The momentum principle:

$$d\vec{\mathbf{p}} = \vec{\mathbf{F}}_{net}\,dt$$

$$\vec{\mathbf{p}} = \frac{m\vec{\mathbf{v}}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\vec{\mathbf{p}}_{new} = \vec{\mathbf{p}}_{old} + \vec{\mathbf{F}}_{net}\Delta t$$

$$\vec{\mathbf{r}}_{new} = \vec{\mathbf{r}}_{old} + \frac{1}{\sqrt{1+\left(\frac{p}{mc}\right)^2}}\left(\frac{\vec{\mathbf{p}}}{m}\right)\Delta t$$

Equations of motion in 1D for constant acceleration and low velocities ($v << c$):

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v(t) = \frac{dx(t)}{dt} = v_0 + at$$

$$a(t) = \frac{dv(t)}{dt} = a = \text{constant}$$

Requirement for uniform circular motion:

$$F_r = \frac{mv^2}{r}$$

Rotational motion:

$$d = \theta r$$

$$v = \omega r \quad \omega = \frac{d\theta}{dt}$$

$$a = \alpha r \quad \alpha = \frac{d\omega}{dt}$$

Gravitational force:

$$\vec{\mathbf{F}} = G\frac{m_1 m_2}{r^2}\hat{\mathbf{r}}$$

$$\vec{\mathbf{F}} = m\vec{\mathbf{g}} \quad \text{(close to the surface of the Earth)}$$

Electrostatic force:

$$\vec{\mathbf{F}} = -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$$

Harmonic motion:

$$F = -kx$$

$$x(t) = x_{\max}\cos(\omega t + \phi) \quad \text{where} \quad \omega = \sqrt{\frac{k}{m}}$$

$$T = \frac{2\pi}{\omega}$$

Damped harmonic motion:

$$x(t) = x_m e^{-\frac{bt}{2m}} e^{it\sqrt{\frac{k}{m}}}$$

Driven harmonic motion:

$$x(t) = \frac{F_0}{{\omega_0}^2 - \omega^2}\cos(\omega t + \phi)$$

Stress and strain:

$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

$$Y = \frac{k_s}{d}$$

Friction forces:

$$f_s \le \mu_s N$$

$$f_k = \mu_k N$$

Drag force (air):

$$\vec{\mathbf{F}}_{air} = -\frac{1}{2}C\rho A v^2\hat{\mathbf{v}}$$

APPENDIX

A Mathematical Formulas

A–1 Quadratic Formula

If $\quad ax^2 + bx + c = 0$

then $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

A–2 Binomial Expansion

$$(1 \pm x)^n = 1 \pm nx + \frac{n(n-1)}{2!}x^2 \pm \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

$$(x + y)^n = x^n\left(1 + \frac{y}{x}\right)^n = x^n\left(1 + n\frac{y}{x} + \frac{n(n-1)}{2!}\frac{y^2}{x^2} + \cdots\right)$$

A–3 Other Expansions

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots$$

$$\tan\theta = \theta + \frac{\theta^3}{3} + \frac{2}{15}\theta^5 + \cdots \qquad |\theta| < \frac{\pi}{2}$$

In general: $\quad f(x) = f(0) + \left(\dfrac{df}{dx}\right)_0 x + \left(\dfrac{d^2f}{dx^2}\right)_0 \dfrac{x^2}{2!} + \cdots$

A–4 Exponents

$$(a^n)(a^m) = a^{n+m} \qquad \frac{1}{a^n} = a^{-n}$$

$$(a^n)(b^n) = (ab)^n \qquad a^n a^{-n} = a^0 = 1$$

$$(a^n)^m = a^{nm} \qquad a^{\frac{1}{2}} = \sqrt{a}$$

A–5 Areas and Volumes

Object	Surface area	Volume
Circle, radius r	πr^2	—
Sphere, radius r	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Right circular cylinder, radius r, height h	$2\pi r^2 + 2\pi r h$	$\pi r^2 h$
Right circular cone, radius r, height h	$\pi r^2 + \pi r\sqrt{r^2 + h^2}$	$\frac{1}{3}\pi r^2 h$

A–8 Vectors

Vector addition is covered in Sections 3–2 to 3–5.
Vector multiplication is covered in Sections 3–3, 7–2, and 11–2.

