Physics 121 Review
Midterm Exam # 2

Surviving Phy 121 Exams.

- Time your work:
  - Exam has 10 MC + 3 analytical questions.
  - Work 15 minutes on the MC questions.
  - Work 15 minutes on each of the analytical questions (45 minutes total).
  - You now have 20 minutes left to finish those questions you did not finish in the first 15 minutes.
- Write neatly:
  - You cannot earn credit if we cannot read what you wrote!
  - You cannot earn credit if we cannot read your name (list first and last name, and student ID).
- Write enough so that we can see your line of thought – you cannot earn credit for what you are thinking!

Surviving Phy 121 Exams.

- Most solutions should start with a diagram, showing all forces (direction and approximate magnitude) and dimensions. All forces and dimensions should be labeled with the variables that will be used in your solution. You also need to define the coordinate system you will be using,
- Indicate what variables are known and what variables are unknown.
- Indicate which variable(s) needs to be determined.
- Indicate the principle(s) that you use to solve the problem.
- If you make any approximations, indicate them.
- Check your units!

Preparing for Exam # 2

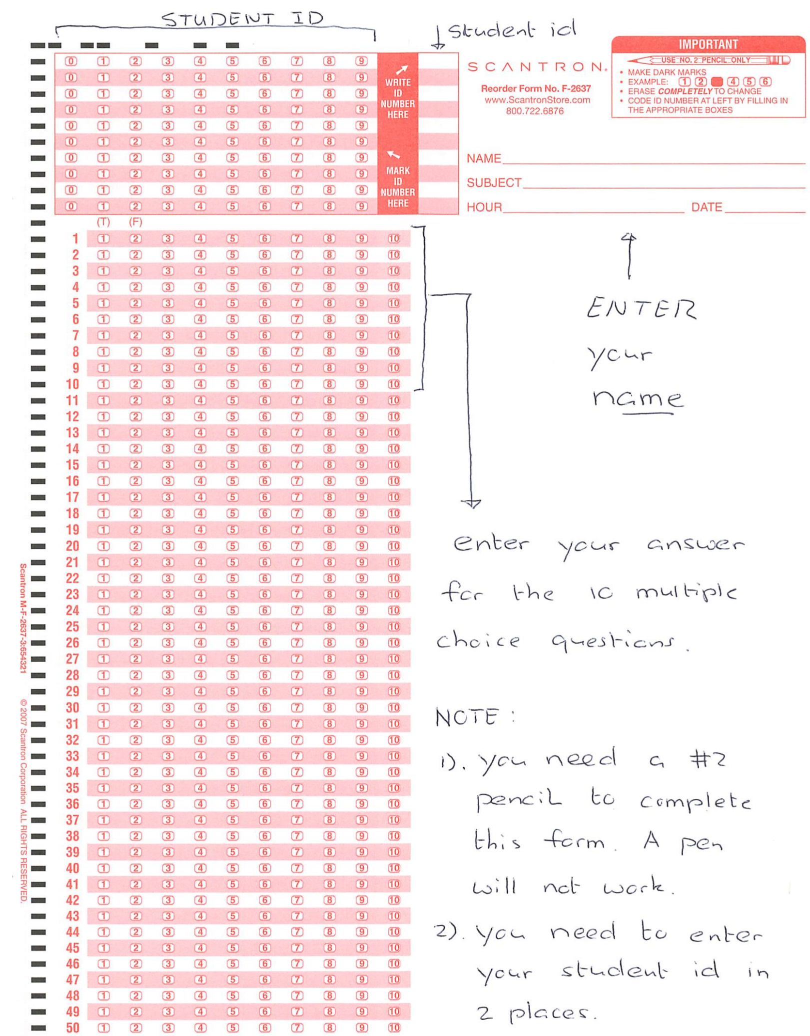
- Take the practice exam as if it was a real exam: take 80 minutes to complete it. Compare your work to the posted solutions to help you focus on specific areas.

- There will be extra office hours on Sunday 3/22 and Monday 3/23:
  - Cole Jerum: 12 pm - 2pm, Sunday 3/22, POA
  - Addison Price: 5 - 7 pm, Sunday 3/22, POA
  - Catherine Lei: 12.30 pm - 2.30 pm, Monday 3/23, POA
  - Thomas Qian: 6 pm - 8 pm, Monday 3/23, POA

- The workshops on Monday 3/23 should be considered Q&A sessions for Exam # 2. Anyone can attend any of the workshops on Monday 3/23.

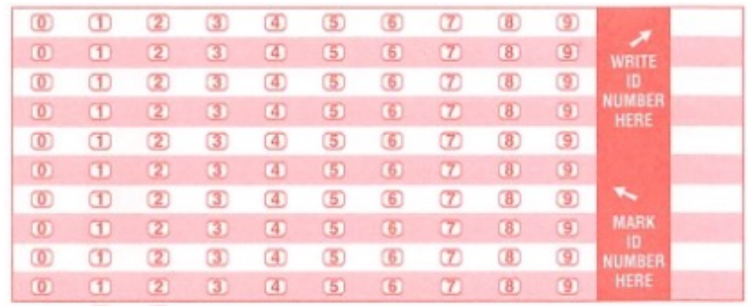
Preparing for Exam # 2

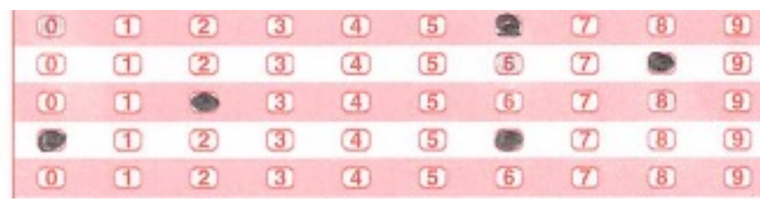
- You need a number 2 pencil to complete the multiple-choice part of the exam.
- You need to know your student ID #.
- You do NOT need a calculator on the exam.
- The exam starts at 8 am and ends at 9.20 am.
- If you are late to start, you will have less time to finish.
- No one can leave before 8.45 am. No one can start after 8.45 am.

