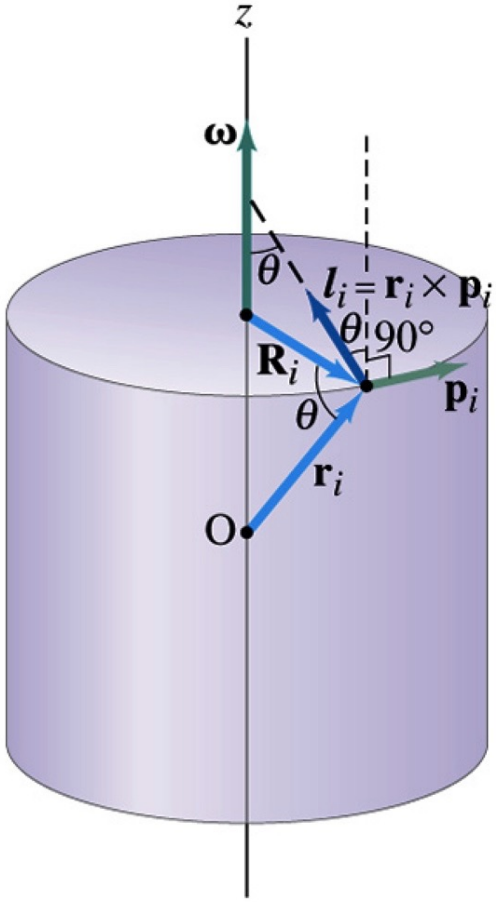
Note the direction of $\vec{l}_i$!!!!
(perpendicular to $\vec{r}_i$ and $\vec{p}_i$)

Angular momentum of rotating rigid objects.

- The z component of the angular momentum is
  $$l_{i,z} = r_i p_i \cos(\theta) = R_i p_i$$
  $$= m_i R_i v_i$$
- The total angular momentum of the rotating object is the sum of the angular momenta of the individual components:

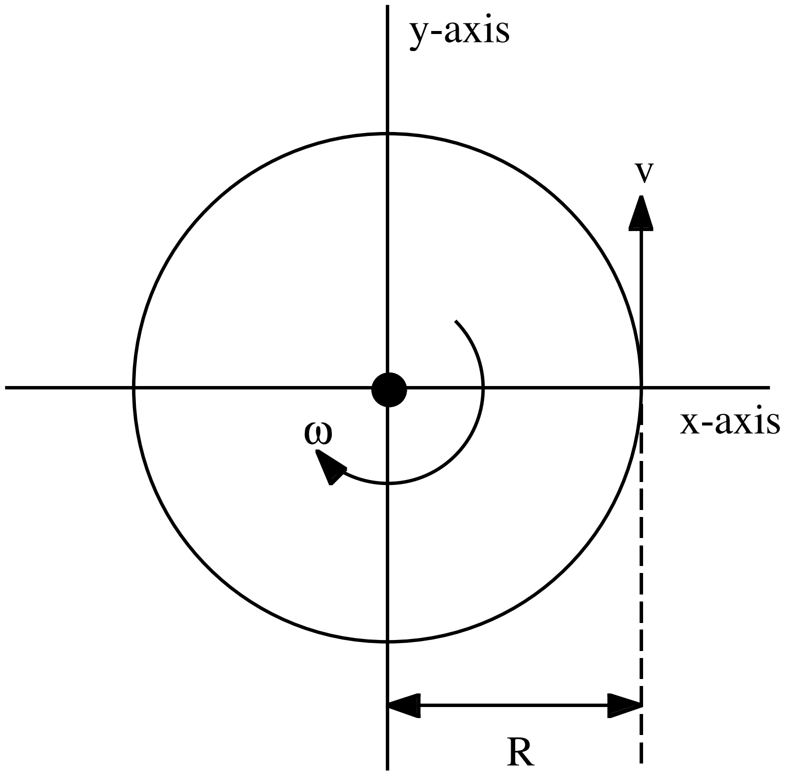
$$L_z = \sum_i l_{i,z} = \sum_i m_i R_i^2 \omega = I\omega$$

- The total angular momentum is thus equal to
  $$\vec{L} = I\omega\hat{z}$$



Conservation of angular momentum. Sample problem.

