Do not turn the pages of the exam until you are instructed to do so.

Exam rules: You may use **only** a writing instrument while taking this test. You may **not** consult any calculators, computers, books, nor each other.

Problems 1 and 10 must be answered on the scantron form. Problems 11, 12, and 13 must be answered in exam booklet 1. Problems 14 and 15 must be answered in exam booklet 2. Problems 16 and 17 must be answered in exam booklet 3.

Your answers need to be well motivated and expressed in terms of the variables used in the problem. You will receive partial credit where appropriate, but only when we can read your solution. Answers that are not motivated will not receive any credit, even if correct.

At the end of the exam, you must hand in your exam, the scantron form, the blue exam booklets, and the equation sheet. All items must be clearly labeled with your name, your student ID number, and the day/time of your recitation. **If any of these items are missing, we will not grade your exam, and you will receive a score of 0 points.**

You are required to complete the following *Honor Pledge for Exams*. Copy and sign the pledge before starting your exam.

"I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own."

_____

_____

_____

Name: _____

Signature: _____

INTENTIONALLY LEFT BLANK

$$\cos(30°) = \frac{1}{2}\sqrt{3} \quad \sin(30°) = \frac{1}{2} \quad \tan(30°) = \frac{1}{3}\sqrt{3}$$

$$\cos(45°) = \frac{1}{2}\sqrt{2} \quad \sin(45°) = \frac{1}{2}\sqrt{2} \quad \tan(45°) = 1$$

$$\cos(60°) = \frac{1}{2} \quad \sin(60°) = \frac{1}{2}\sqrt{3} \quad \tan(60°) = \sqrt{3}$$

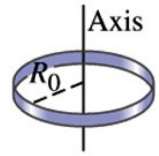
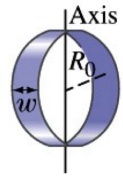
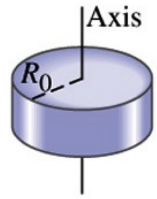
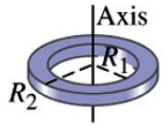
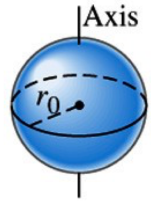
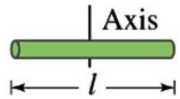
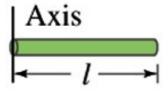
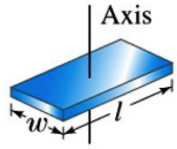
$$\cos\left(\frac{1}{2}\pi - \theta\right) = \sin(\theta) \quad \sin\left(\frac{1}{2}\pi - \theta\right) = \cos(\theta)$$

$$\cos(2\theta) = 1 - 2\sin^2(\theta) \quad \sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

	Circle	Sphere
circumference	$2\pi r$	
(surface) area	πr^2	$4\pi r^2$
volume		$\frac{4}{3}\pi r^3$

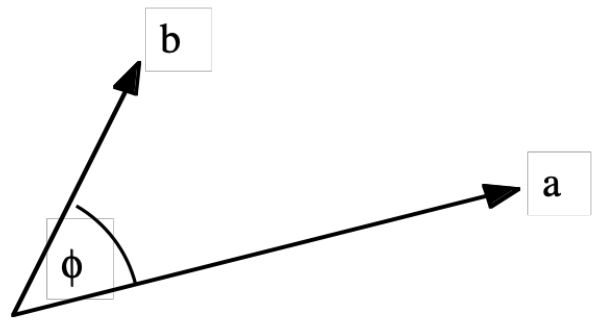
INTENTIONALLY LEFT BLANK

Moments of inertia of various objects of uniform composition.

(a)	Thin hoop of radius R_0	Through center		MR_0^2
(b)	Thin hoop of radius R_0 and width w	Through central diameter		$\frac{1}{2}MR_0^2 + \frac{1}{12}Mw^2$
(c)	Solid cylinder of radius R_0	Through center		$\frac{1}{2}MR_0^2$
(d)	Hollow cylinder of inner radius R_1 and outer radius R_2	Through center		$\frac{1}{2}M(R_1^2 + R_2^2)$
(e)	Uniform sphere of radius r_0	Through center		$\frac{2}{5}Mr_0^2$
(f)	Long uniform rod of length l	Through center		$\frac{1}{12}Ml^2$
(g)	Long uniform rod of length l	Through end		$\frac{1}{3}Ml^2$
(h)	Rectangular thin plate, of length l and width w	Through center		$\frac{1}{12}M(l^2 + w^2)$

