The momentum principle:

$$\Delta d\vec{p} = \vec{F}_{net}\Delta t \tag{1}$$

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} \tag{2}$$

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{3}$$

$$\vec{p}_{new} = \vec{p}_{old} + \vec{F}_{net}\Delta t \tag{4}$$

$$\vec{r}_{new} = \vec{r}_{old} + \frac{1}{\sqrt{1+\left(\frac{p}{mc}\right)^2}}\left(\frac{\vec{p}}{m}\right)\Delta t \tag{5}$$

Equations of motion in 1D for constant acceleration and low velocities ($v << c$):

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2 \tag{6}$$

$$v(t) = \frac{dx(t)}{dt} = v_0 + at \tag{7}$$

$$a(t) = \frac{dv(t)}{dt} = a = \text{constant} \tag{8}$$

Requirement for uniform circular motion:

$$F_r = \frac{mv^2}{r} \tag{9}$$

Rotational motion:

$$d = \theta r \tag{10}$$

$$v = \omega r \quad \omega = \tfrac{d\theta}{dt} \tag{11}$$

$$a = \alpha r \quad \alpha = \tfrac{d\omega}{dt} \tag{12}$$

Gravitational force:

$$\vec{F} = G\frac{m_1 m_2}{r^2}\hat{r} \tag{13}$$

$$\vec{F} = m\vec{g} \quad \text{(close to the surface of the Earth)} \tag{14}$$

Electrostatic force:

$$\vec{F} = -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r} \tag{15}$$

Harmonic motion:

$$F = -kx \tag{16}$$

$$x(t) = x_{\text{max}}\cos(\omega t + \phi) \quad \text{where} \quad \omega = \sqrt{\frac{k}{m}} \tag{17}$$

$$T = \frac{2\pi}{\omega} \tag{18}$$

Damped harmonic motion:

$$x(t) = x_m e^{-\frac{bt}{2m}} e^{it\sqrt{\frac{k}{m}}} \tag{19}$$

Driven harmonic motion:

$$x(t) = \frac{F_0}{{\omega_0}^2 - \omega^2}\cos(\omega t + \phi) \tag{20}$$

Stress and strain:

$$\frac{F}{A} = Y\frac{\Delta L}{L} \tag{21}$$

$$Y = \frac{k_s}{d} \tag{22}$$

Work done by a force:

$$\begin{aligned} W &= \vec{F}\cdot\vec{\mathrm{d}} && \text{constant force} \\ &= \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} && \text{variable force} \end{aligned} \tag{23}$$

Power:

$$P = \frac{dW}{dt} = \vec{F}\cdot\vec{v} \tag{24}$$

Work-energy theorem:

$$\Delta E_{system} = W \tag{25}$$

$$E_{system} = (E_1 + E_2 + E_3 + ...) + U \tag{26}$$

Relativistic energy relations:

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = mc^2 + K \tag{27}$$

$$E^2 - (pc)^2 = \left(mc^2\right)^2 \tag{28}$$

$$K = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 \underset{v \ll c}{\rightarrow} \frac{1}{2}mv^2 \tag{29}$$

Potential energy:

$$\Delta U = -W_{\text{internal}} = -\int \vec{F} \cdot d\vec{r} \tag{30}$$

$$\vec{F} = -\overrightarrow{\nabla} U = \begin{pmatrix} -\frac{\partial U}{\partial x} \\ -\frac{\partial U}{\partial y} \\ -\frac{\partial U}{\partial z} \end{pmatrix} \tag{31}$$

$$U_{gravity} = -G\frac{m_1 m_2}{r} \tag{32}$$

$$U_{gravity} = mgh \tag{33}$$

$$U_{electric} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} \tag{34}$$

$$U_{spring} = \frac{1}{2}kx^2 \tag{35}$$

Heat capacity:

$$\Delta E_{thermal} = mC\Delta T \tag{36}$$

Friction forces:

$$f_s \leq \mu_s N \tag{37}$$

$$f_k = \mu_k N \tag{38}$$

Drag force (air):

$$\vec{F}_{air} = -\frac{1}{2}C\rho A v^2 \hat{v} \tag{39}$$

Energy levels for the Hydrogen atom:

$$E_N = \frac{-13.6}{N^2} \text{ eV}, N = 1, 2, 3, \ldots \tag{40}$$

Vibrational energy levels:

$$E_N = E_0 + N\hbar\omega_0 = E_0 + N\hbar\sqrt{\frac{k_s}{m}},\ N = 0, 1, 2, .. \tag{41}$$