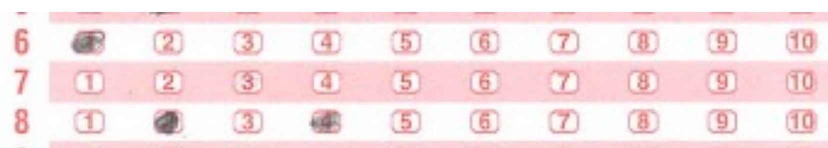


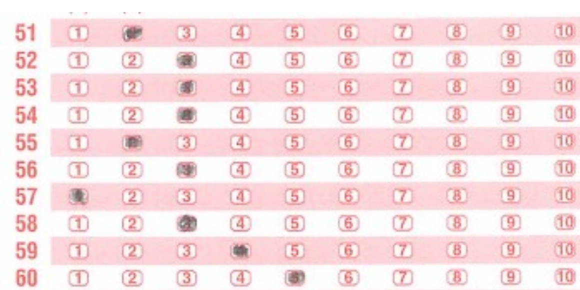
Lessons from Exam # 1.
15% did not (correctly) enter the ID #.

- Scantron:
  - 14 students did not enter their student ID on the scantron form.
  - 9 students did not enter their student ID correctly.
  - 4 students entered the incorrect student ID.
  - Students entered their solutions incorrectly.
  - Answers not on front.
- For exam # 1, I added missing student IDs were possible, but **there will be no adjustments for Exam # 2.**
- Note: your ID has 8 numbers.









Lessons from Exam # 1.

- Questions 11 and 12 must be answered in Book 1.
- Question 13 must be answered in Book 2.
- By putting a 1 or 2 on the front of the booklet, you define which one is Book 1 and which one is Book 2.
- If you answer Question 13 in Book 1, it will not be graded.
- Answers that are not motivated will not receive any credit, even if correct.
- Write neatly!!!

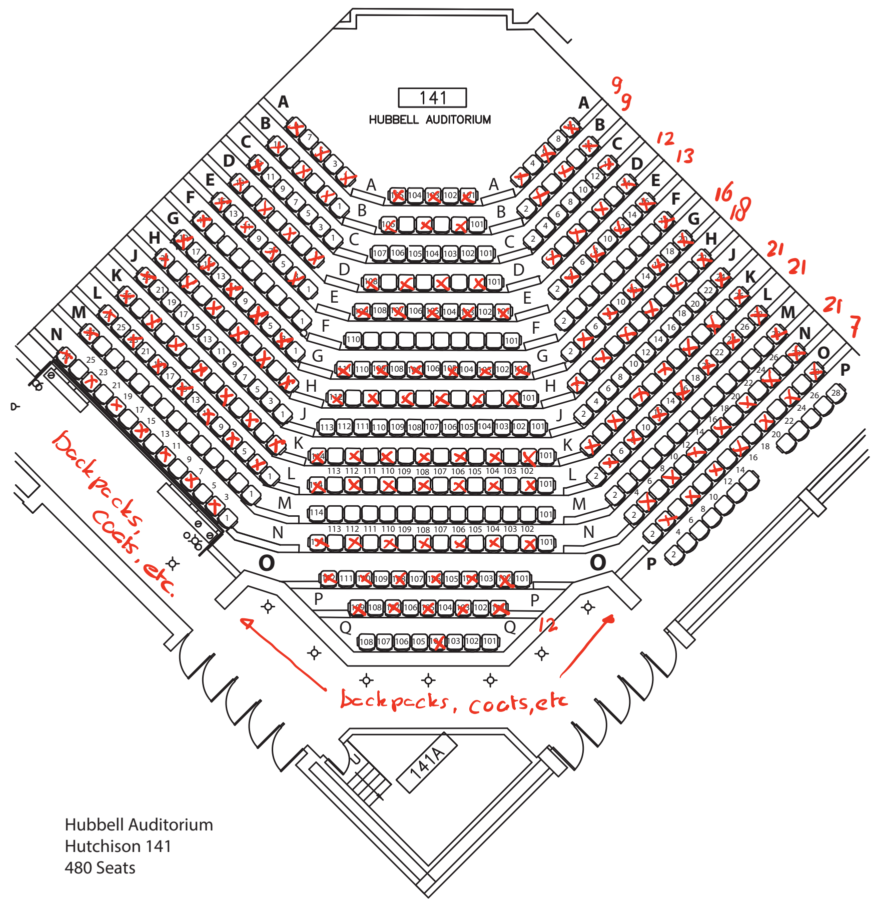
Problems 1 and 10 must be answered on the scantron form. **Problems 11 and 12 must be answered in exam booklet 1. Problem 13 must be answered in exam booklet 2.** The answers need to be well motivated and expressed in terms of the variables used in the problem. You will receive partial credit where appropriate, but only when we can read your solution. **Answers that are not motivated will not receive any credit, even if correct.**

At the end of the exam, you must hand in your exam, the scantron form, the blue exam booklets, and the equation sheet. **All items must be clearly labeled with your name, your student ID number, and the day/time/location of your workshop. If any of these items are missing, we will not grade your exam, and you will receive a score of 0 points.**

Front cover exam

Exam Location: Hubbell.

- You can not enter Hubbel before 7.45 am. You can only sit at locations where there is a brown envelope located. Your backpack, coat, and books need to be left at the entrance of Hubbel.
- You will only need your pen, a number 2 pencil, and an eraser. Being awake might also help!



Warning.

- I cannot cover everything I discussed in lectures 7 – 14 in this review.
- If I skip over certain topics, it does not mean you should not understand that material.
- Your TAs will not see the exam until you see it.
- **There will be lecture at 12.30 pm on Tuesday 3/24.**

Physics 121.
Topics for today.