INTENTIONALLY LEFT BLANK

Properties of the scalar product



$$\vec{a} \bullet \vec{b} = |\vec{a}||\vec{b}|\cos\phi \tag{1}$$

$$\vec{a} \bullet \vec{a} = |\vec{a}|^2 \tag{2}$$

$$\left(\vec{a}+\vec{b}\right) \bullet \left(\vec{a}+\vec{b}\right) = \vec{a} \bullet \vec{a} + \vec{a} \bullet \vec{b} + \vec{b} \bullet \vec{a} + \vec{b} \bullet \vec{b} = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \bullet \vec{b} \tag{3}$$

INTENTIONALLY LEFT BLANK

Good Luck !

INTENTIONALLY LEFT BLANK

Problem 1 (2.5 points) **Answer on Scantron form**

What did **NOT** happen on December 5, 2023?

Figure 1: Sinterklaas.

1. Good Dutch children received presents.
2. Professor Wolfs taught a lecture and told us about Sinterklaas.
3. Sinterklaas celebrated his birthday.
4. Naughty Dutch children were put in a bag.

Problem 2 (2.5 points) **Answer on Scantron form**

If all three collisions shown in Fig. 2 are totally inelastic, which collisions cause(s) the most damage?

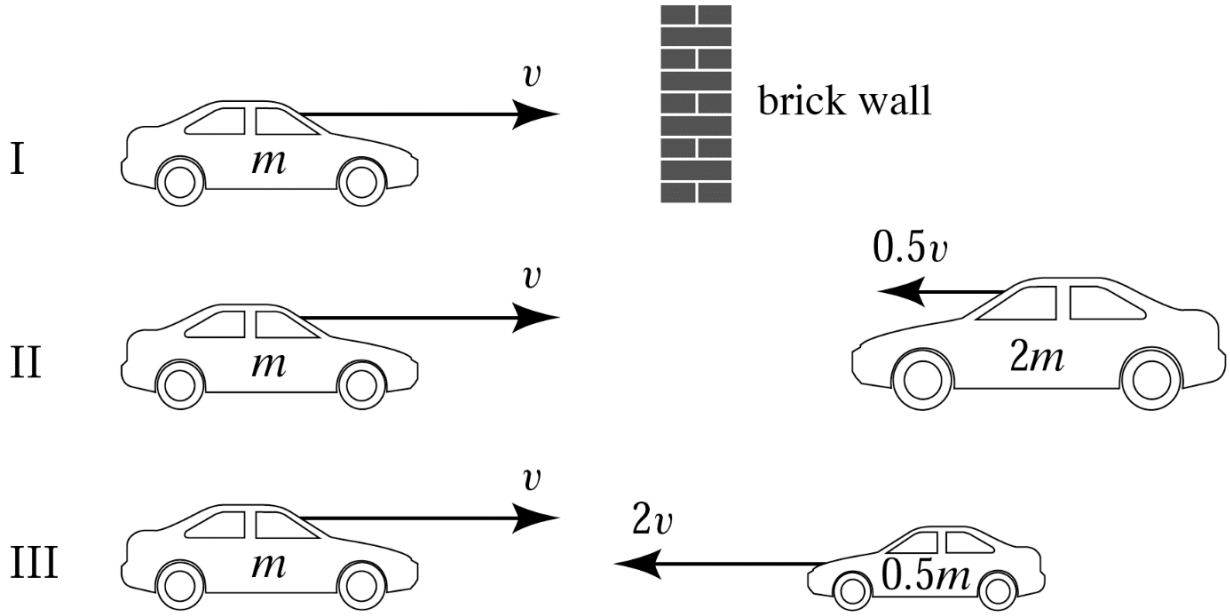


Figure 2: Various car collisions.

1. I.
2. II.
3. III.
4. I and II.
5. I and III.
6. II and III.
7. All three.

Problem 3 (2.5 points) **Answer on Scantron form**

Which of the following statements is **false**?

1. The buoyant force in a liquid is much larger than the buoyant force in air.
2. The buoyant force is a result of small differences in the molecular density of the air/liquid across the surface of the object.
3. The buoyant force is a result of differences in the average molecular velocity of the air/liquid molecules across the surface of the object.
4. The magnitude of the buoyant force increases with increasing temperature, due to the corresponding increase in the average molecular velocities.