A–9 Trigonometric Functions and Identities

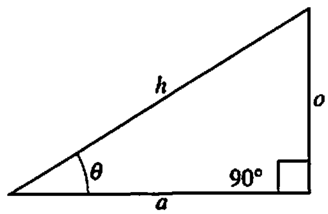

FIGURE A–5

The trigonometric functions are defined as follows (see Fig. A–5, o = side opposite, a = side adjacent, h = hypotenuse. Values are given in Table A–2):

$$\sin\theta = \frac{o}{h} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{h}{o}$$

$$\cos\theta = \frac{a}{h} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{h}{a}$$

$$\tan\theta = \frac{o}{a} = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{a}{o}$$

and recall that

$$a^2 + o^2 = h^2 \qquad \text{[Pythagorean theorem].}$$

FIGURE A–6

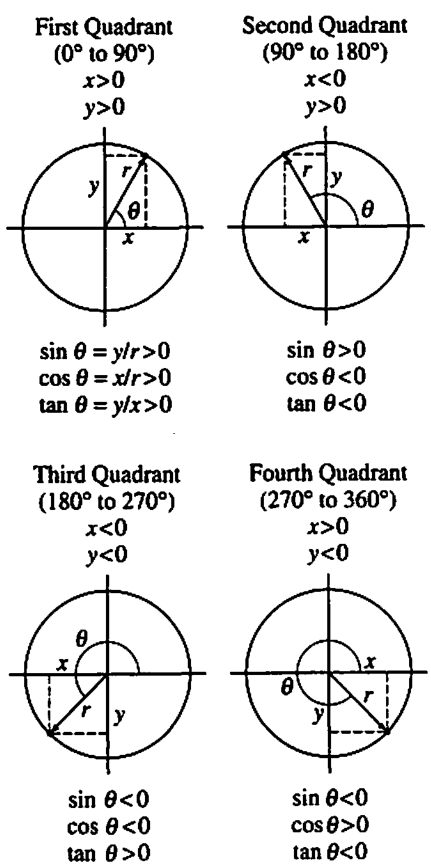

Figure A–6 shows the signs (+ or −) that cosine, sine, and tangent take on for angles θ in the four quadrants (0° to 360°). Note that angles are measured counterclockwise from the x axis as shown; negative angles are measured from *below* the x axis, clockwise: for example, $-30° = +330°$, and so on.

The following are some useful identities among the trigonometric functions:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sec^2\theta - \tan^2\theta = 1, \quad \csc^2\theta - \cot^2\theta = 1$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\sin(180° - \theta) = \sin\theta$$

$$\cos(180° - \theta) = -\cos\theta$$

$$\sin(90° - \theta) = \cos\theta$$

$$\cos(90° - \theta) = \sin\theta$$

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\sin\tfrac{1}{2}\theta = \sqrt{\frac{1-\cos\theta}{2}}, \quad \cos\tfrac{1}{2}\theta = \sqrt{\frac{1+\cos\theta}{2}}, \quad \tan\tfrac{1}{2}\theta = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$\sin A \pm \sin B = 2\sin\left(\frac{A \pm B}{2}\right)\cos\left(\frac{A \mp B}{2}\right).$$

FIGURE A–7

For any triangle (see Fig. A–7):

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} \qquad \text{[Law of sines]}$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma. \qquad \text{[Law of cosines]}$$

Values of sine, cosine, tangent are given in Table A–2.