Energy and wavelength of light:

$$E_{photon} = \frac{hc}{\lambda_{light}} \tag{42}$$

Center of mass:

$$\vec{r}_{cm} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i} = \frac{1}{M}\sum_i m_i \vec{r}_i \tag{43}$$

$$\vec{r}_{cm} = \frac{\int \vec{r} dm}{\int dm} = \frac{1}{M}\int \vec{r} dm \tag{44}$$

Motion of the center of mass:

$$M\vec{a}_{cm} = \vec{F}_{net,ext} \tag{45}$$

Gravitational potential energy of a multi-particle system:

$$U = Mgy_{cm} \tag{46}$$

Kinetic energy of a multi-particle system:

$$K = K_{trans} + K_{rel} = \frac{1}{2}M{v_{cm}}^2 + K_{rel} \tag{47}$$

Impulse of a force:

$$\vec{J} = \int \vec{F} dt \tag{48}$$

Momentum and impulse:

$$\vec{J} = \vec{p}_f - \vec{p}_i \tag{49}$$

Conservation of linear momentum:

$$\Delta\vec{p}_{system} + \Delta\vec{p}_{surroundings} = 0 \tag{50}$$

Elastic collision in one dimension (mass 2 at rest before the collision):

$$v_{2f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \tag{51}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} \tag{52}$$

Completely inelastic collision in one dimension (mass 2 at rest before the collision):

$$v_f = \frac{m_1}{m_1 + m_2} v_i \tag{53}$$

Moment of inertia for a discreet mass distribution:

$$I = \sum_i m_i {r_i}^2 \tag{54}$$

Moment of inertia for a discreet mass distribution:

$$I = \int_{Volume} r^2 dm \tag{55}$$

Kinetic energy of a rotating rigid object:

$$K = \frac{1}{2} I \omega^2 \tag{56}$$

Torque:

$$\vec{\tau} = \vec{r} \times \vec{F} \tag{57}$$

Newton's "second" law for rotational motion:

$$\vec{\tau} = I \vec{\alpha} \tag{58}$$

Angular momentum of a single particle:

$$\vec{\mathrm{L}} = \vec{r} \times \vec{p} \tag{59}$$

Angular momentum of a rotating rigid object:

$$\vec{\mathrm{L}} = I \vec{\omega} \tag{60}$$

The angular momentum principle:

$$\frac{d\vec{\mathrm{L}}}{dt} = \left(\vec{r} \times \vec{F}\right)_{\text{net, ext}} = \vec{\tau}_{\text{net, ext}} \tag{61}$$

Number of micro states:

$$\Omega = \frac{(q + N - 2)!}{q!(N-2)!} \tag{62}$$

Definition of entropy S:

$$S = k \ln \Omega \tag{63}$$

Definition of temperature T:

$$\frac{1}{T} = \frac{dS}{dE_{\text{int}}} \tag{64}$$

The Boltzmann distribution:

$$P(\Delta E) \propto e^{-\Delta E/kT} \tag{65}$$

The Maxwell-Boltzmann velocity distribution:

$$P(v) = 4\pi\left(\frac{M}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-\frac{1}{2}Mv^2/(kT)} \tag{66}$$

Root-mean-square speed:

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{M}} \tag{67}$$

Average translational kinetic energy of an ideal gas:

$$\overline{K}_{trans} = \frac{3}{2}kT \tag{68}$$

Mean-free path d:

$$\frac{N}{V}\left[\pi(R+r)^2 d\right] \approx 2 \tag{69}$$

Number of gas molecules hitting an area A per second:

$$\text{molecules/s} = \frac{1}{4}\frac{N}{V}A\overline{v} \tag{70}$$

Ideal gas law:

$$pV = NkT \tag{71}$$

Work done by a gas:

$$W = \int_{V_1}^{V_2} p\,dV \tag{72}$$

First law of thermodynamics:

$$\Delta E_{system} = W + Q \tag{73}$$

Isothermal compression:

$$W = NkT\ln\left(\frac{V_1}{V_2}\right) \tag{74}$$

Adiabatic compression:

$$pV^{\gamma} = pV^{C_p/C_V} = \text{constant} \tag{75}$$

Heat capacity C defined:

$$\Delta Q = C\Delta T \tag{76}$$

Heat capacities per molecule for ideal monatomic gasses:

