Physics 141, Final Exam  
December 20, 2016, 4.00 pm - 7.00 pm

Do not turn the pages of the exam until you are instructed to do so.

Exam rules: You may use only a writing instrument while taking this test. You may not consult any calculators, computers, books, nor each other.

1. Answer the multiple-choice questions (problems 1 – 10) by marking your answer on the scantron form. For each multiple-choice question (problems 1 – 10), select only one answer. Questions with more than one answer selected will be considered incorrect. If your student ID is not listed properly on the Scantron form, the form will not be processed and you loose points for all multiple-choice questions.

2. The analytical problems (11 – 19) must be answered in three blue exam booklets. You must answer problems 11, 12, and 13 in blue booklet # 1, problems 14, 15, and 16 in blue booklet # 2, and problems 17, 18, and 19 in blue booklet # 3. If you do not follow this convention there is no guarantee that the problems that appear in the wrong booklet will be graded.

3. The answer to each analytical problem must be well motivated and expressed in terms of the variables used in the problem. You will receive partial credit where appropriate, but only when we can read your solution. Answers that are not motivated will not receive any credit, even if correct.

4. At the end of the exam, you must hand in the blue exam booklets, the scantron form, and the exam. All items must be clearly labeled with your name and student ID number. If any of these items is missing, we will not grade your exam, and you will receive a score of 0 points.

5. You are not allowed to leave the exam room until 5.30 pm.

You are required to complete the following Honor Pledge for Exams. Copy and sign the pledge before starting your exam.

“I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.”

____________________________________________________________________

____________________________________________________________________

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Name:  __________________________________________________

Signature  _____________________________________________________________
Useful Relations:

\[
\begin{align*}
\cos(30^\circ) &= \frac{1}{2} \sqrt{3} \quad \sin(30^\circ) = \frac{1}{2} \quad \tan(30^\circ) = \frac{1}{3} \sqrt{3} \\
\cos(45^\circ) &= \frac{1}{2} \sqrt{2} \quad \sin(45^\circ) = \frac{1}{2} \sqrt{2} \quad \tan(45^\circ) = 1 \\
\cos(60^\circ) &= \frac{1}{2} \quad \sin(60^\circ) = \frac{1}{2} \sqrt{3} \quad \tan(60^\circ) = \sqrt{3} \\
\cos\left(\frac{1}{2} \pi - \theta\right) &= \sin(\theta) \quad \sin\left(\frac{1}{2} \pi - \theta\right) = \cos(\theta) \\
\cos(2\theta) &= 1 - 2 \sin^2(\theta) \quad \sin(2\theta) = 2 \sin(\theta) \cos(\theta)
\end{align*}
\]

Circle \hspace{1cm} Sphere

- **Circumference**: $2\pi r$
- **Surface Area**: $\pi r^2$ \hspace{1cm} $4\pi r^2$
- **Volume**: $\frac{4}{3} \pi r^3$
Properties of the scalar product:

\[ \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \phi \]

\[ \mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2 \]

\[ (\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b}) = \mathbf{a} \cdot \mathbf{a} + \mathbf{b} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{a} = |\mathbf{a}|^2 + 2 \mathbf{a} \cdot \mathbf{b} + |\mathbf{b}|^2 \]
Moments of inertia of various objects of uniform composition.

(a) Thin hoop of radius $R_0$
Through center
\[ MR_0^2 \]

(b) Thin hoop of radius $R_0$ and width $w$
Through central diameter
\[ \frac{1}{2} MR_0^2 + \frac{1}{12} Mw^2 \]

(c) Solid cylinder of radius $R_0$
Through center
\[ \frac{1}{2} MR_0^2 \]

(d) Hollow cylinder of inner radius $R_1$ and outer radius $R_2$
Through center
\[ \frac{1}{2} M (R_1^2 + R_2^2) \]

(e) Uniform sphere of radius $r_0$
Through center
\[ \frac{2}{5} Mr_0^2 \]

(f) Long uniform rod of length $l$
Through center
\[ \frac{1}{12} Ml^2 \]

(g) Long uniform rod of length $l$
Through end
\[ \frac{1}{3} Ml^2 \]

(h) Rectangular thin plate, of length $l$ and width $w$
Through center
\[ \frac{1}{12} M(l^2 + w^2) \]
Problem 1 (2.5 points)

A child rides on a merry-go-round, travelling at constant speed from location $A$ to location $B$.