- Review of the material covered on Exam # 2.
  - The gravitational force (chapter 6):
    - Newton's law of gravitation.
    - Gravitational force close to the surface of the Earth.
    - Orbital motion.
    - Kepler's laws.
  - Work and Energy (chapters 7 - 8):
    - Work and Work-Energy Theorem.
    - Kinetic and Potential Energy.
    - Conservative and Non-Conservative Forces.
    - Conservation of Energy.
  - Linear Momentum and Collisions (chapter 9):
    - Center-of-Mass and Motion of the Center-of-Mass.
    - Conservation of Linear Momentum.
    - Rocket Equations.
    - Elastic and Inelastic Collisions.

Chapter 6

The gravitational force.

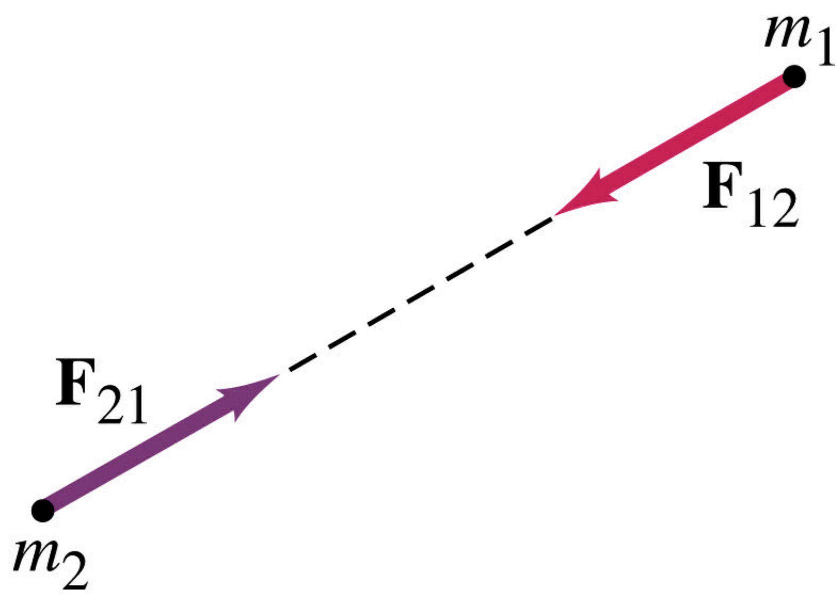
Excluded Sections:
6.8
6.9

Chapter 6.
The gravitational force.

- The magnitude of the gravitational force is given by the following relation:

$$\vec{F}_{grav} = G\frac{m_1 m_2}{r^2}\hat{r}$$

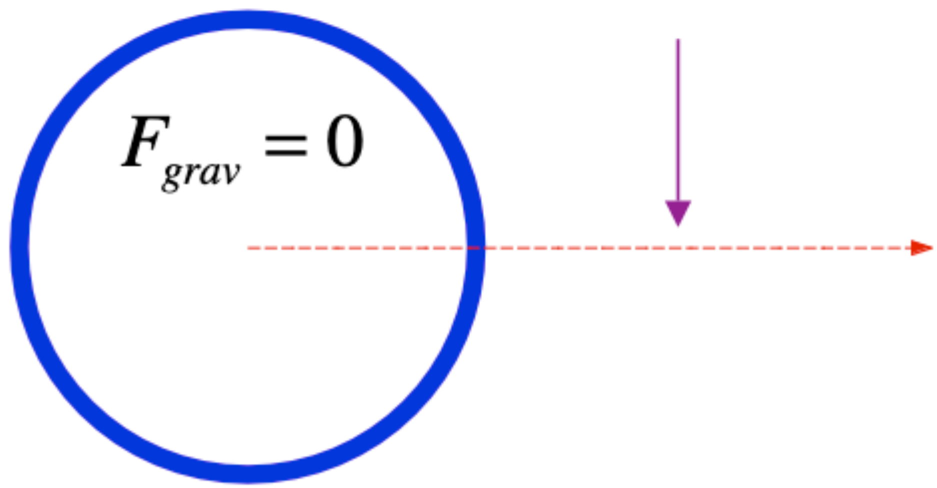
- The constant G is the gravitational constant which is equal to 6.67 x 10^{-11} N m^2/kg^2.
- The gravitational force is always attractive.
- It is directed along the line connecting the two masses.



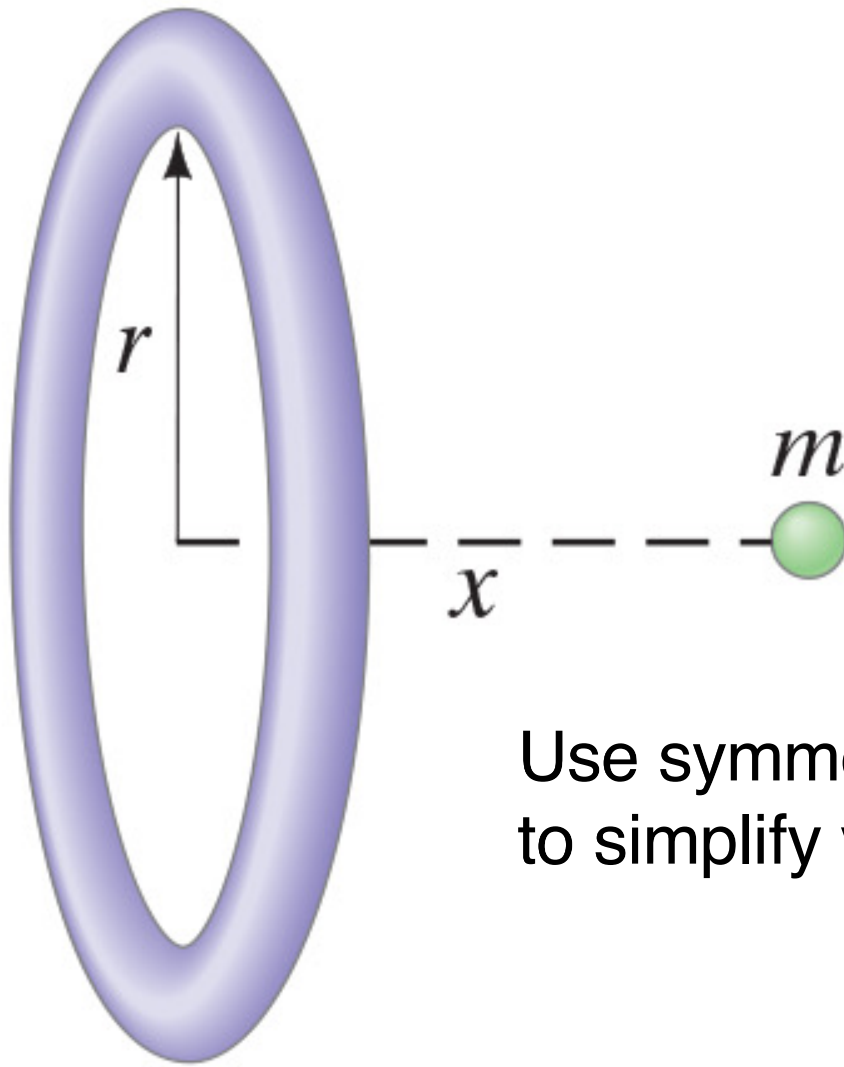
Chapter 6.
The shell theorem.

