

**Do not turn the pages of the exam until you are instructed to do so.**

**Exam rules:** You may use **only** a writing instrument while taking this test. You may **not** consult any calculators, computers, books, nor each other.

**Problems 1 and 10 must be answered on the scantron form. Problems 11, 12, and 13 must be answered in exam booklet 1. Problems 14 and 15 must be answered in exam booklet 2. Problems 16 and 17 must be answered in exam booklet 3.**

Your answers need to be well motivated and expressed in terms of the variables used in the problem. You will receive partial credit where appropriate, but only when we can read your solution. Answers that are not motivated will not receive any credit, even if correct.

At the end of the exam, you must hand in your exam, the scantron form, the blue exam booklets, and the equation sheet. All items must be clearly labeled with your name, your student ID number, and the day/time of your recitation. **If any of these items are missing, we will not grade your exam, and you will receive a score of 0 points.**

**You are required to complete the following *Honor Pledge for Exams*. Copy and sign the pledge before starting your exam.**

“I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.”

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$$\cos(30^\circ) = \frac{1}{2}\sqrt{3} \quad \sin(30^\circ) = \frac{1}{2} \quad \tan(30^\circ) = \frac{1}{3}\sqrt{3}$$

$$\cos(45^\circ) = \frac{1}{2}\sqrt{2} \quad \sin(45^\circ) = \frac{1}{2}\sqrt{2} \quad \tan(45^\circ) = 1$$

$$\cos(60^\circ) = \frac{1}{2} \quad \sin(60^\circ) = \frac{1}{2}\sqrt{3} \quad \tan(60^\circ) = \sqrt{3}$$

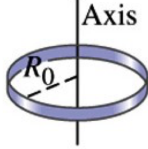
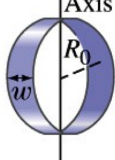
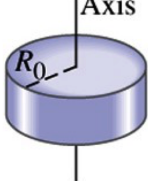
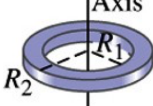
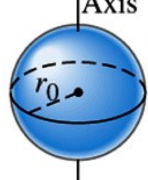
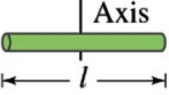
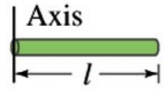
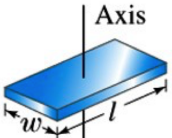
$$\cos\left(\frac{1}{2}\pi - \theta\right) = \sin(\theta) \quad \sin\left(\frac{1}{2}\pi - \theta\right) = \cos(\theta)$$

$$\cos(2\theta) = 1 - 2\sin^2(\theta) \quad \sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

	Circle	Sphere
circumference	$2\pi r$	
(surface) area	$\pi r^2$	$4\pi r^2$
volume		$\frac{4}{3}\pi r^3$

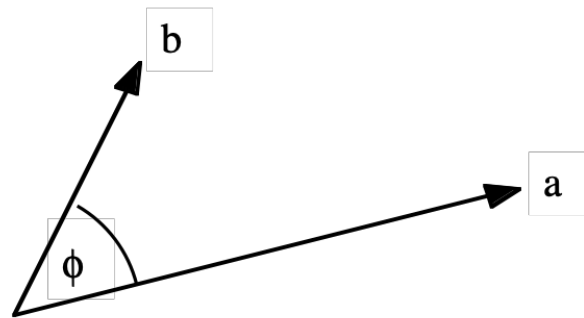
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Moments of inertia of various objects of uniform composition.

(a)	Thin hoop of radius $R_0$	Through center		$MR_0^2$
(b)	Thin hoop of radius $R_0$ and width $w$	Through central diameter		$\frac{1}{2}MR_0^2 + \frac{1}{12}Mw^2$
(c)	Solid cylinder of radius $R_0$	Through center		$\frac{1}{2}MR_0^2$
(d)	Hollow cylinder of inner radius $R_1$ and outer radius $R_2$	Through center		$\frac{1}{2}M(R_1^2 + R_2^2)$
(e)	Uniform sphere of radius $r_0$	Through center		$\frac{2}{5}Mr_0^2$
(f)	Long uniform rod of length $l$	Through center		$\frac{1}{12}Ml^2$
(g)	Long uniform rod of length $l$	Through end		$\frac{1}{3}Ml^2$
(h)	Rectangular thin plate, of length $l$ and width $w$	Through center		$\frac{1}{12}M(l^2 + w^2)$

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### Properties of the scalar product



$$a \cdot b = |\mathbf{a}| |\mathbf{b}| \cos \phi \quad (1)$$

$$a \cdot a = |\mathbf{a}|^2 \quad (2)$$

$$(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b}) = a \cdot a + b \cdot b + 2 \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}|^2 + |\mathbf{b}|^2 + 2 \mathbf{a} \cdot \mathbf{b} \quad (3)$$

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**Good Luck !**

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**Problem 1 (2.5 points)****Answer on Scantron form**What did **NOT** happen on December 5, 2024?

Figure 1: Sinterklaas.

1. Good Dutch children received presents.
2. Professor Wolfs taught a lecture and told us about Sinterklaas.
3. Sinterklaas celebrated his birthday.
4. Naughty Dutch children were put in a bag.

**Problem 2 (2.5 points)****Answer on Scantron form**

What can you say about the total energy of the object for the various trajectories shown in the Fig. 2?