What is the direction of $\Delta \vec{p}$, the change in the child’s linear momentum between locations $A$ and $B$?

9 zero magnitude
Problem 2 (2.5 points)

You have two identical springs, connected in parallel. When you hang a mass \( m \) from this system, as shown in the Figure below, the new equilibrium position of the system is a distance \( d \) below the equilibrium position when no mass is connected to the system.

Now you connect the two springs in series. The system is in equilibrium when you connect mass \( m \) to the end of the lower spring. What is the displacement of mass \( m \) when it has reached its new equilibrium position?

1. \( 4d \)
2. \( 2d \)
3. \( \sqrt{2}d \)
4. \( d \)
5. \( d/\sqrt{2} \)
6. \( d/2 \)
7. \( d/4 \)
Problem 3 (2.5 points)

Two wheels, initially at rest, roll the same distance without slipping down identical inclined planes. Wheel B has twice the radius but the same mass as wheel A. All the mass is concentrated in their rims, so that the rotational inertias are $I = mR^2$. Which wheel has more translational kinetic energy when it gets to the bottom?

1. Wheel A.
2. Wheel B.
3. The kinetic energies are the same.
4. Need more information.
Problem 4 (2.5 points)

Two springs with spring constants $k_1$ and $k_2$ are connected as shown in the Figure below.

What is the displacement $y$ of the connection point from its initial equilibrium position when the two springs are stretched a distance $d$ as a result of the application of force $F$.

1. $y = d \frac{k_1}{k_1 + k_2}$
2. $y = d \frac{k_1}{k_2}$
3. $y = d \frac{k_2}{k_1 + k_2}$
4. $y = d \frac{k_2}{k_1}$
5. $y = d$
Problem 5 (2.5 points)  

Which of the diagrams in the following Figure corresponds to a system of two electrons that start out far apart, moving straight toward each other with nonzero initial velocities?

1. (a)  
2. (b)  
3. (c)  
4. (d)  
5. (e)  
6. (f)
Problem 6 (2.5 points) SCANTRON FORM

The Figure below shows the potential energy distribution of a star-planet system. Three different types of motion of the planet are represented by the three energy states indicated (A, B, and C). Which of these states represents the planet in an elliptical orbit?

1. A.  
2. B.  
3. C.
Problem 7 (2.5 points)

Initially the entropy of object A is 100 J/K and the entropy of object B is also 100 J/K. Both objects are immersed in large vats of hot water. When the thermal energy of A has increased by 1000 joules, its entropy is 200 J/K. When the thermal energy of B has increased by 2000 joules, its entropy is 300 J/K. Which object is at a higher temperature?

1. Object A is at a higher temperature than Object B.
2. Object B is at a higher temperature than Object A.
3. Their temperatures are the same.

Problem 8 (2.5 points)

A gas is made up of diatomic molecules. At temperature $T$, the ratio of the number of molecules in vibrational energy state 2 to the number of molecules in the ground state is measured to be 0.35. The difference in energy between state 2 and the ground state is $\Delta E$. Which of the following conclusions is correct?

1. $\Delta E \approx k_B T$
2. $\Delta E \ll k_B T$
3. $\Delta E \gg k_B T$
Problem 9 (2.5 points)  

A string is wrapped around a uniform disk of mass $M$ and radius $R$. Attached to the disk are four low-mass rods of length $b$, each with a small mass $m$ at its end.