- Consider a shell of material of mass m_1 and radius R.
- In the region outside the shell, the gravitational force will be identical to what it would have been if all the mass of the shell was located at its center.
- In the region inside the shell, the gravitational force on a point mass m_2 is equal to 0 N.

$$F_{grav} = G\frac{m_1 m_2}{r^2}$$



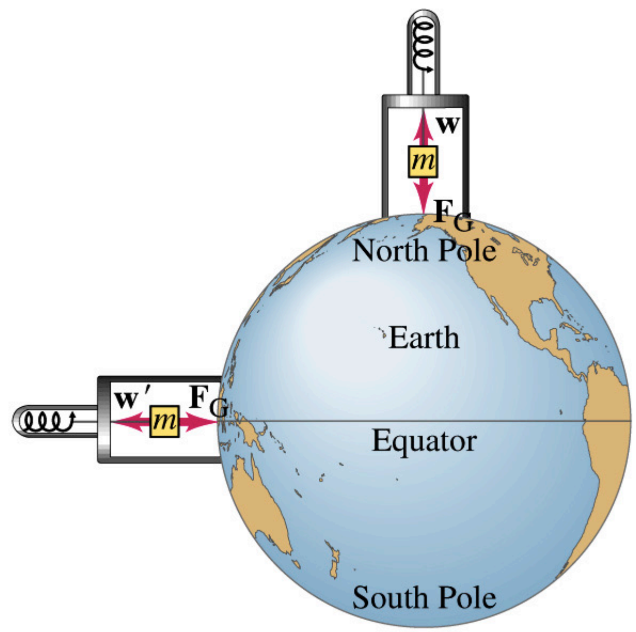
Chapter 6.
Calculating the gravitational force.



Use symmetry arguments
to simplify your calculations.

Chapter 6.
Variations in the gravitational force.

- The gravitational force on the surface of the earth is not uniform for a number of reasons:
  - The effect of the rotation of the earth.
  - The earth is not a perfect sphere.
  - The mass is not distributed uniformly, and significant variations in density can be found (in fact using variations in the gravitational force is one way to discover oil fields).



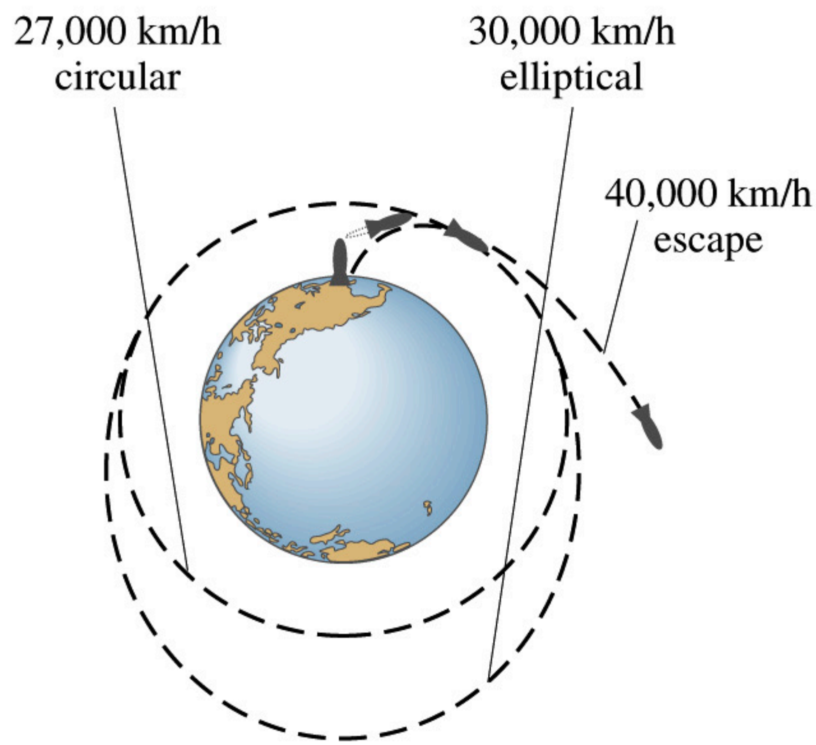
Chapter 6.
Orbital motion.

- Consider an object of mass m moving in a circular orbit of radius r around the earth.
- In order for this motion to be possible, a net force must be acting on this object with a magnitude of mv^2/r, directed towards the center of the earth.
- The only force that acts in this direction is the gravitational force and we must thus require that

$$G\frac{mM_{earth}}{r^2} = \frac{mv^2}{r}$$

or

$$v^2 = G\frac{M_{earth}}{r}$$



Chapter 6.
Orbital motion.