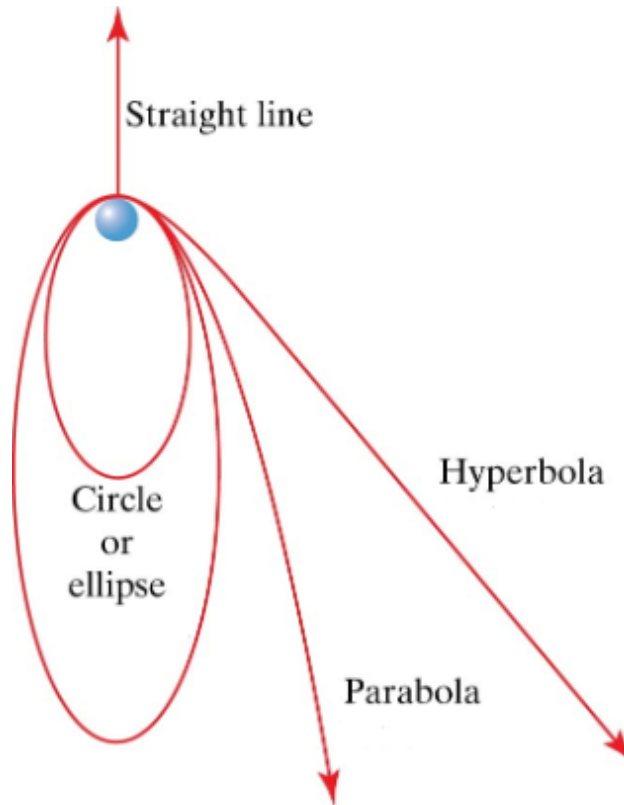


Figure 2: Possible orbital trajectories.

1.  $E_{\text{elliptical trajectory}} > 0 \text{ J}$ ,  $E_{\text{hyperbolic trajectory}} = 0 \text{ J}$ , and  $E_{\text{parabolic trajectory}} < 0 \text{ J}$ .
2.  $E_{\text{elliptical trajectory}} > 0 \text{ J}$ ,  $E_{\text{hyperbolic trajectory}} < 0 \text{ J}$ , and  $E_{\text{parabolic trajectory}} = 0 \text{ J}$ .
3.  $E_{\text{elliptical trajectory}} = 0 \text{ J}$ ,  $E_{\text{hyperbolic trajectory}} > 0 \text{ J}$ , and  $E_{\text{parabolic trajectory}} < 0 \text{ J}$ .
4.  $E_{\text{elliptical trajectory}} = 0 \text{ J}$ ,  $E_{\text{hyperbolic trajectory}} < 0 \text{ J}$ , and  $E_{\text{parabolic trajectory}} > 0 \text{ J}$ .
5.  $E_{\text{elliptical trajectory}} < 0 \text{ J}$ ,  $E_{\text{hyperbolic trajectory}} = 0 \text{ J}$ , and  $E_{\text{parabolic trajectory}} > 0 \text{ J}$ .
6.  $E_{\text{elliptical trajectory}} < 0 \text{ J}$ ,  $E_{\text{hyperbolic trajectory}} > 0 \text{ J}$ , and  $E_{\text{parabolic trajectory}} = 0 \text{ J}$ .

**Problem 3 (2.5 points)****Answer on Scantron form**

Consider the arrangements of 25 quanta of energy among 100 oscillators shown in Fig. 3.

10 by 10 array of 100 oscillators, 25 quanta of energy

1	1	1	1	1					
1	1	1	1	1					
1	1	1	1	1					
1	1	1	1	1					
1	1	1	1	1					

A

		1							
			1		1	1			
1				1					
	1	1							1
					1				
1			1						
								3	
		1		1					
		1			1				
1				2			2	1	

B

					25				

C

5									
5									
5									
5									
5									

D

Figure 3: A few possible ways to distribute 25 quanta over 100 oscillators.

Which arrangement is least probable?

1. A.
2. B.
3. C.
4. D.
5. They are equally probable.

## Problem 4 (2.5 points)

Answer on Scantron form

Figure 4 shows a graph of the potential energy as function of the interatomic distance for a particular molecule.