- A cockroach with mass m runs counterclockwise around the rim of a lazy Susan (a circular dish mounted on a vertical axle) of radius R and rotational inertia I with frictionless bearings. The cockroach's speed (with respect to the earth) is v, whereas the lazy Susan turns clockwise with angular speed ω_0. The cockroach finds a bread crumb on the rim and, of course, stops. (a) What is the angular speed of the lazy Susan after the cockroach stops? (b) Is mechanical energy conserved ?



Conservation of angular momentum. Sample problem.

- The initial angular momentum of the cockroach is

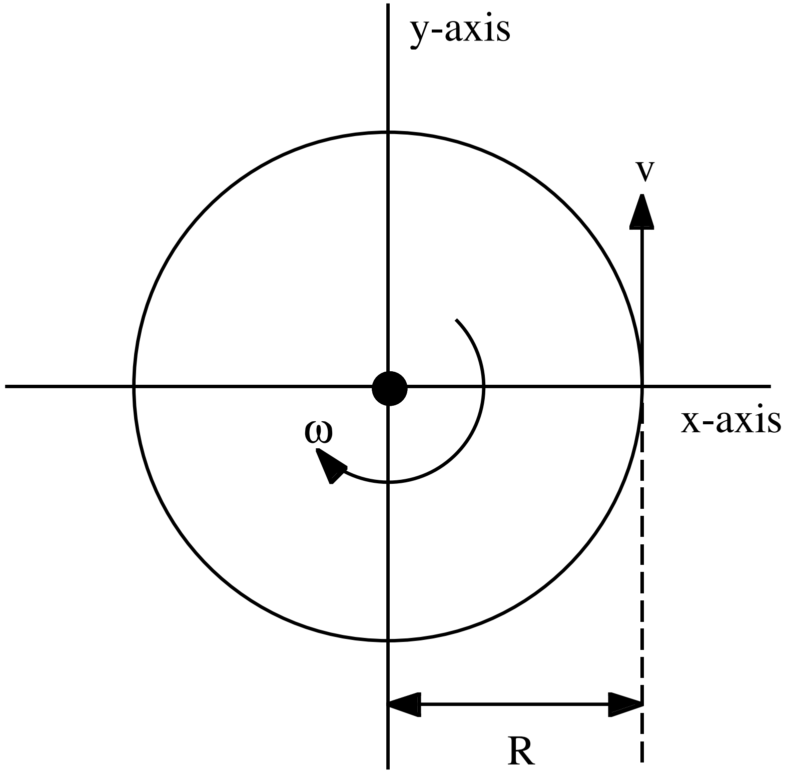
$$\vec{L}_c = \vec{r}_c \times \vec{p}_c = Rmv\hat{z}$$