The apparatus is initially at rest on a nearly frictionless surface. Then you pull the string with a constant force $F$. At the instant that the center of the disk has moved a distance $d$, an additional length $w$ of string has unwound off the disk. What is the rotational kinetic energy of the apparatus at this instant?

1. $RF/(MR^2)$
2. $RF/((1/2)MR^2)$
3. $RF/(MR^2 + mb^2)$
4. $RF/((1/2)MR^2 + mb^2)$
5. $Fd$
6. $Fw$
7. $F(w + d)$
8. $F(w - d)$
Problem 10 (2.5 points)

Which of the following teams is the favorite sport team of your Physics 141 instructor?

1. The Yankees
2. The Buffalo Bills
3. The Red Sox
4. AJAX
Problem 11 (25 points)  

An ideal, monatomic gas is allowed to expand quasi-statically and adiabatically from an initial volume $V_i$ and initial temperature $T_i$ to a final volume $V_f$. The volume contains $n$ moles of gas.

a. What is the final temperature $T_f$?

b. What is the final pressure $p_f$?

c. What is the work done by the gas?

d. What is the change in the internal energy of the gas?

e. If the gas expands quasi-statically and isothermally, instead of adiabatically, what would be the final pressure $p_f$?

f. Calculate the heat added to the system during the isothermal expansion.

Express all your answers in terms of the variables provided. Your answers must be well motivated.
Problem 12 (25 points)  

Consider a system with four degrees of freedom. One degree of freedom is characterized by an energy level scheme that has an energy gap of $2E$ between the energy levels. The other three degrees of freedom are characterized by an energy level scheme that has energy gaps of $E$ between the energy levels. The lowest energy levels of this system are shown in the Figure below.

![Energy Level Scheme](image)

a. What is the most probable macro state(s) in which the system can be found if the system has three units of energy?

b. What is the probability to find the system in the most probable macro state?

c. What is the most probable macro state(s) in which the system can be found if the system has two units of energy?

d. What is the temperature of the system when it is in a state with two units of energy?

Express all your answers in terms of the variables provided. Your answers must be well motivated.
Problem 13 (20 points)

Some material consisting of a collection of microscopic systems is kept at a high temperature. A photon detector capable of detecting photon energies from infrared through ultraviolet observes photons emitted with energies of 0.3 eV, 0.5 eV, 0.8 eV, 2.0 eV, 2.5 eV, and 2.8 eV. These are the only photon energies observed.

a. Draw and label a possible energy-level diagram for one of the microscopic systems, which has 4 bound states. On the diagram, indicate the transitions corresponding to the emitted photons. Explain briefly.

b. Would a spring-mass model be a good model for this microscopic system? Why or why not?

c. The material is now cooled down to a very low temperature, and the photon detector stops detecting photon emissions. Next, a beam of light with a continuous range of energies from infrared through ultraviolet shines on the material, and the photon detector observes the beam of light after it passes through the material. What photon energies in this beam of light are observed to be significantly reduced in intensity (“dark absorption lines”)? Explain briefly.

Express all your answers in terms of the variables provided. Your answers must be well motivated.
Problem 14 (20 points)  

There is an amusement park ride that some people love and others hate. A bunch of people stand against the wall of a cylindrical room of radius $R$ and the room starts to rotate at higher and higher speed (see Figure below). The surface of the wall is designed to maximize friction between the person and the wall. When a certain critical speed is reached, the floor drops away, leaving the people stuck against the wall as they whirl around at constant speed.

a. If the critical speed is $v$, what is the minimum value of the static friction coefficient between the wall and the people that will ensure that they do not slide down the wall when the floor drops?

b. What happens to the minimum value of the static friction coefficient when the speed doubles to $2v$?