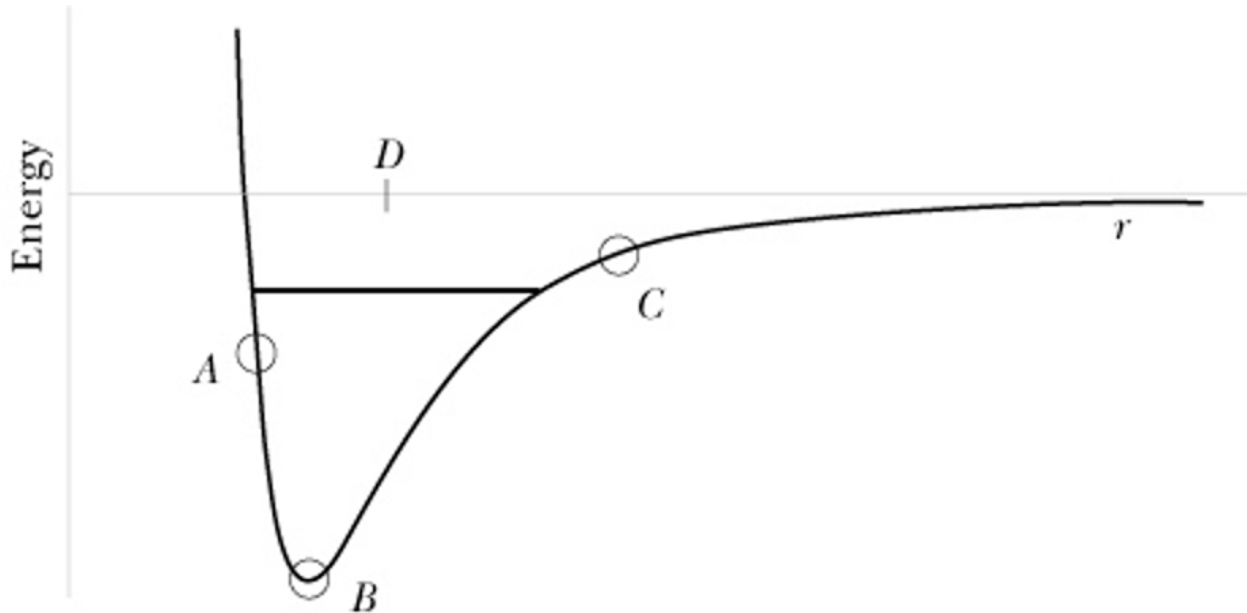


Figure 4: Potential energy as a function of the interatomic distance for a particular molecule.

Rank the magnitudes of the forces at locations  $A$ ,  $B$ , and  $C$ .

1.  $|F_A| > |F_B| > |F_C|$
2.  $|F_A| > |F_C| > |F_B|$
3.  $|F_B| > |F_A| > |F_C|$
4.  $|F_B| > |F_C| > |F_A|$
5.  $|F_C| > |F_A| > |F_B|$
6.  $|F_C| > |F_B| > |F_A|$

**Problem 5 (2.5 points)****Answer on Scantron form**

A cart rolls with low friction on a track. A fan is mounted on the cart. When the fan is turned on, there is a constant force acting on the cart. Three different experiments are performed:

- Fan off: The cart is originally at rest. You give it a brief push, and it coasts a long distance along the  $+x$  direction, slowly coming to a stop.
- Fan forward: The fan is turned on and you hold the cart stationary. You then take your hand away and the cart moves forward in the  $+x$  direction. After travelling a long distance along the track, you quickly stop and hold the cart.
- Fan backward: The fan is turned on facing the "wrong" direction and you hold the cart stationary. You give it a brief push and the cart moves in the  $+x$  direction, slowing down, turning around, and returning to the starting position where you quickly stop and hold the cart.

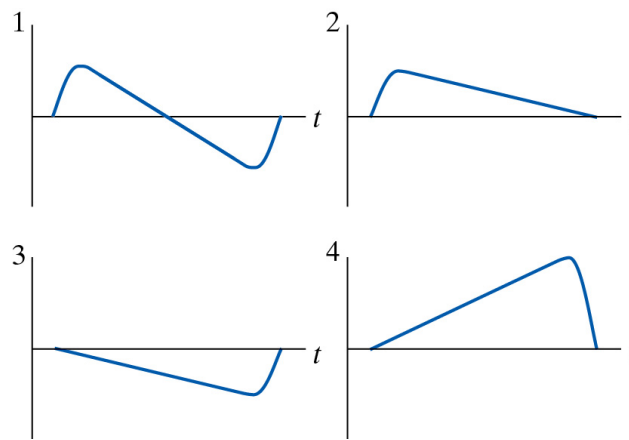


Figure 5: Linear momentum versus time.

Figure 5 shows the  $x$  component of the linear momentum of the car as a function of time. Match the three experiments with the correct graphs.

- |                          |                          |                           |
|--------------------------|--------------------------|---------------------------|
| 1. $a = 1, b = 2, c = 3$ | 5. $a = 2, b = 3, c = 1$ | 9. $a = 4, b = 2, c = 3$  |
| 2. $a = 1, b = 3, c = 2$ | 6. $a = 2, b = 4, c = 1$ | 10. $a = 4, b = 2, c = 1$ |
| 3. $a = 1, b = 4, c = 2$ | 7. $a = 3, b = 4, c = 1$ |                           |
| 4. $a = 2, b = 1, c = 4$ | 8. $a = 3, b = 1, c = 4$ |                           |

**Problem 6 (2.5 points)****Answer on Scantron form**

A cart rolls with low friction on a track. A fan is mounted on the cart. When the fan is turned on, there is a constant force acting on the cart. Three different experiments are performed:

- Fan off: The cart is originally at rest. You give it a brief push, and it coasts a long distance along the  $+x$  direction, slowly coming to a stop.
- Fan forward: The fan is turned on and you hold the cart stationary. You then take your hand away and the cart moves forward in the  $+x$  direction. After travelling a long distance along the track, you quickly stop and hold the cart.
- Fan backward: The fan is turned on facing the "wrong" direction and you hold the cart stationary. You give it a brief push and the cart moves in the  $+x$  direction, slowing down, turning around, and returning to the starting position where you quickly stop and hold the cart.