- The initial angular momentum of the lazy Susan is

$$\vec{L}_d = -I\omega_0\hat{z}$$

- The total initial angular momentum is thus equal to

$$\vec{L} = \vec{L}_c + \vec{L}_d = (Rmv - I\omega_0)\hat{z}$$



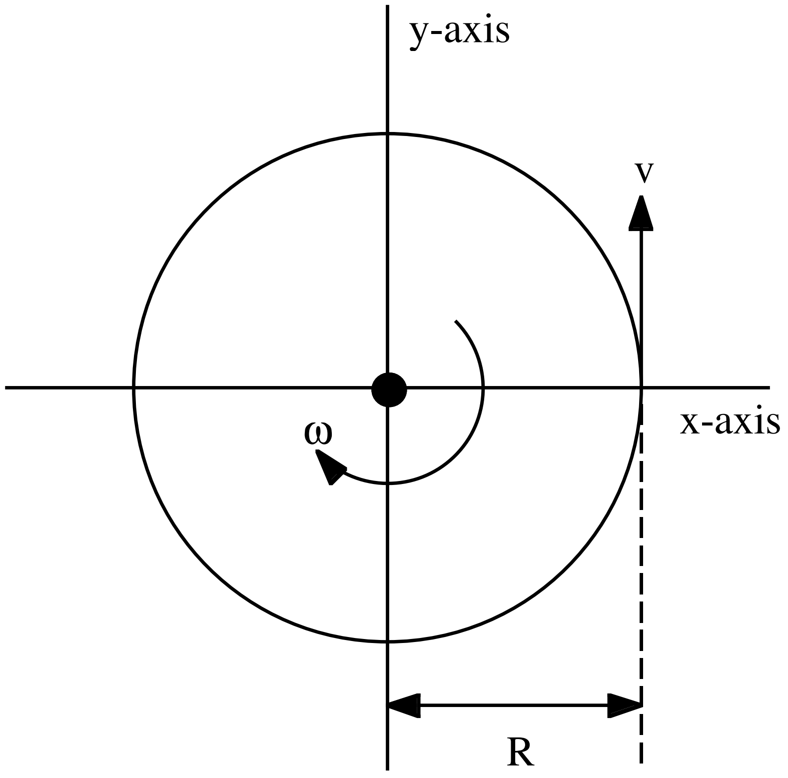
Conservation of angular momentum. Sample problem.

- When the cockroach stops, it will move in the same way as the rim of the lazy Susan. The forces that bring the cockroach to a halt are internal forces, and angular momentum is thus conserved.
- The moment of inertia of the lazy Susan + cockroach is equal to

$$I_f = I + mR^2$$

- The final angular velocity of the system is thus equal to

$$\omega = \frac{L_f}{I_f} = \frac{Rmv - I\omega_0}{I + mR^2}$$



Conservation of angular momentum. Sample problem.

- The initial kinetic energy of the system is equal to

$$K_i = \frac{1}{2}\underset{\underset{\textbf{Cockroach}}{\uparrow}}{mv^2} + \frac{1}{2}\underset{\underset{\textbf{Lazy Susan}}{\uparrow}}{I\omega_0^2} = \frac{1}{2}\frac{Imv^2 + I^2\omega_0^2 + m^2R^2v^2 + mR^2I\omega_0^2}{I + mR^2}$$

- The final kinetic energy of the system is equal to

$$K_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}(I + mR^2)\left(\frac{Rmv - I\omega_0}{I+mR^2}\right)^2 = \frac{1}{2}\frac{(Rmv - I\omega_0)^2}{I+mR^2}$$

- The change in the kinetic energy is thus equal to

$$\Delta K = \frac{1}{2}\frac{mI(-2Rv\omega_0 - v^2 - R^2\omega_0^2)}{I + mR^2} = -\frac{1}{2}\frac{mI}{I + mR^2}(v + R\omega_0)^2$$

3 Minute 55 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 3 minute 55 second intermission.

- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.
  - Solve a WeBWorK problem.

Conservation and angular momentum at the atomic and nuclear level.

- Particles at the atomic and nuclear level have two different forms of angular momentum:
  - Translational angular momentum: the angular momentum associated with the "orbital" motion of the particles. This angular momentum is also called the **orbital angular momentum**.
  - Rotational angular momentum: the angular momentum associated with the rotation of the particles around their symmetry axis. This angular momentum is called the **spin of the particle**.
- The angular momentum at the atomic and nuclear level is quantized; its projection along the x, y, or z axis is an integer or half-integer multiple of $\hbar = h/2\pi$.