Problem 4 (2.5 points) **Answer on Scantron form**

The coefficient of performance of a Carnot engine operated as a heat pump is

1. $1 - T_L/T_H$
2. $(1 - T_L/T_H)^{-1}$
3. $(T_H/T_L - 1)^{-1}$
4. $T_H/T_L - 1$

In these equations, T_H is the temperature of the hot reservoir and T_L is the temperature of the cold reservoir.

Problem 5 (2.5 points) **Answer on Scantron form**

You have two identical springs, connected in parallel. When you hang a mass m from this system, as shown in Fig. 3, the new equilibrium position of the system is a distance d below the equilibrium position when no mass is connected to the system.

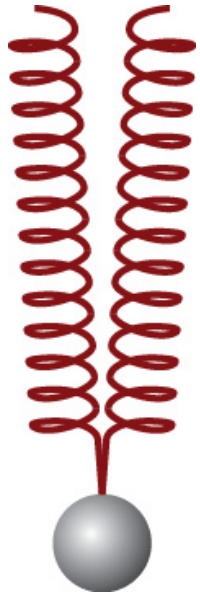


Figure 3: A parallel spring system.

Now you connect the two springs in series. The system is in equilibrium when you connect mass m to the end of the lower spring. What is the displacement of mass m when it has reached its new equilibrium position?

1. $4d$
2. $2d$
3. $\sqrt{2}d$
4. d
5. $d/\sqrt{2}$
6. $d/2$
7. $d/4$

Problem 6 (2.5 points) **Answer on Scantron form**

A cart rolls with low friction on a track. A fan is mounted on the cart. When the fan is turned on, there is a constant force acting on the cart. Three different experiments are performed:

a) Fan off: The cart is originally at rest. You give it a brief push, and it coasts a long distance along the $+x$ direction, slowly coming to a stop.

b) Fan forward: The fan is turned on and you hold the cart stationary. You then take your hand away and the cart moves forward in the $+x$ direction. After travelling a long distance along the track, you quickly stop and hold the cart.

c) Fan backward: The fan is turned on facing the "wrong" direction and you hold the cart stationary. You give it a brief push and the cart moves in the $+x$ direction, slowing down, turning around, and returning to the starting position where you quickly stop and hold the cart.

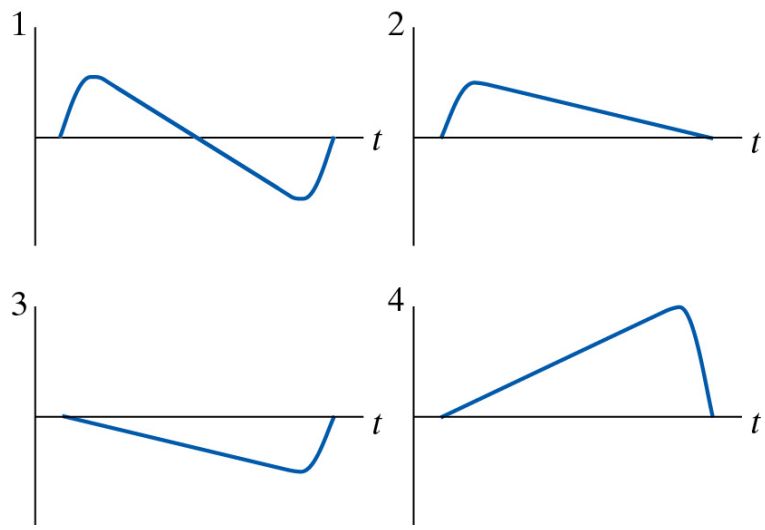


Figure 4: Linear momentum versus time.

Figure 4 shows the x component of the linear momentum of the car as a function of time. Match the three experiments with the correct graphs.

1. a = 1, b = 2, c = 3
2. a = 1, b = 3, c = 2
3. a = 1, b = 4, c = 2
4. a = 2, b = 1, c = 4
5. a = 2, b = 3, c = 1
6. a = 2, b = 4, c = 1
7. a = 3, b = 4, c = 1
8. a = 3, b = 1, c = 4
9. a = 4, b = 2, c = 3
10. a = 4, b = 2, c = 1

Problem 7 (2.5 points) **Answer on Scantron form**

Two masses M_1 and M_2 sit on a table connected by a rope, as shown in Fig. 5. A second rope is attached to the opposite side of M_2. Both masses are pulled along the table with the tension in the second rope equal to T_2.