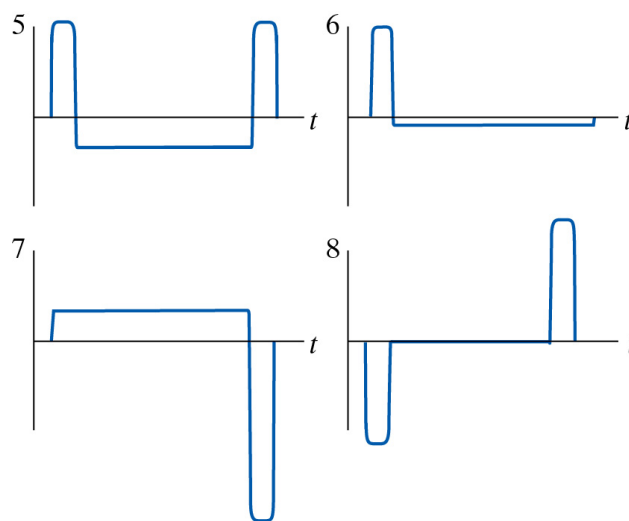


Figure 6: Force versus time.

Figure 6 shows the  $x$  component of the net force acting on the car as a function of time. Match the three experiments with the correct graphs.

- |                          |                          |                           |
|--------------------------|--------------------------|---------------------------|
| 1. $a = 5, b = 6, c = 7$ | 5. $a = 6, b = 7, c = 5$ | 9. $a = 8, b = 6, c = 7$  |
| 2. $a = 5, b = 7, c = 8$ | 6. $a = 6, b = 8, c = 5$ | 10. $a = 8, b = 6, c = 5$ |
| 3. $a = 5, b = 8, c = 7$ | 7. $a = 7, b = 8, c = 5$ |                           |
| 4. $a = 6, b = 5, c = 7$ | 8. $a = 7, b = 5, c = 8$ |                           |

**Problem 7 (2.5 points)****Answer on Scantron form**

A cart rolls with low friction on a track. A fan is mounted on the cart. When the fan is turned on, there is a constant force acting on the cart. Three different experiments are performed:

- Fan off: The cart is originally at rest. You give it a brief push, and it coasts a long distance along the  $+x$  direction, slowly coming to a stop.
- Fan forward: The fan is turned on and you hold the cart stationary. You then take your hand away and the cart moves forward in the  $+x$  direction. After travelling a long distance along the track, you quickly stop and hold the cart.
- Fan backward: The fan is turned on facing the "wrong" direction and you hold the cart stationary. You give it a brief push and the cart moves in the  $+x$  direction, slowing down, turning around, and returning to the starting position where you quickly stop and hold the cart.

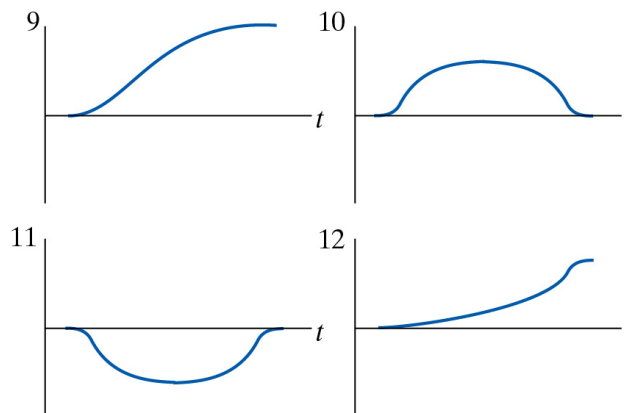


Figure 7: Position versus time.

Figure 6 shows the  $x$  position of the cart as a function of time. The graphs start when the cart is at rest and end when the cart is again at rest. Match the three experiments with the correct graphs.

- |                            |                            |                             |
|----------------------------|----------------------------|-----------------------------|
| 1. $a = 9, b = 10, c = 11$ | 5. $a = 10, b = 11, c = 9$ | 9. $a = 12, b = 10, c = 11$ |
| 2. $a = 9, b = 11, c = 10$ | 6. $a = 10, b = 12, c = 9$ | 10. $a = 12, b = 11, c = 9$ |
| 3. $a = 9, b = 12, c = 10$ | 7. $a = 11, b = 9, c = 10$ |                             |
| 4. $a = 10, b = 9, c = 12$ | 8. $a = 11, b = 10, c = 9$ |                             |

## Problem 8 (2.5 points)

Answer on Scantron form

For each graph in Fig. 8, choose the letter (a-i) corresponding to the appropriate description of the motion of a fan cart moving along a linear track. **Not all descriptions will be used.**

- The cart moves to the left, gradually slowing down.
- The cart moves to the right, gradually speeding up.
- The cart moves to the left at constant speed.
- The cart moves to the left, gradually slowing down, stop, and move to the right, speeding up.
- The cart remains stationary and does not move.
- The cart moves to the right, gradually slowing down.
- The cart moves to the right, gradually slowing down, stop, and move to the left, speeding up.
- The cart moves to the left, gradually speeding up.
- The cart moves to the right at constant speed.