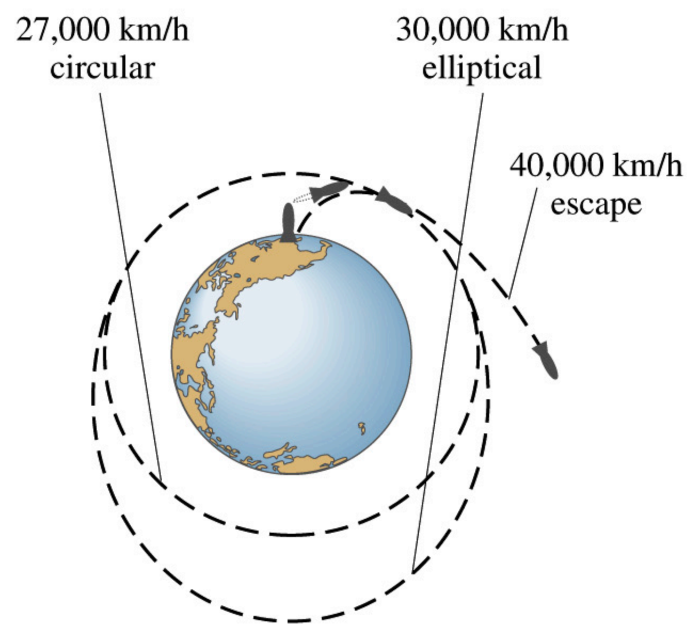
- The orbital velocity is related to the period of motion:

$$v = \frac{2\pi r}{T}$$

and the relation between v and r can be rewritten as a relation between T and r:

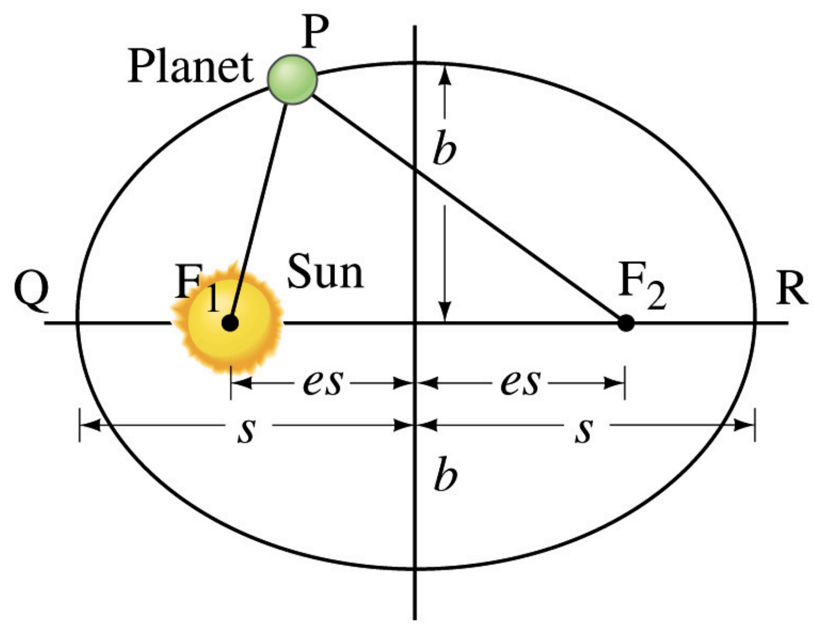
$$r^3 = GM_{earth}\frac{T^2}{4\pi^2}$$

- This relation shows that based on the orbital properties of the moon we can determine the mass of the earth.



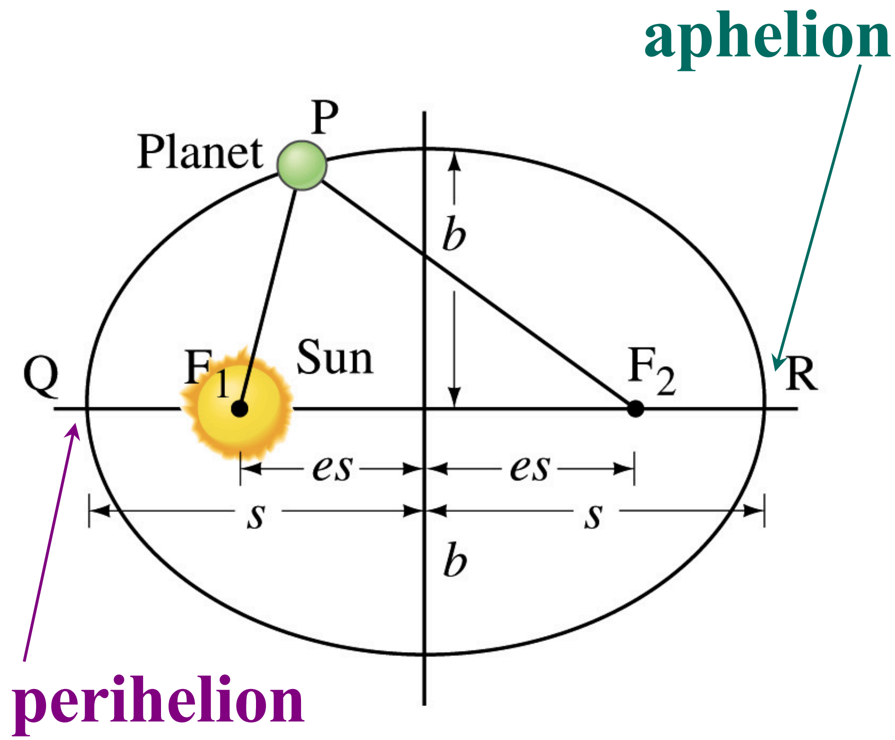
Chapter 6. Orbital shapes.

- Stable planetary motion does not require a perfect circular orbit.
- The shape of the orbit of a planet is described by an ellipse (note: a circle is a special type of ellipse). The ellipse is determined by specifying its semimajor axis s and its seminor axis b.
- The foci of an ellipse are special points for which the sum of the distance F_1 to P and the distance F_2 to P is the same for every point on the ellipse.



Chapter 6.
Kepler's first law.

- Note: for a circle $s = b$ and $F_1 = F_2$.
- The sun is located at one focus on the ellipse.
- The eccentricity e of the defined such that es is the distance from the center of the ellipse to either focus. Note: for a circle $e = 0$ m.
- The properties of the shape of the orbit of the planets and the location of the sun are part of what we call **Kepler's First Law**.