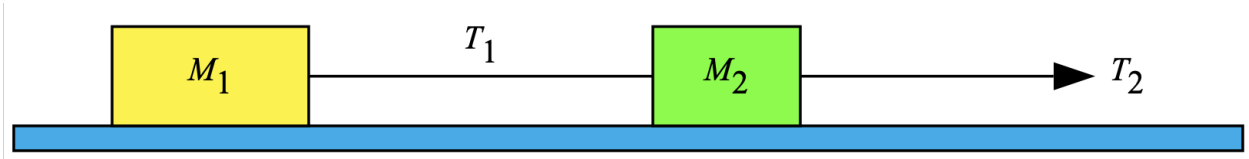


Figure 5: Two masses being pulled.

Let T_1 denote the tension in the rope connecting the two masses. Which of the following statements is true?

1. $T_1 = T_2$
2. $T_1 > T_2$
3. $T_1 < T_2$
4. We need to know the relative values of M_1 and M_2 to answer this question.

Problem 8 (2.5 points) **Answer on Scantron form**

The linear density of a long thin rod of cross-sectional area A and length L decreases linearly from a value of ρ_0 at the left end to zero at the right end. How far from the left end is the rod's center-of-mass located?

1. $(1/5)L$
2. $(1/4)L$
3. $(1/3)L$
4. $(2/5)L$
5. $(1/2)L$
6. $(3/5)L$
7. $(2/3)L$
8. $(3/4)L$
9. $(4/5)L$

Problem 9 (2.5 points) **Answer on Scantron form**

The specific heat capacity of a diatomic gas as function of temperature is shown in Fig. 6.

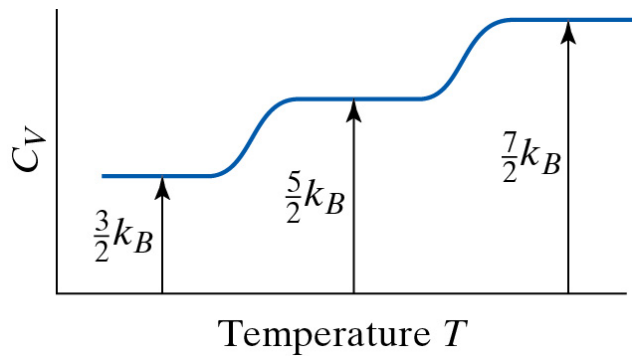


Figure 6: Heat capacity C_V of a diatomic gas as function of temperature T.

At a certain temperature T, the specific heat capacity C_V of the diatomic gas is measured to be $(5/2)k_B$. What degrees of freedom are accessible at this temperature?

1. Translational and rotational.
2. Translational and vibrational.
3. Rotational and vibrational.
4. Translational and electronic.
5. Electronic and rotational.
6. Electronic and vibrational.
7. Translational, electronic, rotational, and vibrational.
8. Electronic, rotational, and vibrational

Problem 10 (2.5 points) **Answer on Scantron form**

A gas is made up of diatomic molecules. At temperature T, the ratio of the number of molecules in vibrational energy state 2 to the number of molecules in the ground state is measured to be 0.35. The difference in energy between state 2 and the ground state is ΔE. Which of the following conclusions is correct?

1. $\Delta E \approx k_B T$
2. $\Delta E \ll k_B T$
3. $\Delta E \gg k_B T$

Problem 11 (25 points) **Answer in booklet 1**

A cylinder with cross sectional area A contains N molecules of nitrogen gas at pressure p_0. The gas is in thermal equilibrium with a heat bath of temperature T_0. A piston confines the gas inside a region of volume V_0. The entire system is contained in a vacuum vessel, and only the nitrogen gas exerts a pressure on the piston.

a) You quickly pull up the piston to increase the volume of the gas to V_f. What is the temperature T_f of the gas immediately after you finish pulling up the piston? What approximations did you make?

b) What is the work done by the gas during this expansion?

c) What is the force you must exert on the piston, immediately after you finish pulling it up, in order to hold it into its final position?

d) You wait while the nitrogen returns back to its original temperature T_0. What is now the force you must exert on the piston in order to hold it into its final position?

e) You now very slowly move the piston back to its original position such that the gas is contained again in a volume V_0. How much work must you do to move the piston back to this position? What approximations did you make?