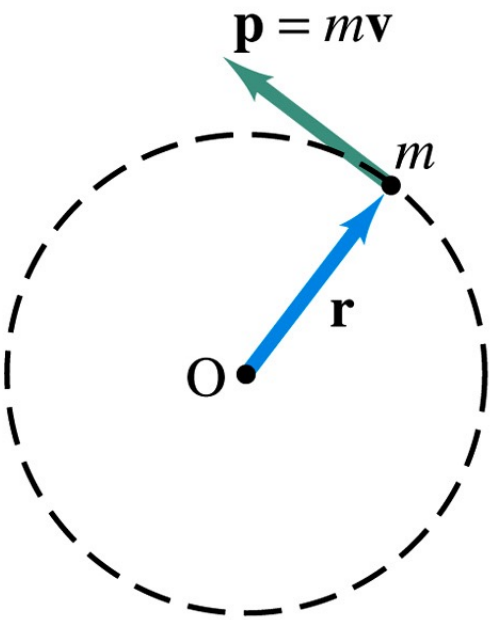
Implications of quantization of angular momentum.

- Consider the "classical" picture of the motion of electrons in atoms.
- If the angular momentum is a integer multiple of $\hbar$, the orbit must be such that

$$rp = N\hbar$$

- In order to carry out circular motion, the force on the electron must be equal to

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{mv^2}{r} = \frac{p^2}{mr}$$



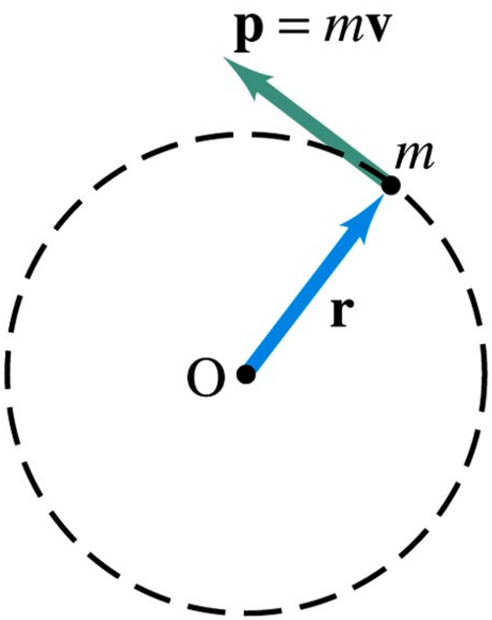
Implications of quantization of angular momentum.

- Eliminating p from the force equation shows us that

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{p^2}{mr} = \frac{\left(\frac{N\hbar}{r}\right)^2}{mr} = \frac{N^2\hbar^2}{mr^3}$$

- This equation can be used to determine the radius r:

$$r = 4\pi\varepsilon_0 \frac{N^2\hbar^2}{me^2}$$



Implications of quantization of angular momentum.

- The quantization of r results in a quantization of both the potential and the kinetic energy of the electron:

$$U = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} = -\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{me^4}{N^2\hbar^2}$$

$$K = \frac{1}{2}\frac{p^2}{m} = \frac{1}{2m}\frac{N^2\hbar^2}{r^2} = \frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{me^4}{N^2\hbar^2}$$

- The total energy of the electron is thus equal to

$$E = K + U = -\frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{me^4}{N^2\hbar^2} = -\frac{13.6}{N^2}\text{ eV}$$