$$C_V = \frac{3}{2}k \tag{77}$$

Heat capacities per molecule for ideal non-monatomic gasses:

$$C_V \geq \frac{3}{2} \tag{78}$$

Relation between C_p and C_V:

$$C_p = C_V + k \tag{79}$$

Rate of thermal energy transfer:

$$\frac{dQ}{dt} = \sigma A \frac{T_H - T_L}{L} \tag{80}$$

Efficiency of a heat engine:

$$\text{efficiency} = \frac{|W|}{Q_H} = 2 - \frac{T_L}{T_H} \tag{81}$$

Quality factor of a heat pump:

$$K = \frac{|Q_C|}{|W|} \tag{82}$$

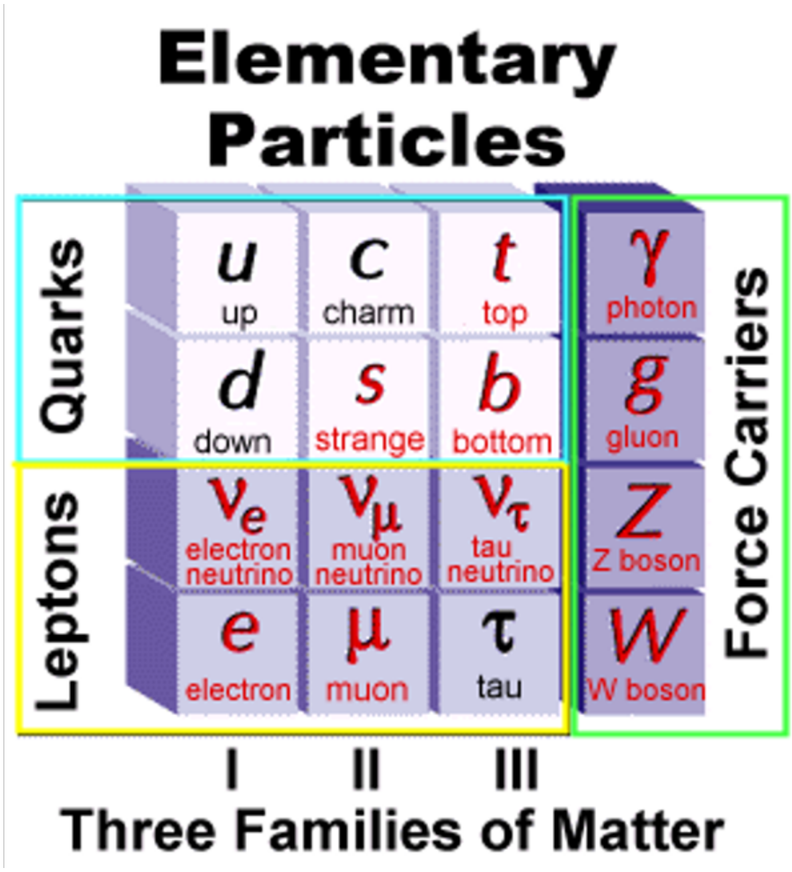
Important additional information:

Best baseball team in the USA:
Yankees

Best soccer team in the world:
AJAX

Best airline in the world:
KLM (Koninklijke Luchtvaart Maatschappij = Royal Dutch Airlines)

If in doubt, the correct answer may be:
Yankees, AJAX, the Netherlands, or KLM.


Elementary Particles
Quarks
u up
c charm
t top
d down
s strange
b bottom
Leptons
νe electron neutrino
νμ muon neutrino
ντ tau neutrino
e electron
μ muon
τ tau
Force Carriers
γ photon
g gluon
Z Z boson
W W boson
I II III
Three Families of Matter

FERMIONS

matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	–1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	–1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	–1

Quarks spin = 1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	175	2/3
b bottom	4.3	–1/3

BOSONS

force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.4	–1
W^+	80.4	+1
Z^0	91.187	0

Strong (color) spin = 1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$

Baryons are fermionic hadrons.
There are about 120 types of baryons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
p	proton	uud	1	0.938	1/2
$\bar{p}$	anti-proton	$\bar{u}\bar{u}\bar{d}$	–1	0.938	1/2
n	neutron	udd	0	0.940	1/2
Λ	lambda	uds	0	1.116	1/2
Ω^-	omega	sss	–1	1.672	3/2

Mesons $q\bar{q}$

Mesons are bosonic hadrons.
There are about 140 types of mesons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	–1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.770	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2 .980	0