Your answer needs to be well motivated and expressed in terms of the variables provided and $\gamma = C_p/C_V$.

Problem 12 (25 points) **Answer in booklet 1**

Three bodies of identical mass m rotate in circular orbits of radius r around their center of mass, as shown in Fig. 7. The system is held together by their mutual gravitational forces.

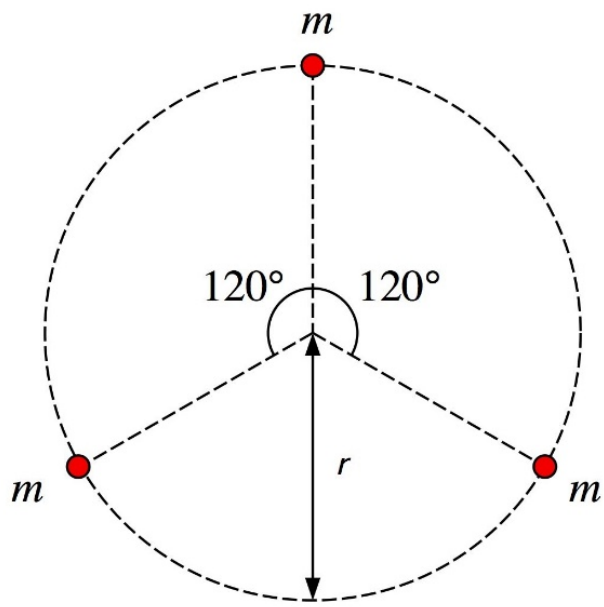


Figure 7: A three-body system.

a) Using a diagram, indicate the direction of the net force acting on each body.

b) What is the magnitude of the net force acting on each body?

c) What is the orbital period of each body?

Your answer needs to be well motivated and expressed in terms of the variables provided and G.

Problem 13 (25 points) **Answer in booklet 1**

Consider a two-dimensional elastic collision involving particles of equal mass m in which one of the particles is initially at rest, as shown in Fig. 8.

Figure 8: The collision system before the collision.

After the collision, the angle between the directions of these two particles is A, as shown in Fig. 9.

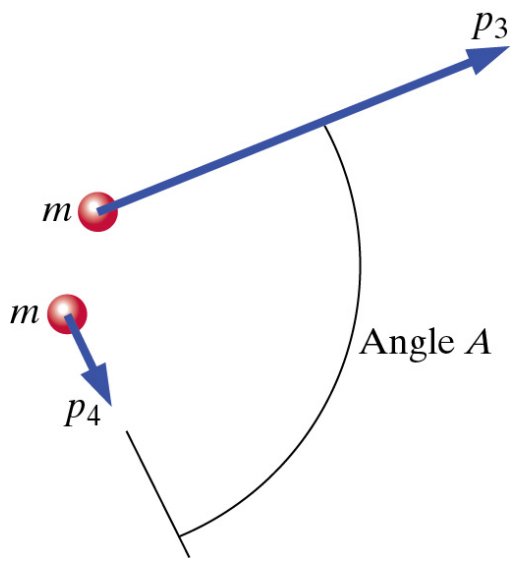


Figure 9: The collision system after the collision.

a) Use vector notation to write down the relation between the initial linear momentum of the incident particle, $\vec{p}_1$, and the linear momenta of the two particles after the collision, $\vec{p}_3$ and $\vec{p}_4$.

b) Use the scalar product to obtain an expression for the magnitude of $\vec{p}_1$ in terms of the magnitudes of $\vec{p}_3$ and $\vec{p}_4$ and the angle A. Note: see page 7 for details of the scalar product.

c) Use the relation you derived in part b) to obtain an expression that relates the kinetic energy of the incident particle, K_1, to the kinetic energies of the outgoing particles, K_3 and K_4.

d) Since the collision is elastic, what can you conclude about the angle A?

Your answer needs to be well motivated and expressed in terms of the variables provided.

Problem 14 (25 points) **Answer in booklet 1**

A small block of mass m slides along a friction-less loop-the-loop track, as shown in Fig. 10. The block is released from rest at point P.