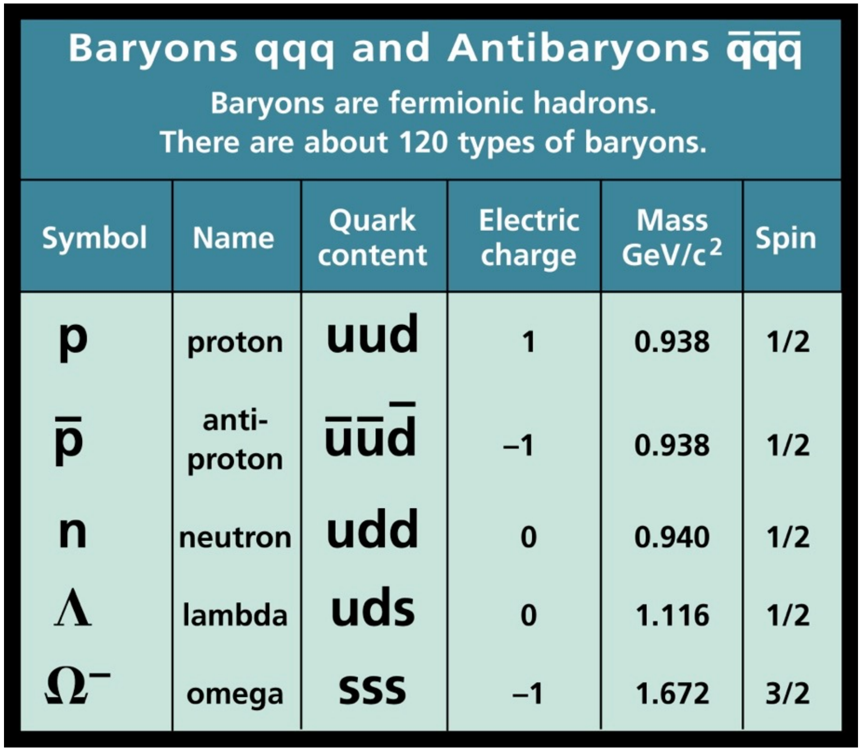
Implications of quantization of angular momentum.

- The energy levels of an electron in the hydrogen atom exactly match the levels predicted using this simple model, and the quantization of the energy levels is thus a direct consequence of the quantization of angular momentum.
- In addition to the orbital angular momentum of the electrons in the atom, they also poses spin. The projection of the spin of the electron on a particular axis will be either $+\frac{1}{2}\hbar$ or $-\frac{1}{2}\hbar$. It will **never** be zero. The electron is said to be a spin $\frac{1}{2}$ particle.
- Many other particles, such as muons, neutrinos, and quarks, are spin $\frac{1}{2}$ particles.

Implications of quantization of angular momentum.

- Since quarks are the building blocks of hadrons, we also expect that hadrons have a well-defined spin.
  - Hadrons that contain three quarks can either be spin $\frac{1}{2}$ or spin $\frac{3}{2}$.
  - Hadrons that contain two quarks can either be spin 0 or spin 1.
- The total spin of a particle limits how particles can be distributed across the various energy levels of the system.
  - If the spin is a half integer, the particle is called a **Fermion**, and it must obey the Pauli exclusion principle (two fermions can not be in the exact same quantum state).
  - If the spin is an integer, the particle is called a **Boson**, and it is not subject to the Pauli exclusion principle (there is not limit to the number of Bosons that can be in the exact same quantum state).

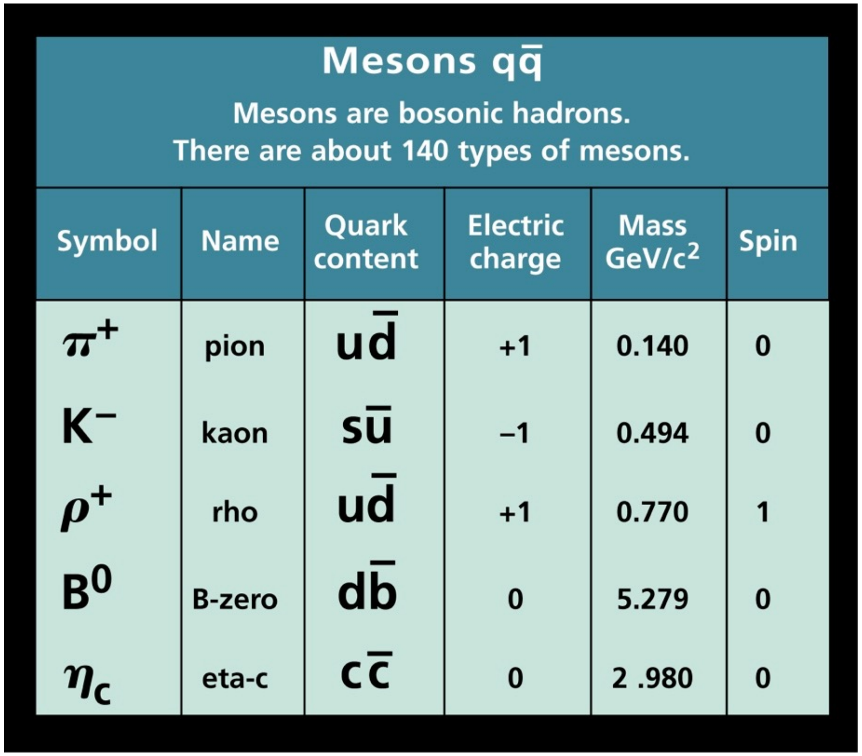
The building blocks of matter: combining quarks.



Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$

Baryons are fermionic hadrons.
There are about 120 types of baryons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
p	proton	uud	1	0.938	1/2
$\bar{p}$	anti-proton	$\bar{u}\bar{u}\bar{d}$	–1	0.938	1/2
n	neutron	udd	0	0.940	1/2
Λ	lambda	uds	0	1.116	1/2
Ω^-	omega	sss	–1	1.672	3/2



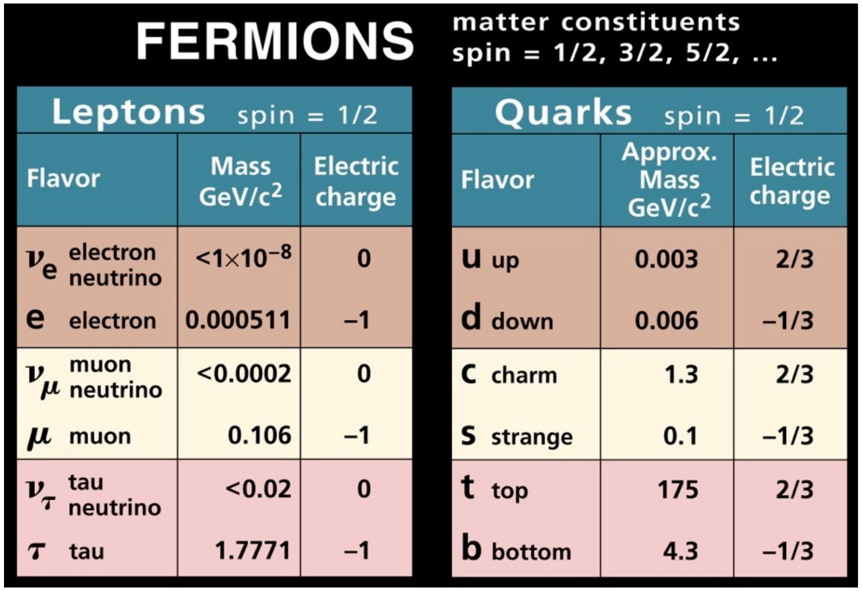
Mesons $q\bar{q}$

Mesons are bosonic hadrons.
There are about 140 types of mesons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	–1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.770	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2 .980	0

http://particleadventure.org/particleadventure/frameless/chart.html

The building blocks of matter: grouped according to spin.



