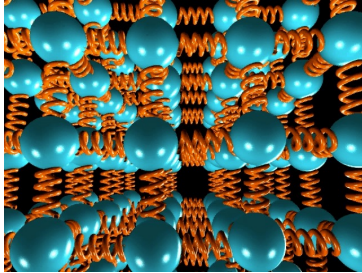


Physics 141, Lecture 7.



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1

New York Times 9/21/2023.
I directed the author to lecture 5.



...his year in space on a Thursday. At a news conference on Tuesday, Mr. Rubio spoke on video from the International Space Station about what he was most looking forward to when he returns home on Sept. 27: his family, fresh food and silence.

"For me, honestly, obviously, hugging my wife and kids, going to be paramount, and I'll probably focus on that for the first couple days," Mr. Rubio said as he bobbed gently in zero gravity.

He said he was also looking forward to being back in his quiet backyard and enjoying the trees and the silence.

His return home will be even more because of Mr. Rubio

...been asked to do a mission before train would have declined his family. But, he had asked him to deeper into his twing, he would have cause it was his job.

He acknowledged space, away from taken a psychology it was important mentally because tion's "very unromantic."

"One thing that and hopefully has certainly haven't — is just to kind and stay steady

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Outline.

- Course information:
 - Exam # 1
- Quiz.
- Continuation of the discussion of Chapter 4:
 - Simple harmonic motion.
 - Damped harmonic motion.
 - Driven harmonic motion.

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Exam 1.

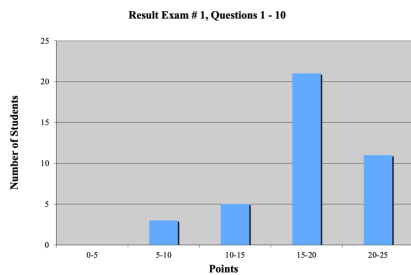
- The results of exam # 1 will be distributed via email on Monday.
- The exam will be returned next week during recitations.
- If you do only one thing: compare the points you see on the blue booklets with the points in your email. It is easy to make a mistake during data entry.
- Compare your solutions with the posted solutions.
- If you are unhappy with how your exam was graded:
 - Write a note explaining why you feel you deserve more points.
 - Hand you note and your exam booklet(s) to me before or during class on October 1.
 - Do **not** ask your TA to modify your grade.

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First results Exam # 1: 10% of the scantron forms had incorrect student IDs.



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First results Exam # 1: 50% of all students failed problem 2.

Problem 2 (2.5 points)

Answer on Scantron form

You measure the length of a plate using a ruler, as shown in Fig. 2.

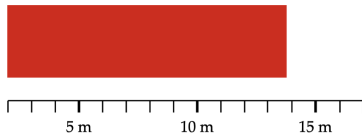


Figure 2: The measurement of the length of a plate.

What is your best estimate of the length of the plate (in units of meters)?

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Quiz lecture 07.
[PollEv.com/frankwolfs050](https://pollEv.com/frankwolfs050)

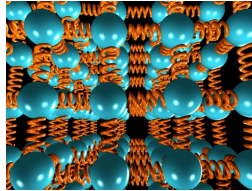
- The quiz today will have three questions.
- I will collect your answers electronically using the Poll Everywhere system.
- The answers for each question will be entered in sequence (first 60 s for question 1, followed by 60 s for question 2, etc.).



7

A quick review of the material discussed in
Lecture 6.

- We can visualize a solid as a collection of atoms of mass m , interconnected by springs.
- The atoms are not at rest in a solid, but continuously vibrate around an equilibrium position.
- The temperature of the solid is a measure of the kinetic energy associated with the motion of the atoms.
- This simple model can explain many important properties of matter, but many others can only be explained in terms of quantum mechanics.



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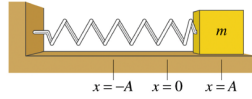
The spring-mass system.

- The key to the understanding of the atomic model of matter is the understanding of the spring-like interaction between the atoms.
- Since matter will never be at the absolute zero temperature, the atoms will have a non-zero average kinetic energy (proportional to the temperature of the matter).
- Since the atoms will move, the "springs" in our model will carry out a dynamic motion which we will need to understand in more detail.

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The spring-mass system.

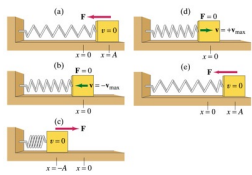
- For the spring force we know:
 - Its direction is opposite to the displacement.
 - Its magnitude is $k|x|$.
- Consider the force acting on mass m when it is located at position x :
 - $F = -kx$
 - But we also know that $F = ma$
 - Thus $a = \frac{d^2x}{dt^2} = -\frac{k}{m}x$



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The spring-mass system.



- The displacement of the spring as function of time can thus be determined if we can solve the following equation:

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$
- This equation has two possible solutions:
 - $x(t) = A\cos(\omega t + \phi)$
 - $x(t) = A\sin(\omega t + \phi)$
 where $\omega^2 = k/m$.
- This motion is an example of simple harmonic motion.