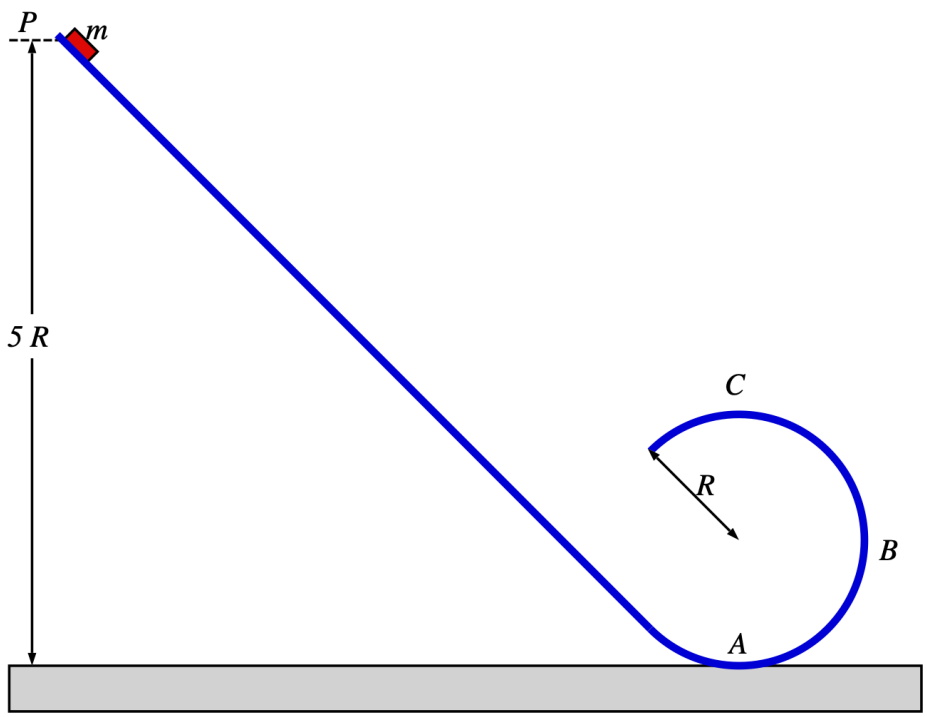


Figure 10: Motion on the loop-the-loop.

a) Draw a force diagram of all forces acting on the block at point A.

b) What is the speed of the block at point A?

c) What is the net force acting on the block at point B (specify direction and magnitude)?

d) At what height above the bottom of the loop should the block be released so that it is on the verge of losing contact with the track at the top of the loop (point C)?

e) What would your answer to part d) be if instead of a block sliding down the track you would use a sphere of radius r and mass m?

Your answer needs to be well motivated and expressed in terms of the variables provided and g.

Problem 15 (25 points) **Answer in booklet 2**

Consider the classical model of a hydrogen atom in which an electron of mass m is in a circular orbit of radius r around a proton of mass M. In this model, we will assume that the proton remains at rest at the center of the circular orbit. Since the electron and the proton have charges with opposite signs, the electric force between them is attractive.

a) The electric force between the electron and the proton is attractive and is much larger than the gravitational force acting between them. The electric force is directed in the same direction as the gravitational force, and has a magnitude equal to

$$|F_{el}| = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

where e is the magnitude of the charge of the electron. What is the velocity of the electron in an orbit of radius r, assuming uniform circular motion? You can ignore relativistic effects and the rest mass of the electron.

b) The electric potential energy of the electron is equal to

$$U_{el} = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

What is the total energy of the electron in an orbit of radius r? You can ignore relativistic effects and the rest mass of the electron.

c) What is the magnitude of the angular momentum of the electron in an orbit of radius r?

d) Quantum mechanics tells us that the orbital angular momentum of the electron is quantized and is an integer multiple of $h/2\pi$:

$$\left|\vec{L}\right| = N\hbar = N\frac{h}{2\pi}$$

where N is 1, 2, 3, ... The quantization of angular momentum leads to a quantization of the energy of the electron. Based on this model, what are the energies of the photons that can be emitted by the hydrogen atom when it decays from an excited state with $N = 3$ to its ground state ($N = 1$)?

Your answer needs to be well motivated and expressed in terms of the variables provided.

Problem 16 (25 points) **Answer in booklet 3**

A block with mass m is pushed along a horizontal floor by a force P that makes an angle A with the horizontal, as shown in Fig. 11a. The coefficients of kinetic and static friction between the block and the floor are μ_k and μ_s, respectively.