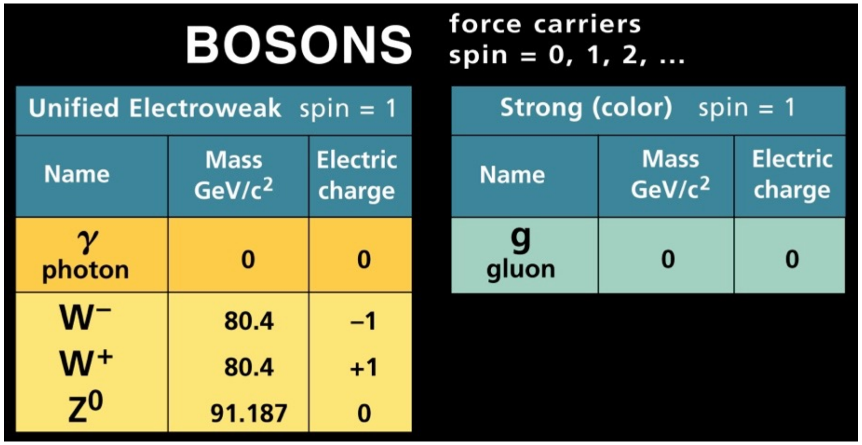
FERMIONS matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c^2	Electric charge
ν_e electron neutrino	<1×10^{-8}	0
e electron	0.000511	–1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	–1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	–1

Quarks spin = 1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	175	2/3
b bottom	4.3	–1/3



BOSONS force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W$^-$	80.4	–1
W$^+$	80.4	+1
Z^0	91.187	0

Strong (color) spin = 1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

http://particleadventure.org/particleadventure/frameless/chart.html

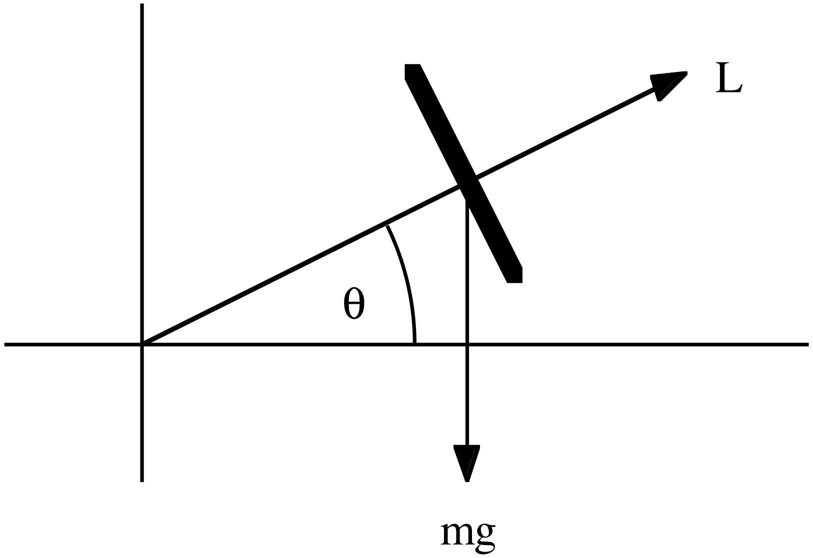
Implications of quantization of angular momentum.

- A final remark (before talking about precession):

  The spin of macroscopic objects will also be quantized, but the difference between different spin states is so small that it is impossible to observed effects of this quantization.

Precession.

- Consider a rotating rigid object spinning around its symmetry axis.
- The object carries a certain angular momentum *L*.
- Consider what will happen when the object is balanced on the tip of its axis (which makes an angle θ with the horizontal plane).
- The gravitational force, which is an external force, will generate a toques with respect of the tip of the axis.



Precession.