APPENDIX

A Mathematical Formulas

A–1 Quadratic Formula

If $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

A–2 Binomial Expansion

$$(1 \pm x)^n = 1 \pm nx + \frac{n(n-1)}{2!}x^2 \pm \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

$$(x + y)^n = x^n\left(1 + \frac{y}{x}\right)^n = x^n\left(1 + n\frac{y}{x} + \frac{n(n-1)}{2!}\frac{y^2}{x^2} + \cdots\right)$$

A–3 Other Expansions

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots$$

$$\tan\theta = \theta + \frac{\theta^3}{3} + \frac{2}{15}\theta^5 + \cdots \qquad |\theta| < \frac{\pi}{2}$$

In general: $f(x) = f(0) + \left(\dfrac{df}{dx}\right)_0 x + \left(\dfrac{d^2f}{dx^2}\right)_0 \dfrac{x^2}{2!} + \cdots$

A–4 Exponents

$$(a^n)(a^m) = a^{n+m}$$

$$(a^n)(b^n) = (ab)^n$$

$$(a^n)^m = a^{nm}$$

$$\frac{1}{a^n} = a^{-n}$$

$$a^n a^{-n} = a^0 = 1$$

$$a^{\frac{1}{2}} = \sqrt{a}$$

A–5 Areas and Volumes

Object	Surface area	Volume
Circle, radius r	πr^2	—
Sphere, radius r	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Right circular cylinder, radius r, height h	$2\pi r^2 + 2\pi rh$	$\pi r^2 h$
Right circular cone, radius r, height h	$\pi r^2 + \pi r\sqrt{r^2 + h^2}$	$\frac{1}{3}\pi r^2 h$

A–8 Vectors

Vector addition is covered in Sections 3–2 to 3–5.
Vector multiplication is covered in Sections 3–3, 7–2, and 11–2.

A–9 Trigonometric Functions and Identities

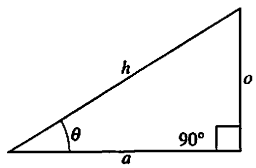


FIGURE A–5

The trigonometric functions are defined as follows (see Fig. A–5, o = side opposite, a = side adjacent, h = hypotenuse. Values are given in Table A–2):

$$\sin\theta = \frac{o}{h} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{h}{o}$$

$$\cos\theta = \frac{a}{h} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{h}{a}$$

$$\tan\theta = \frac{o}{a} = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{a}{o}$$

and recall that

$$a^2 + o^2 = h^2 \qquad \text{[Pythagorean theorem]}.$$

FIGURE A–6

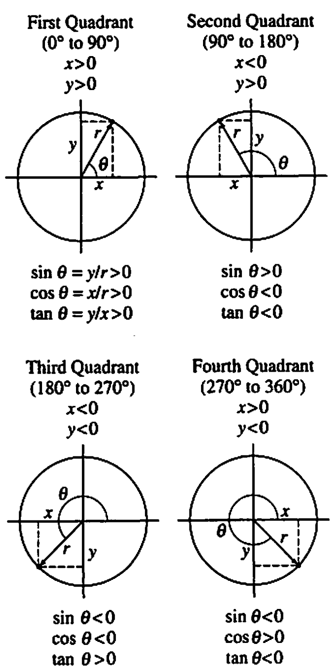


Figure A–6 shows the signs (+ or −) that cosine, sine, and tangent take on for angles θ in the four quadrants (0° to 360°). Note that angles are measured counterclockwise from the x axis as shown; negative angles are measured from *below* the x axis, clockwise: for example, $-30° = +330°$, and so on.

The following are some useful identities among the trigonometric functions:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sec^2\theta - \tan^2\theta = 1, \quad \csc^2\theta - \cot^2\theta = 1$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\sin(180° - \theta) = \sin\theta$$

$$\cos(180° - \theta) = -\cos\theta$$

$$\sin(90° - \theta) = \cos\theta$$

$$\cos(90° - \theta) = \sin\theta$$

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\sin\tfrac{1}{2}\theta = \sqrt{\frac{1 - \cos\theta}{2}}, \quad \cos\tfrac{1}{2}\theta = \sqrt{\frac{1 + \cos\theta}{2}}, \quad \tan\tfrac{1}{2}\theta = \sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}$$

$$\sin A \pm \sin B = 2\sin\left(\frac{A \pm B}{2}\right)\cos\left(\frac{A \mp B}{2}\right).$$

FIGURE A–7

For any triangle (see Fig. A–7):

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} \qquad \text{[Law of sines]}$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma. \qquad \text{[Law of cosines]}$$

Values of sine, cosine, tangent are given in Table A–2.