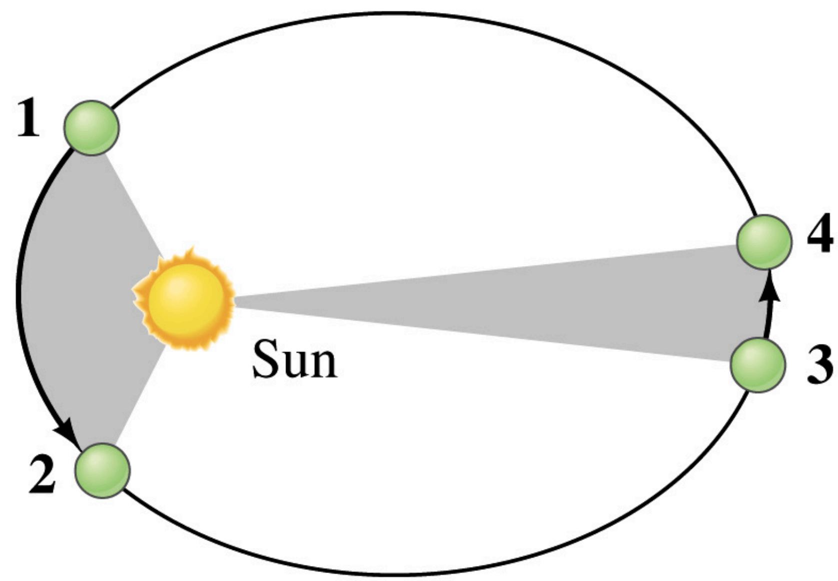
s = semi-major axis
b = semi-minor axis

Chapter 6.
Kepler's second law.

- Kepler's Second Law states:

  "Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal periods of time."

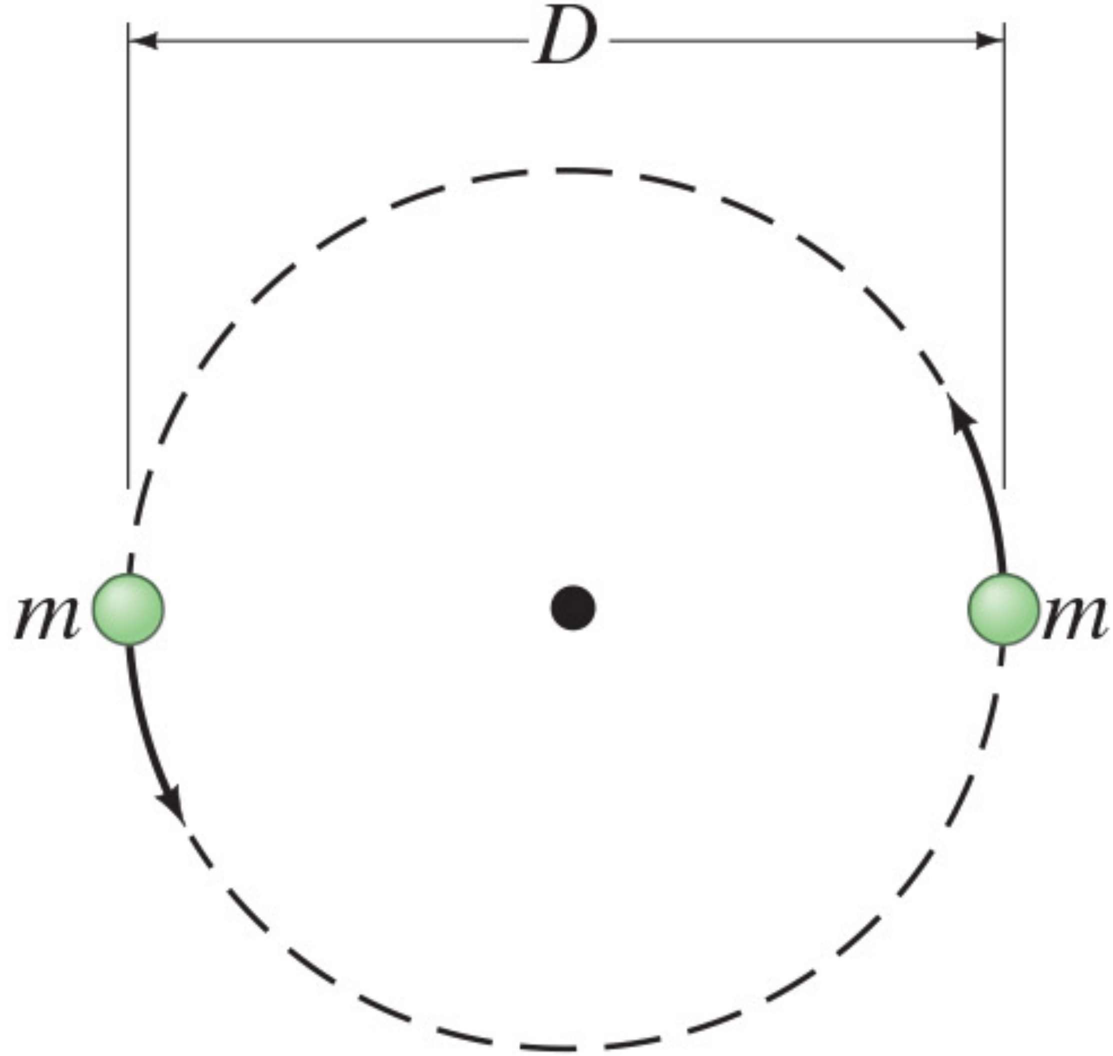
- Important consequences of Kepler's Second Law:
  - The velocity of the planet will increase the closer the planet is to the Sun (e.g. $v_{12} > v_{34}$).
  - The details of the orbit provide information about the mass of the sun.



Chapter 6.
Example problem.

A binary star.

What is the period of rotation?