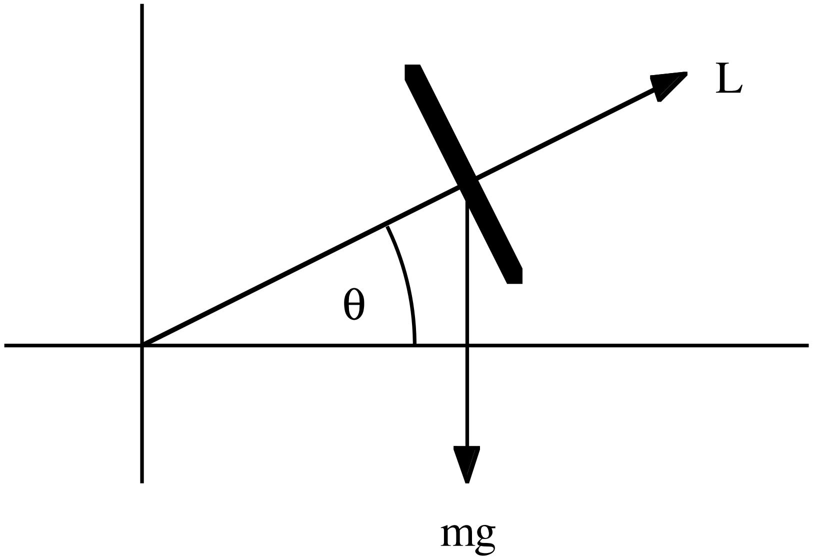
- The external torque is equal to

$$|\vec{\tau}| = \left|\vec{r}\times\vec{F}\right| = |\vec{r}|\left|\vec{F}\right|\sin\left(\frac{\pi}{2}+\theta\right)$$
$$= Mgr\cos(\theta)$$

- The external torque causes a change in the angular momentum:

$$d\vec{L} = \vec{\tau}dt$$

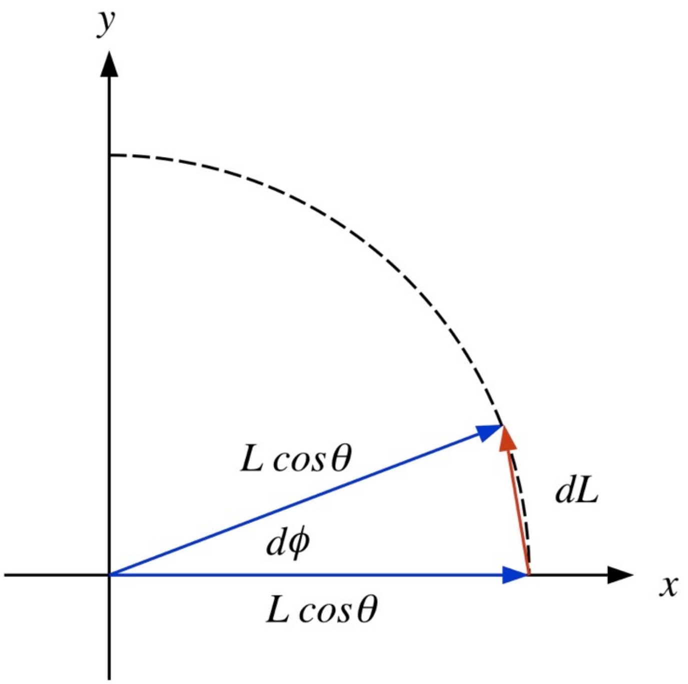
- Thus:
  - The change in the angular momentum points in the same direction as the direction of the torque.
  - The torque will thus change the direction of $\vec{L}$ but not its magnitude.



Precession.

- The effect of the torque can be visualized by looking at the motion of the projection of the angular momentum in the *xy* plane.
- The angle of rotation of the projection of the angular momentum vector when the angular momentum changes by $d\vec{L}$ is equal to

$$d\phi = \frac{dL}{L\cos(\theta)} = \frac{Mgr\cos(\theta)\,dt}{L\cos(\theta)}$$
$$= \frac{Mgrdt}{L}$$



Precession.

- Since the projection of the angular momentum during the time interval dt rotates by an angle $d\phi$, we can calculate the rate of precession:

$$\Omega = \frac{d\phi}{dt} = \frac{Mgr}{L} = \frac{Mgr}{I\omega}$$

- We conclude the following:
  - The rate of precessions does not depend on the angle θ.
  - The rate of precession decreases when the angular momentum increases.

Up next: equilibrium.