A-4 APPENDIX A

APPENDIX

B Derivatives and Integrals

Derivatives: General Rules

(See also Section 2–3.)

$$\frac{dx}{dx} = 1$$

$$\frac{d}{dx}[af(x)] = a\frac{df}{dx} \qquad [a = \text{constant}]$$

$$\frac{d}{dx}[f(x) + g(x)] = \frac{df}{dx} + \frac{dg}{dx}$$

$$\frac{d}{dx}[f(x)g(x)] = \frac{df}{dx}g + f\frac{dg}{dx}$$

$$\frac{d}{dx}[f(y)] = \frac{df}{dy}\frac{dy}{dx} \qquad [\text{chain rule}]$$

$$\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)} \qquad \text{if } \frac{dy}{dx} \neq 0.$$

Derivatives: Particular Functions

$$\frac{da}{dx} = 0 \qquad [a = \text{constant}]$$

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}\sin ax = a\cos ax$$

$$\frac{d}{dx}\cos ax = -a\sin ax$$

$$\frac{d}{dx}\tan ax = a\sec^2 ax$$

$$\frac{d}{dx}\ln ax = \frac{1}{x}$$

$$\frac{d}{dx}e^{ax} = ae^{ax}$$

Indefinite Integrals: General Rules

(See also Section 7–3.)

$$\int dx = x$$

$$\int af(x)\,dx = a\int f(x)\,dx \qquad [a = \text{constant}]$$

$$\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int u\,dv = uv - \int v\,du$$ [integration by parts: see also B

Indefinite Integrals: Particular Functions

(An arbitrary constant can be added to the right side of each equation.)

$$\int a\,dx = ax \qquad [a = \text{constant}]$$

$$\int x^m\,dx = \frac{1}{m+1}x^{m+1} \qquad [m \neq -1]$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax$$

$$\int \tan ax\,dx = \frac{1}{a}\ln|\sec ax|$$

$$\int \frac{1}{x}\,dx = \ln x$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left(x + \sqrt{x^2 \pm a^2}\right)$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) = -\cos^{-1}\left(\frac{x}{a}\right) \qquad [\text{if } x^2 \leq a^2]$$

$$\int \frac{dx}{(x^2 \pm a^2)^{\frac{3}{2}}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}}$$

$$\int \frac{x\,dx}{(x^2 \pm a^2)^{\frac{3}{2}}} = \frac{-1}{\sqrt{x^2 \pm a^2}}$$

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$\int xe^{-ax}\,dx = -\frac{e^{-ax}}{a^2}(ax + 1)$$

$$\int x^2e^{-ax}\,dx = -\frac{e^{-ax}}{a^3}(a^2x^2 + 2ax + 2)$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right) \qquad [x^2 > a^2]$$

$$= -\frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right) \qquad [x^2 < a^2]$$

A Few Definite Integrals

$$\int_0^\infty x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty e^{-ax^2}\,dx = \sqrt{\frac{\pi}{4a}}$$

$$\int_0^\infty xe^{-ax^2}\,dx = \frac{1}{2a}$$

$$\int_0^\infty x^2e^{-ax^2}\,dx = \sqrt{\frac{\pi}{16a^3}}$$

$$\int_0^\infty x^3e^{-ax^2}\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty x^{2n}e^{-ax^2}\,dx = \frac{1\cdot3\cdot5\cdots(2n-1)}{2^{n+1}a^n}\sqrt{\frac{\pi}{a}}$$

Integration by Parts

Sometimes a difficult integral can be simplified by carefully choosing the functions u and v in the identity:

$$\int u\,dv = uv - \int v\,du. \qquad [\text{Integration by parts}]$$

This identity follows from the property of derivatives

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

or as differentials: $d(uv) = u\,dv + v\,du$.

For example $\int xe^{-x}\,dx$ can be integrated by choosing $u = x$ and $dv = e^{-x}\,dx$ in the "integration by parts" equation above:

$$\int xe^{-x}\,dx = (x)(-e^{-x}) + \int e^{-x}\,dx$$

$$= -xe^{-x} - e^{-x} = -(x+1)e^{-x}.$$

SECTION B–6 **A-7**