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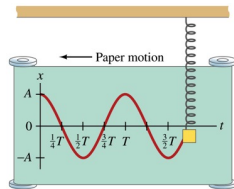
Simple harmonic motion.

Phase Constant

Amplitude

$$x(t) = A\cos(\omega t + \phi)$$

Angular Frequency



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Simple harmonic motion.

- Instead of the angular frequency ω the motion can also be described in terms of its period T or its frequency ν .
- The period T is the time required to complete one oscillation:

$$x(t) = x(t + T)$$

or

$$A\cos(\omega t + \phi) = A\cos(\omega t + \omega T + \phi)$$

- In order for this to be true, we must require that $\omega T = 2\pi$. The period T is thus equal to $2\pi/\omega$.
- The frequency ν is the number of oscillations carried out per second ($\nu = 1/T$). The unit of frequency is the Hertz (Hz). Per definition, $1 \text{ Hz} = 1 \text{ s}^{-1}$.

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Simple harmonic motion. What forces are required?

- Consider we observe simple harmonic motion.
- The observation of the equation of motion can be used to determine the nature of the force that generates this type of motion.
- In order to do this, we need to determine the acceleration of the object carrying out the harmonic motion:

$$x(t) = A\cos(\omega t + \phi)$$

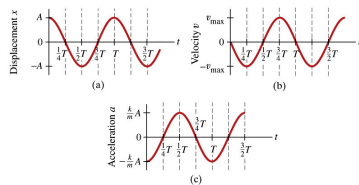
$$v(t) = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi)$$

$$a(t) = \frac{dv}{dt} = -\omega^2 A\cos(\omega t + \phi) = -\omega^2 x(t)$$

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Simple harmonic motion. What forces are required?



Note: maxima in displacement correlate with minima in acceleration.

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Simple harmonic motion. What forces are required?

- Using Newton's second law we can determine the force responsible for the harmonic motion:

$$F = ma = -m\omega^2 x$$

- We conclude:

Simple harmonic motion is the motion executed by a particle of mass m , subject to a force F that is proportional to the displacement of the particle, but opposite in sign.

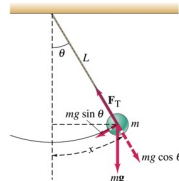
- Any force that satisfies this criterion **can** produce simple harmonic motion. If more than one force is present, you need to examine the net force, and make sure that the net force is proportional to the displacements, but opposite in sign.

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Simple harmonic motion (SHM). The simple pendulum.

- Consider a simple pendulum.

- A simple pendulum is a pendulum for which all the mass is located at a single point at the end of a massless string.
- There are two forces acting on the mass: the tension T and the gravitational force mg .
- The tension T cancels the radial component of the gravitational force when $|x|$ and $|\theta|$ reach their maxima. At all other positions, the net radial force is pointing in the same direction as the tension T and provides the required centripetal acceleration.



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Simple harmonic motion (SHM). The simple pendulum.

- The net force acting on the mass is directed perpendicular to the string and is equal to

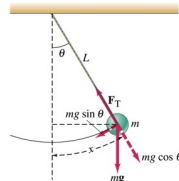
$$F = -mg \sin \theta$$

The minus sign indicates that the force is directed opposite to the angular displacement.

- When the angle θ is small, we can approximate $\sin \theta$ by θ :

$$F \approx -mg\theta = -mg \frac{x}{L}$$

- Note: the force is again proportional to the displacement.



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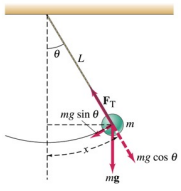
Simple harmonic motion (SHM). The simple pendulum.

- The equation of motion for the pendulum is thus

$$F = m \frac{d^2xy}{dt^2} = -mg \frac{x}{L}$$
 or

$$\frac{d^2xy}{dt^2} = -\frac{g}{L}x$$
- The equation of motion is the same as the equation of motion for a SHM, and the pendulum will thus carry out SHM with an angular frequency $\omega = \sqrt{g/L}$.
- The period T of the pendulum is

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{L}{g}}$$
- T is independent of mass m.



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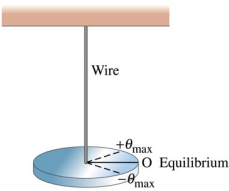
19

Simple harmonic motion (SHM). The torsion pendulum.

- What is the angular frequency of the SHM of a torsion pendulum:
 - When the base is rotated, it twists the wire and the wire generates a torque which is proportional to the twist angle:

$$\tau = -K\theta$$
 The torque generates an angular acceleration α :


$$\alpha = \frac{d^2\theta}{dt^2} = \frac{\tau}{I} = -\frac{K}{I}\theta$$
- The resulting motion is harmonic motion with an angular frequency $\omega = \sqrt{K/I}$.