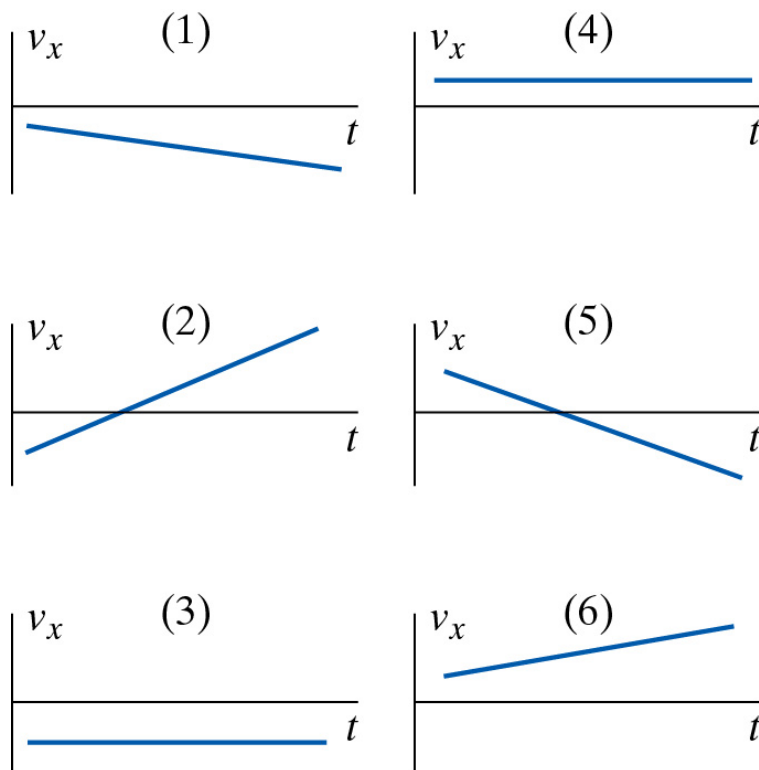


Figure 8: Velocity versus time.

**Problem 8 continued (2.5 points)****Answer on Scantron form**

1. 1 = a, 2 = d, 3 = c, 4 = b, 5 = e, 6 = i
2. 1 = h, 2 = d, 3 = c, 4 = i, 5 = g, 6 = b
3. 1 = d, 2 = h, 3 = i, 4 = c, 5 = b, 6 = g
4. 1 = c, 2 = d, 3 = h, 4 = i, 5 = g, 6 = b
5. 1 = a, 2 = i, 3 = d, 4 = c, 5 = g, 6 = b
6. 1 = h, 2 = d, 3 = i, 4 = c, 5 = g, 6 = b
7. 1 = d, 2 = h, 3 = c, 4 = i, 5 = b, 6 = g
8. 1 = c, 2 = h, 3 = d, 4 = i, 5 = b, 6 = g

## Problem 9 (2.5 points)

Answer on Scantron form

Consider the two phase diagrams shown in Fig. 9 and Fig. 10.

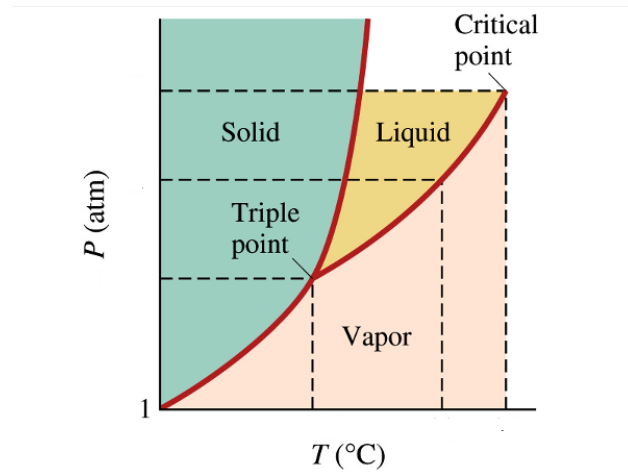


Figure 9: Phase diagram 1.

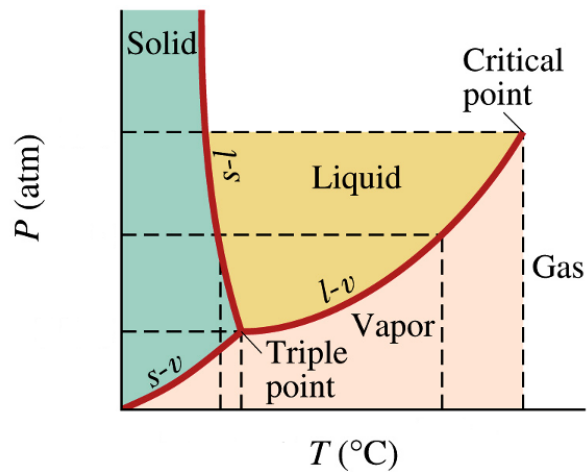


Figure 10: Phase diagram 2.

Which of these phase diagrams is the phase diagram of water?

1. Phase diagram 1.
2. Phase diagram 2.

**Problem 10 (2.5 points)****Answer on Scantron form**

In the Rutherford experiment, what was surprising to the experimenters?

1. Sometimes the alpha particles passed right through the gold foil without being deflected.
2. Sometimes the alpha particles were deflected slightly when they passed through the gold foil.
3. Sometimes the alpha particles bounced back from the gold foil.

**Problem 11 (25 points)****Answer in booklet 1**

A thin box in outer space contains a large ball of clay of mass  $M$ , connected to an initially relaxed spring of stiffness  $k_s$ , as shown in the Fig. 11. The mass of the box and spring are negligible compared to  $M$ .