Express all your answers in terms of the variables provided. Your answers must be well motivated.
Problem 15 (20 points)  

Two blocks of mass $m_1$ and $m_3$, connected by a rod of mass $m_2$, are sitting on a frictionless surface. You push to the left on the right block with a constant force $F$.

\begin{center}
\begin{tikzpicture}
  \draw[thick] (0,0) -- (2,0);
  \draw[fill=blue!20] (0,0) rectangle (1,1);
  \draw[fill=blue!20] (1,0) rectangle (2,1);
  \draw[->, red] (1.5,0.5) -- (2,0);
  \node at (0.5,0.5) {$m_2$};
  \node at (0,1) {$m_3$};
  \node at (2,1) {$m_1$};
  \node at (1.5,0.5) {$F$};
\end{tikzpicture}
\end{center}

a. What is the acceleration of the blocks?

b. What is the force exerted by the rod on block 3? Specify magnitude and direction.

Suppose that instead of pushing on the right block, you pull on the left block with the same constant force $F$.

c. What is now the force exerted by the rod on block 3? Specify magnitude and direction.

Express all your answers in terms of the variables provided. Your answers must be well motivated.
Problem 16 (25 points)

Consider a two-dimensional elastic collision involving particles of equal mass in which one of the particles is initially at rest, as shown in the Figure below.

After the collision, the angle between the directions of these two particles is \( A \), as shown in the Figure below.

![Diagram](image.png)

a. Use vector notation to write down the relation between the initial linear momentum of the incident particle, \( \vec{p}_1 \), and the linear momenta of the two particles after the interaction, \( \vec{p}_3 \) and \( \vec{p}_4 \).

b. Use the scalar product to obtain an expression for the magnitude of \( \vec{p}_1 \) in terms of the magnitude of \( \vec{p}_3 \) and \( \vec{p}_4 \) and the angle \( A \). Note: see details on the scalar product on page 3.

c. Use the relation you derived in part b) to obtain an expression that relates the kinetic energy of the incident particle, \( K_1 \), to the kinetic energies of the outgoing particles, \( K_3 \) and \( K_4 \).

d. Since the collision is elastic, what can you conclude about the angle \( A \)?

Express all your answers in terms of the variables provided. Your answers must be well motivated.
Problem 17 (20 points)  

A yo-yo is constructed of three disks: two outer disks of mass $M$ and radius $R$, and an inner disk of mass $m$ and radius $r$, as shown in the Figure below. A string is wrapped around the inner disk. The yo-yo is suspended from the ceiling and then released.

The purpose of this problem is to calculate $T$.

a. If you knew the tension $T$, what would be the linear acceleration of the center of mass?

b. If you knew the tension $T$, what would be the angular acceleration of the yo-yo with respect to its center of mass?

c. What is the relation between the linear acceleration of the center of mass and the angular acceleration of the yo-yo?

d. Use your answers to a), b), and c) to calculate the tension $T$.

Express all your answers in terms of the variables provided. Your answers must be well motivated.
Consider a mass $m$ falling near the surface of the Earth. Choose location $A$ on the surface of the Earth at the location shown in the Figure. Neglect air resistance.

**Problem 18 (10 points)**

a. What is the rate of change of the angular momentum of the mass about location $A$ at the instant shown in the Figure? Specify the direction and the magnitude of the change of the angular momentum.

b. Use the result calculated in part a) to determine the linear acceleration of the mass in the vertical direction at the instant shown in the Figure.

Express all your answers in terms of the variables provided. Your answers must be well motivated.
Problem 19 (10 points)  
A small rock of mass \( m \) passes a massive star of mass \( M \), following the path shown in the diagram. When the rock is a distance \( d_1 \) from the center of the star, the magnitude of its linear momentum is \( p_1 \) and its direction is specified by the angle \( \alpha \).

At a later time, when the rock is at a distance \( d_2 \) from the center of the star, its heading is in the negative \( y \) direction. There are no other massive objects nearby. What is the magnitude of its linear momentum \( p_2 \) at this instant?

Express all your answers in terms of the variables provided. Your answers must be well motivated.
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