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3 Minute 35 Second Intermission



- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 3 minute 35 second intermission.
- You can:
 - Stretch out.
 - Talk to your neighbors.
 - Ask me a quick question.
 - Enjoy the fantastic music.
 - Go asleep, as long as you wake up in 3 minutes and 35 seconds.



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Damped harmonic motion.

- Consider what happens when in addition to the restoring force a damping force (such as the drag force) is acting on the system:

$$F = -kx - b \frac{dx}{dt}$$

- The equation of motion is now given by:

$$\frac{d^2x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \frac{k}{m} x = 0$$

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Damped harmonic motion.

- The general solution of this equation of motion is

$$x(t) = Ae^{i\omega t}$$

- If we substitute this solution into the equation of motion we find

$$-\omega^2 Ae^{i\omega t} + i\omega \frac{b}{m} Ae^{i\omega t} + \frac{k}{m} Ae^{i\omega t} = 0$$

- In order to satisfy the equation of motion, the angular frequency must satisfy the following condition:

$$\left(\omega^2 - i\omega \frac{b}{m} - \frac{k}{m} \right) Ae^{i\omega t} = 0$$

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Damped harmonic motion.

- We can solve this equation and determine the two possible values of the angular velocity:

$$\omega = \frac{1}{2} \left(i \frac{b}{m} \pm \sqrt{4 \frac{k}{m} - \frac{b^2}{m^2}} \right) \approx \frac{1}{2} i \frac{b}{m} \pm \sqrt{\frac{k}{m}}$$

- The solution to the equation of motion is thus given by

$$x(t) \approx x_m e^{-\frac{b}{2m} t} e^{i \sqrt{\frac{k}{m}} t}$$

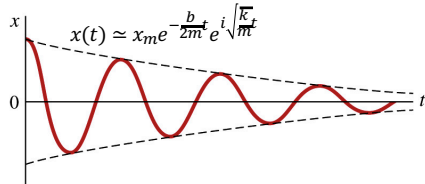
↑ Damping Term
 ↑ SHM Term

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Damped harmonic motion.



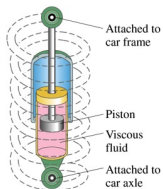
The general solution contains a SHM term, with an amplitude that decreases as function of time

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Damped harmonic motion has many practical applications.



Damping is not always a curse.

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Driven harmonic motion.

- Consider what happens when we apply a time-dependent force $F(t)$ to a system that normally would carry out SHM with an angular frequency ω_0 .

- Assume the external force $F(t) = mF_0 \sin(\omega t)$. The equation of motion can now be written as

$$\frac{d^2x}{dt^2} = -\omega_0^2 x + \frac{mF_0 \sin(\omega t)}{m} = -\omega_0^2 x + F_0 \sin(\omega t)$$

- The steady state motion of this system will be harmonic motion with an angular frequency equal to the angular frequency of the driving force.

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Driven harmonic motion.

- Consider the general solution

$$x(t) = A \cos(\omega t + \phi)$$

- The parameters in this solution must be chosen such that the equation of motion is satisfied. This requires that

$$-\omega^2 A \cos(\omega t + \phi) + \omega_0^2 A \cos(\omega t + \phi) - F_0 \sin(\omega t) = 0$$

- This equation can be rewritten as

$$A(\omega_0^2 - \omega^2)(\cos(\omega t) \cos(\phi) - \sin(\omega t) \sin(\phi)) - F_0 \sin(\omega t) = 0$$

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Driven harmonic motion.

- Our general solution must thus satisfy the following condition:

$$(\omega_0^2 - \omega^2)A \cos(\omega t) \cos(\phi) - \{(\omega_0^2 - \omega^2)A \sin(\phi) - F_0\} \sin(\omega t) = 0$$

- Since this equation must be satisfied at all time, we must require that the coefficients of $\cos(\omega t)$ and $\sin(\omega t)$ are 0. This requires that

$$(\omega_0^2 - \omega^2)A \cos(\phi) = 0$$

and

$$(\omega_0^2 - \omega^2)A \sin(\phi) - F_0 = 0$$

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Driven harmonic motion.

- The interesting solutions are solutions where $A \neq 0$ and $\omega \neq \omega_0$. In this case, our general solution can only satisfy the equation of motion if

$$\cos(\phi) = 0$$

and

$$(\omega_0^2 - \omega^2)A \sin(\phi) - F_0 = (\omega_0^2 - \omega^2)A - F_0 = 0$$

- The amplitude of the motion is thus equal to

$$A = \frac{F_0}{\omega_0^2 - \omega^2}$$

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Driven harmonic motion.

- If the driving force has a frequency close to the natural frequency of the system, the resulting amplitudes can be very large even for small driving amplitudes. The system is said to be in resonance.
- In realistic systems, there will also be a damping force. Whether or not resonance behavior will be observed will depend on the strength of the damping term.

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Driven harmonic motion.

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That's all for today!

Next lecture: force, motion, and energy.

"Rush Hour in Reno"

Credit and Copyright: John Endter of Minden, Nevada

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