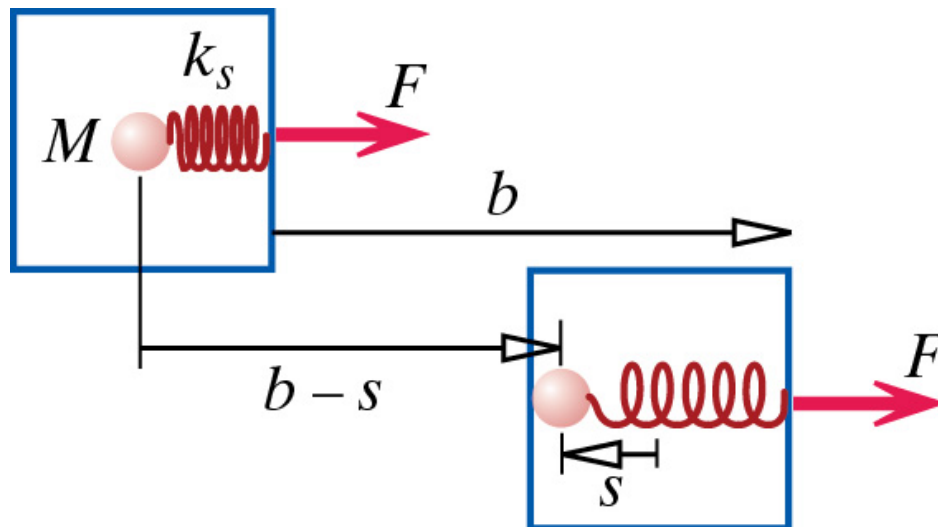


Figure 11: Motion of a thin box containing a spring and a ball in outer space.

The apparatus is initially at rest. Then a force of constant magnitude  $F$  is applied to the box. When the box has moved a distance  $b$ , the clay makes contact with the left side of the box and sticks there, with the spring stretched an amount  $s$ .

- (10 points) Immediately after the clay sticks to the box, how fast is the box moving?
- (15 points) What is the increase in the internal energy of the clay?

Your answers needs to be well motivated and expressed in terms of the variables provided.

**Problem 12 (25 points)****Answer in booklet 1**

A tiny piece of space junk of mass  $m$  strikes a glancing blow to a spherical satellite. After the collision the space junk is traveling in a new direction and moving more slowly. The space junk has negligible rotation both before and after the collision. The velocities of the space junk before and after the collision are shown in Fig. 12.

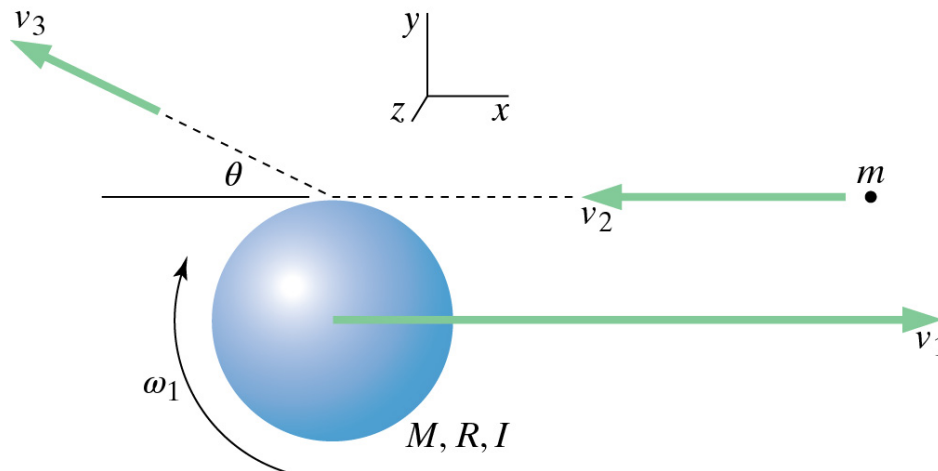


Figure 12: A collision in space.

The center of mass of the satellite is located at its geometrical center. The satellite has mass  $M$ , radius  $R$ , and moment of inertia  $I$  about its center. Before the collision, the satellite was moving and rotating as shown in Fig. 12.

- (5 points) What are the  $x$  and  $y$  components of the center-of-mass velocity of the satellite just after the collision?
- (10 points) What is the rotational speed  $\omega_f$  of the satellite just after the collision?
- (10 points) Calculate the rise in the thermal energy of the satellite and the space junk combined.

**Your answers needs to be well motivated and expressed in terms of the variables provided.**

**Problem 13 (25 points)****Answer in booklet 1**

The operation of an automobile internal combustion engine can be approximated by a reversible cycle known as the Otto cycle, whose  $pV$  diagram is shown in Fig. 13. The gas in cylinder at point  $a$  (pressure  $p_a$ , volume  $V_a$ , temperature  $T_a$ ) is compressed adiabatically to point  $b$  (pressure  $p_b$ , volume  $V_b$ , temperature  $T_b$ ). Between point  $b$  and point  $c$  (pressure  $p_c$ , volume  $V_c$ , temperature  $T_c$ ), heat is added to the gas, and the pressure increases at constant volume. During the power stroke, between point  $c$  and point  $d$  (pressure  $p_d$ , volume  $V_d$ , temperature  $T_d$ ), the gas expands adiabatically. Between point  $d$  and point  $a$ , heat is removed from the system, and the pressure decreases at constant volume. Assume the gas is an ideal monatomic gas.