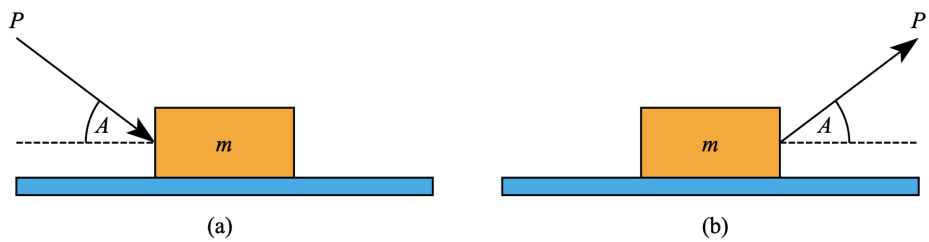


Figure 11: Pushing and pulling a block.

a) Calculate the maximum force that can be applied without moving the block.

b) If the applied force P is larger than the maximum force calculated in a), the block will start to move. Calculate the acceleration of the block.

c) As a result of the applied force P, the block moves over a distance d. What is the work done on the block by force P?

Instead of pushing the block with a force P, we can pull the block with the same force P, as shown in Fig. 11b.

d) Calculate the maximum force that can be applied without moving the block. Is this force larger or smaller than the maximum force calculated in part a)?

e) If the applied force P is larger than the maximum force calculated in part d), the block will start to move. Calculate the acceleration of the block.

f) As a result of the applied force P, the block moves over a distance d. What is the work done on the block by force P?

g) In which situation (block being pulled or pushed) will the block have the highest kinetic energy when it has been displaced over a distance d? Your answer needs to be well motivated.

Your answer needs to be well motivated and expressed in terms of the variables provided and g.

Problem 17 (25 points) **Answer in booklet 3**

A rod of length L and negligible mass is attached to a uniform disk of mass M and radius R, as shown in Fig. 12.

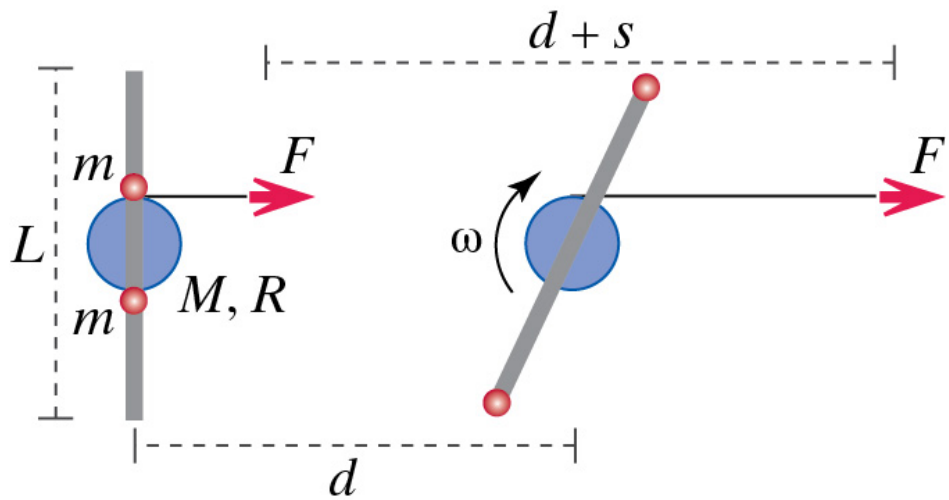


Figure 12: System studied in Problem 17.

A string is wrapped around the disk, and you pull on the string with a constant force F. Two small balls, each of mass m, slide along the rod with negligible friction. The apparatus start from rest, and when the center of the disk has moved a distance d, a length of string s has come off the disk, and the balls have collided with the end of the rod and stuck there. The apparatus slides on a nearly frictionless table.

a) At this instant, what is the speed v of the center of the disk?

b) At this instant, the angular speed of the disk is ω. What is the rotational energy of the system at this instant?

c) What is the change in the internal energy of the system at this instant?

Your answer needs to be well motivated and expressed in terms of the variables provided.

INTENTIONALLY LEFT BLANK

INTENTIONALLY LEFT BLANK

Figure 13: My own part of the skin of a Boeing 747 of the KLM.