Chapter 7

Work and Energy

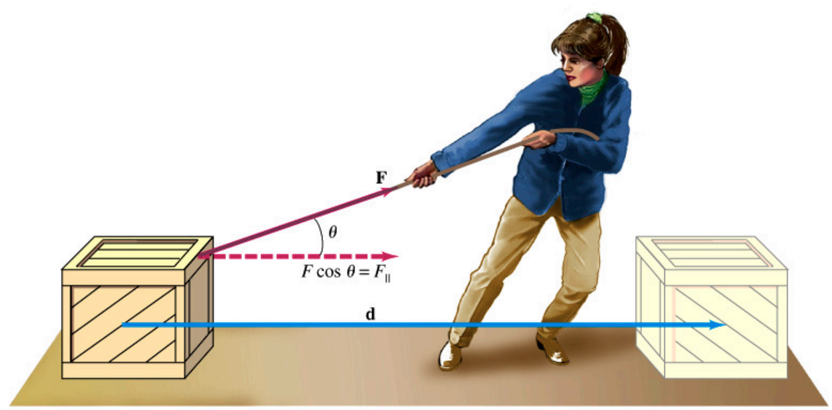
Chapter 7.
Work: Definition.

- When a force F is applied to an object, it may produce a displacement d.

- The work W done by the force F is defined as

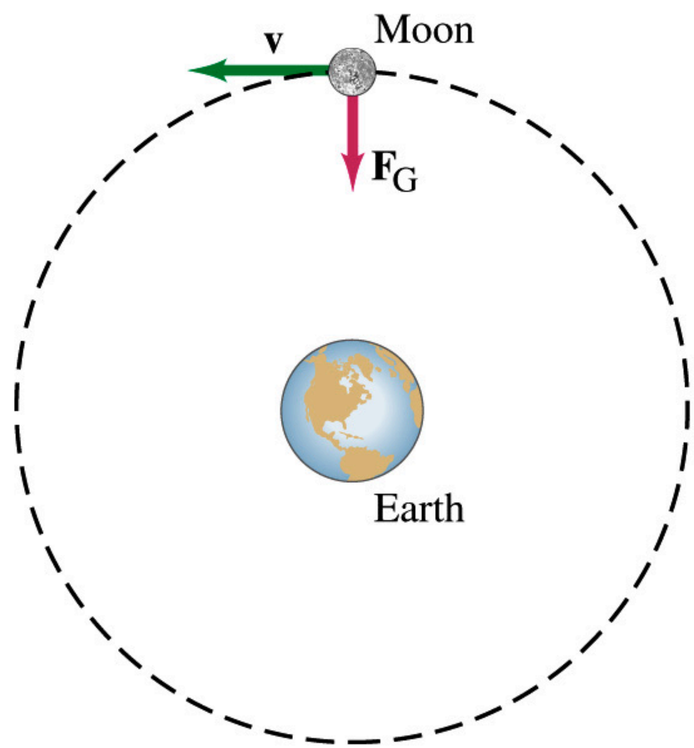
$$W = \vec{F} \cdot \vec{d} = Fd\cos\theta$$

where θ is the angle between the force $\vec{F}$ and the displacement $\vec{d}$.

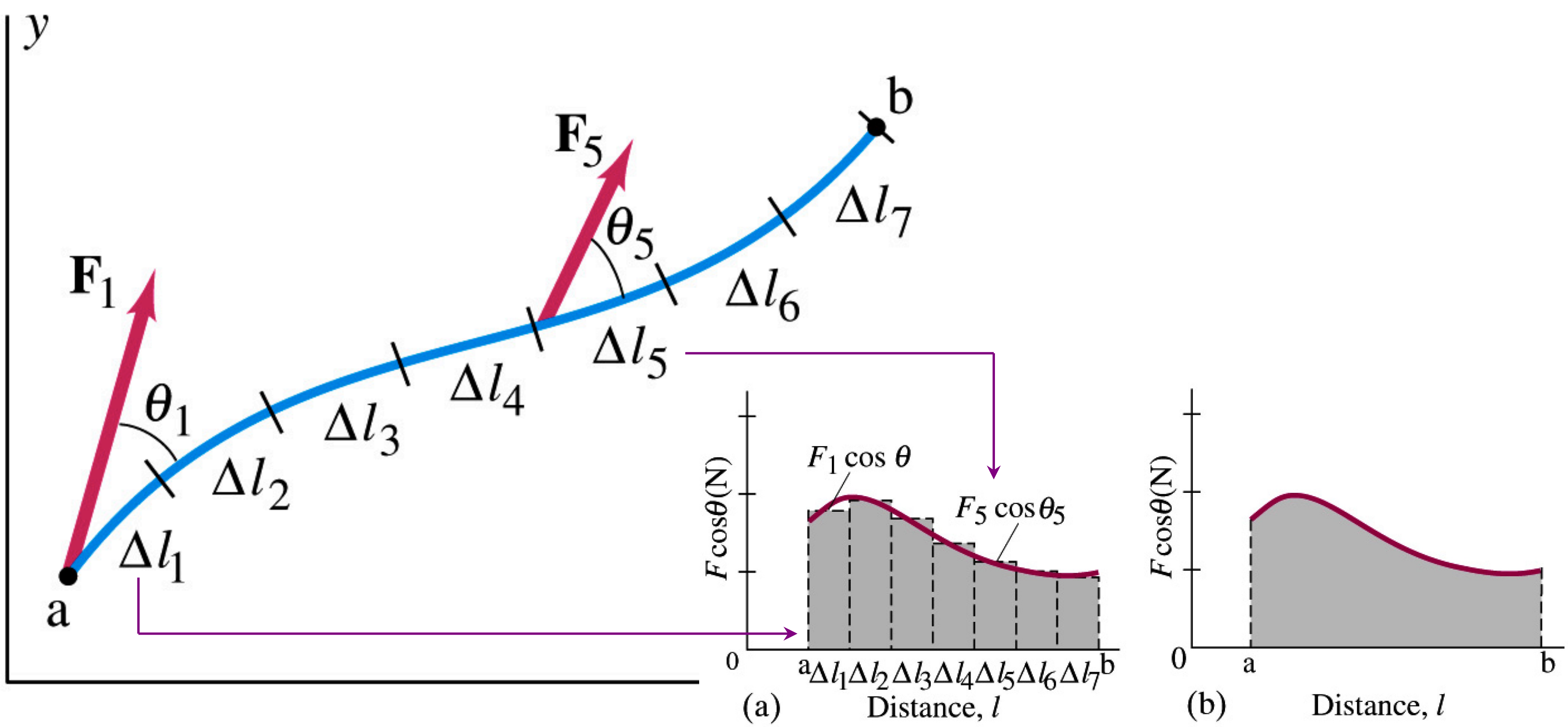


Chapter 7.
Work: Units.