APPENDIX

B Derivatives and Integrals

Derivatives: General Rules

(See also Section 2–3.)

$$\frac{dx}{dx} = 1$$

$$\frac{d}{dx}[af(x)] = a\frac{df}{dx} \qquad [a = \text{constant}]$$

$$\frac{d}{dx}[f(x) + g(x)] = \frac{df}{dx} + \frac{dg}{dx}$$

$$\frac{d}{dx}[f(x)g(x)] = \frac{df}{dx}g + f\frac{dg}{dx}$$

$$\frac{d}{dx}[f(y)] = \frac{df}{dy}\frac{dy}{dx} \qquad [\text{chain rule}]$$

$$\frac{dx}{dy} = \frac{1}{\left(\dfrac{dy}{dx}\right)} \qquad \text{if } \frac{dy}{dx} \neq 0.$$

Derivatives: Particular Functions

$$\frac{da}{dx} = 0 \qquad [a = \text{constant}]$$

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}\sin ax = a\cos ax$$

$$\frac{d}{dx}\cos ax = -a\sin ax$$

$$\frac{d}{dx}\tan ax = a\sec^2 ax$$

$$\frac{d}{dx}\ln ax = \frac{1}{x}$$

$$\frac{d}{dx}e^{ax} = ae^{ax}$$

Indefinite Integrals: General Rules

(See also Section 7–3.)

$$\int dx = x$$

$$\int a\,f(x)\,dx = a\int f(x)\,dx \qquad [a = \text{constant}]$$

$$\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int u\,dv = uv - \int v\,du \qquad [\text{integration by parts: see also B}$$

Indefinite Integrals: Particular Functions

(An arbitrary constant can be added to the right side of each equation.)

$$\int a\,dx = ax \qquad [a = \text{constant}]$$

$$\int x^m\,dx = \frac{1}{m+1}x^{m+1} \qquad [m \neq -1]$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax$$

$$\int \tan ax\,dx = \frac{1}{a}\ln|\sec ax|$$

$$\int \frac{1}{x}\,dx = \ln x$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left(x + \sqrt{x^2 \pm a^2}\right)$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) = -\cos^{-1}\left(\frac{x}{a}\right) \qquad [\text{if } x^2 \leq a^2]$$

$$\int \frac{dx}{(x^2 \pm a^2)^{\frac{3}{2}}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}}$$

$$\int \frac{x\,dx}{(x^2 \pm a^2)^{\frac{3}{2}}} = \frac{-1}{\sqrt{x^2 \pm a^2}}$$

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$\int xe^{-ax}\,dx = -\frac{e^{-ax}}{a^2}(ax + 1)$$

$$\int x^2e^{-ax}\,dx = -\frac{e^{-ax}}{a^3}(a^2x^2 + 2ax + 2)$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left(\frac{x - a}{x + a}\right) \qquad [x^2 > a^2]$$

$$= -\frac{1}{2a}\ln\left(\frac{a + x}{a - x}\right) \qquad [x^2 < a^2]$$

A Few Definite Integrals

$$\int_0^\infty x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty e^{-ax^2}\,dx = \sqrt{\frac{\pi}{4a}}$$

$$\int_0^\infty xe^{-ax^2}\,dx = \frac{1}{2a}$$

$$\int_0^\infty x^2e^{-ax^2}\,dx = \sqrt{\frac{\pi}{16a^3}}$$

$$\int_0^\infty x^3e^{-ax^2}\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty x^{2n}e^{-ax^2}\,dx = \frac{1\cdot3\cdot5\cdots(2n-1)}{2^{n+1}a^n}\sqrt{\frac{\pi}{a}}$$

Integration by Parts

Sometimes a difficult integral can be simplified by carefully choosing the functions u and v in the identity:

$$\int u\,dv = uv - \int v\,du. \qquad [\text{Integration by parts}]$$

This identity follows from the property of derivatives

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

or as differentials: $d(uv) = u\,dv + v\,du$.

For example $\int xe^{-x}\,dx$ can be integrated by choosing $u = x$ and $dv = e^{-x}\,dx$ in the "integration by parts" equation above:

$$\int xe^{-x}\,dx = (x)(-e^{-x}) + \int e^{-x}\,dx$$

$$= -xe^{-x} - e^{-x} = -(x + 1)e^{-x}.$$