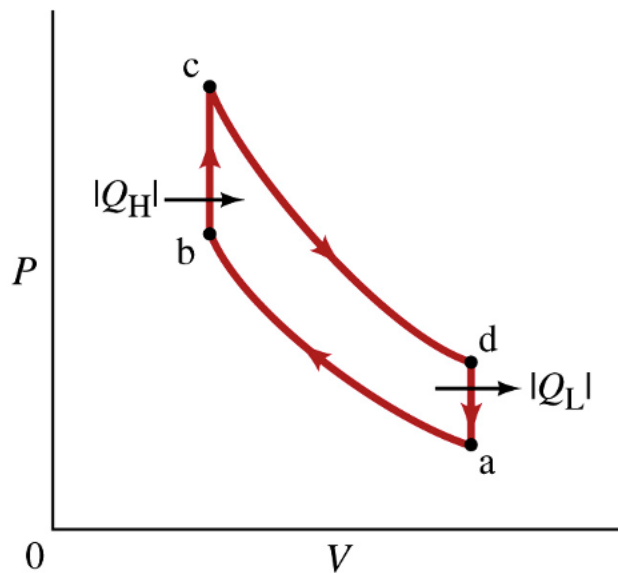


Figure 13: The Otto cycle.

- (5 points) Assuming there are  $N$  molecules of gas in the system, what are the heats  $|Q_H|$  and  $|Q_L|$ ? Express your answer in terms of  $N$ ,  $k$ ,  $T_a$ ,  $T_b$ ,  $T_c$ , and  $T_d$ .
- (5 points) What is the efficiency of the Otto cycle? Express your answer in terms of  $T_a$ ,  $T_b$ ,  $T_c$ , and  $T_d$ .
- (10 points) Express the efficiency of the Otto cycle in terms of just the compression ratio  $V_a=V_b$  and  $\gamma$ .
- (5 points) How does the efficiency change when we replace the monatomic gas with a diatomic gas?

**Your answers needs to be well motivated and expressed in terms of the variables provided.**

**Problem 14 (25 points)****Answer in booklet 2**

A block of mass  $M$  is attached to a spring with spring constant  $k$ , as shown in Fig. 14. The spring-mass system is in equilibrium when it has a length  $L$ . A bullet of mass  $m$  is fired from below and buries itself in the block. The block reaches a maximum height  $h$  above its original position.

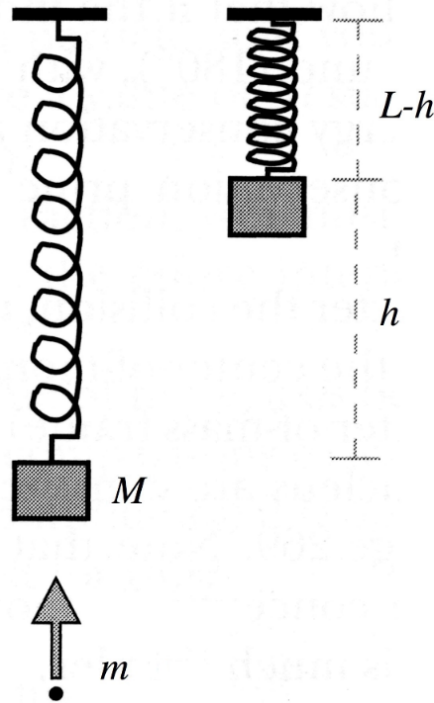


Figure 14: Measuring the speed of a bullet.

- (5 points) What is the rest length of the spring?
- (10 points) What is the speed of the block right after the bullet hits?
- (10 points) What is the speed of the bullet just before it hits the block?

Your answers needs to be well motivated and expressed in terms of the variables provided.

**Problem 15 (25 points)****Answer in booklet 2**

There is no general analytical solution for the motion of a gravitational system consisting of more than two bodies. However, there do exist analytical solutions for multiple-body systems with very special initial conditions. Consider the four-star system shown in the Fig. 15.

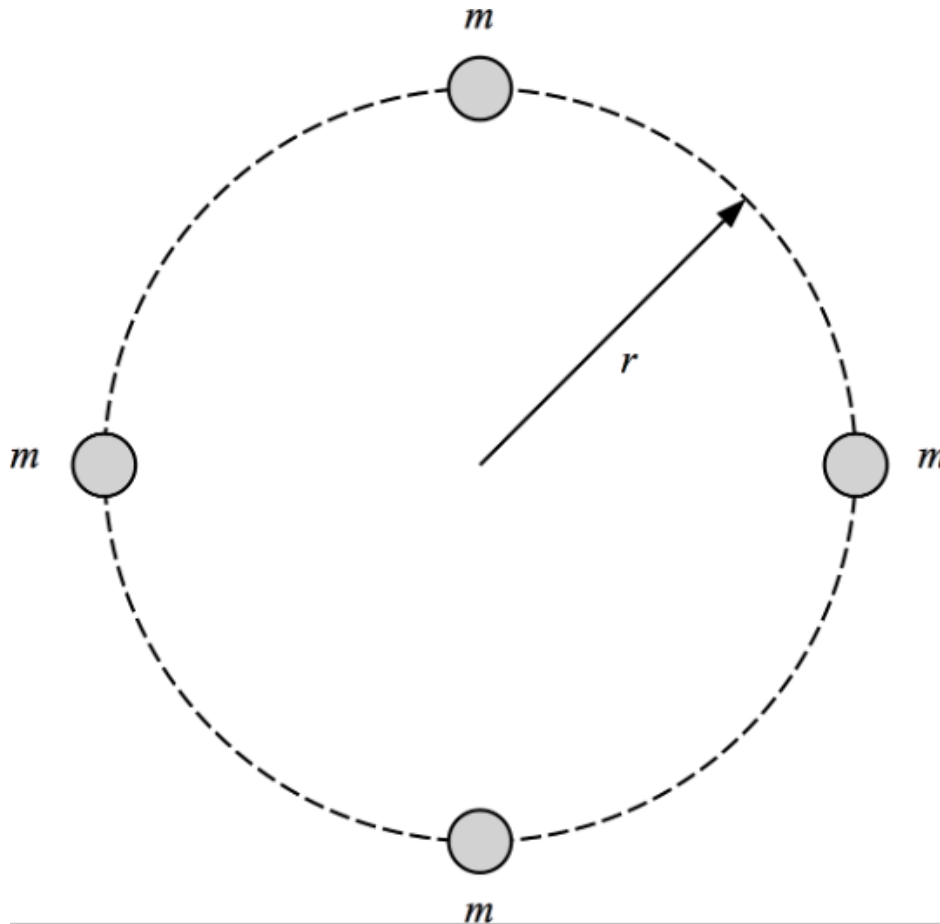


Figure 15: Four star system.

The system consists of four stars, each of mass  $m$ , moving with the same speed in the plane of the page along a circle of radius  $r$ .

- (15 points)** Determine the magnitude and the direction of the gravitational force exerted on one star by the other three stars.
- (10 points)** Calculate how long it takes one star to make one complete revolution.

**Your answers need to be well motivated and expressed in terms of the variables provided. You may assume that  $v \ll c$ .**

**Problem 16 (25 points)****Answer in booklet 3**

There is an amusement park ride that some people love and others hate. A bunch of people stand against the wall of a cylindrical room of radius  $R$  and the room starts to rotate at higher and higher speeds (see Fig. 16). The surface of the wall is designed to maximize friction between the person and the wall. When a certain critical speed is reached, the floor drops away, leaving the people stuck against the wall as they whirl around at constant speed.

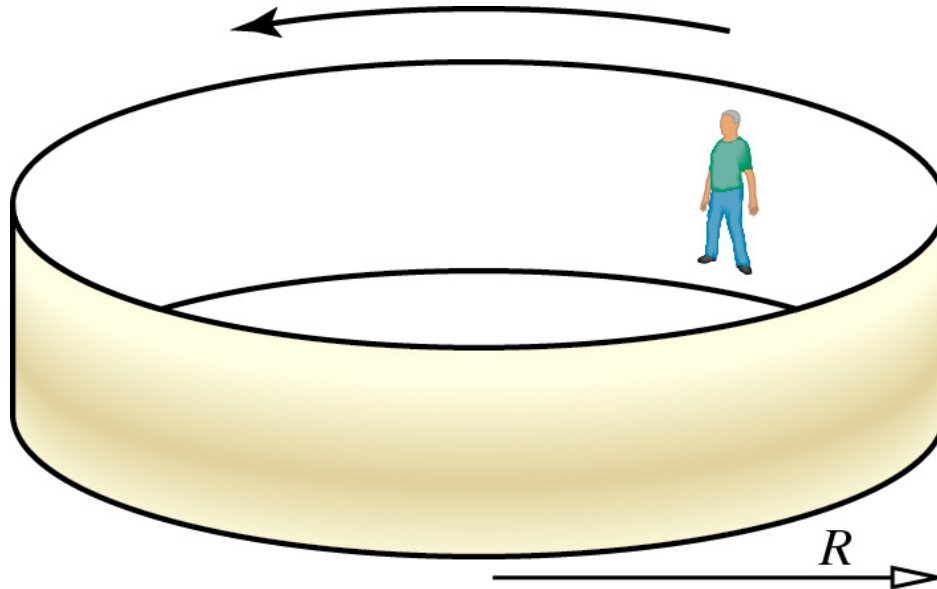


Figure 16: An amusement park ride.

- (20 points) If the critical speed is  $v$ , what is the minimum value of the static friction coefficient between the wall and the people that will ensure that they do not slide down the wall when the floor drops?
- (5 points) What happens to the minimum value of the static friction coefficient when the speed doubles to  $2v$ ?

**Your answers needs to be well motivated and expressed in terms of the variables provided.**

**Problem 17 (25 points)****Answer in booklet 3**

You observe photon emissions from a collection of quantum objects, each of which is known to have just four quantized energy levels. The collection is continually bombarded by an electron beam, and you detect emitted photons using a detector sensitive to photons in the energy range from 2.5 eV to 30 eV. With this detector, you observe photons emitted with energies of 3 eV, 6 eV, 8 eV, and 9 eV, but no other energies.

- a) **(5 points)** It is known that the ground-state energy of these objects is -10 eV. Propose two possible arrangements of energy levels that is consistent with the observations. **Explain in detail, using diagrams.**
- b) **(5 points)** You obtain a second detector that is sensitive to photon energies in the energy range between 0.1 to 2.5 eV. What additional photon energies do you observe? **Explain in detail, using diagrams.**
- c) **(5 points)** You turn off the electron beam so that essentially all the objects are in the ground state. Then you send a beam of photons with a wide range of energies through the material. Using both detectors, what are the energies of the absorption lines? How can this information be used to distinguish between the two proposed level schemes obtained in part a)?

**Your answers need to be well motivated.**

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