- The unit of work is the Joule (abbreviated J).
- Per definition, 1 J = 1 Nm = 1 kg m^2/s^2.
- There are many important examples of forces that do not do any work. For example, the gravitational force between the earth and the moon does not do any work! Note: in this case, the speed of the moon does not change.

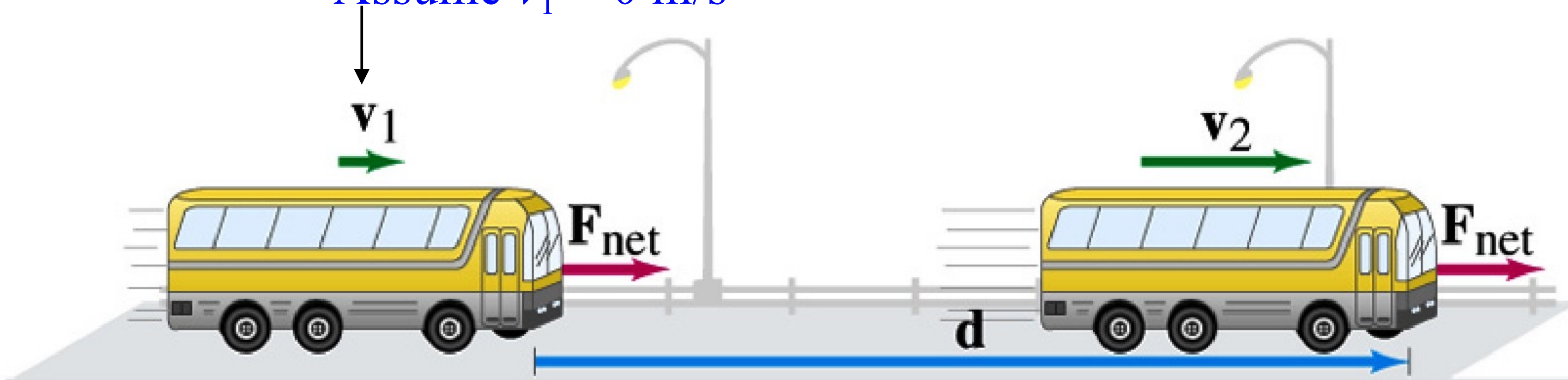


Chapter 7.
Work done by a varying force.



Chapter 7.
Work-Energy theorem.

Assume $v_1 = 0$ m/s

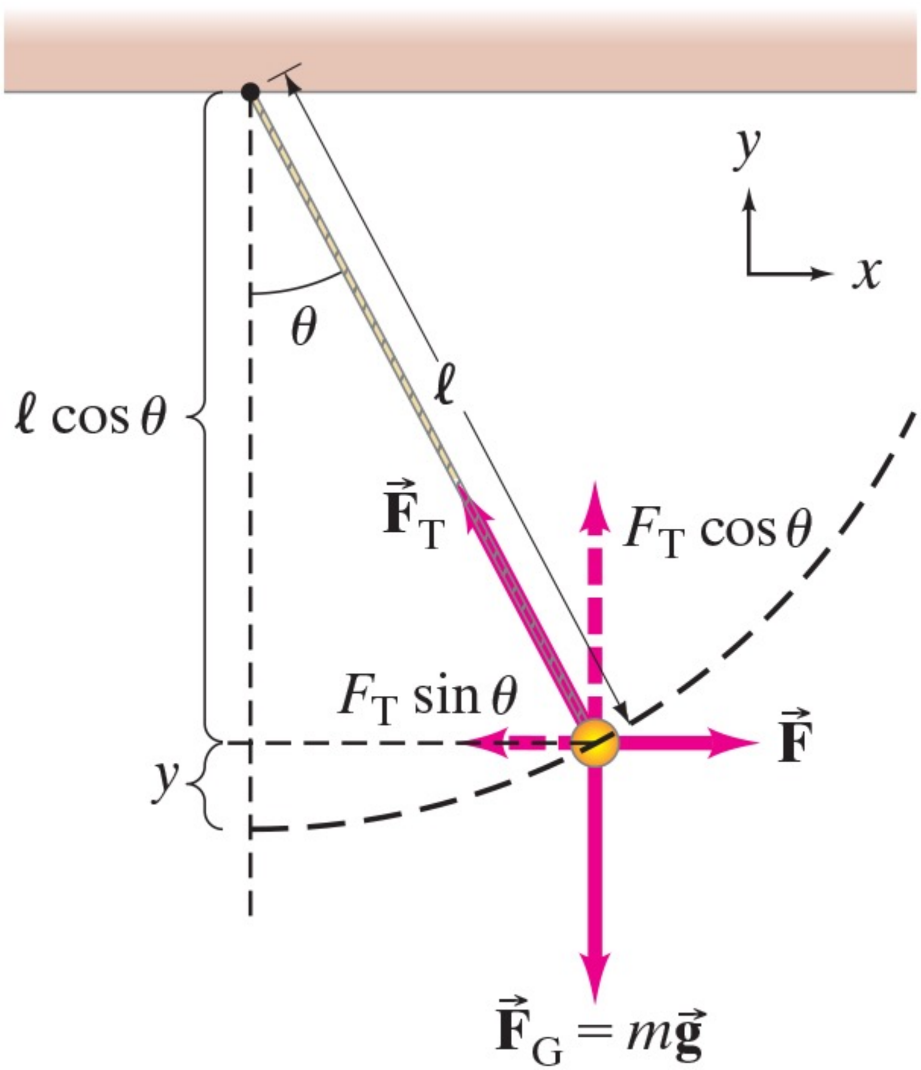


- Assuming that there is no heat flow in or out of the bus, we conclude that for low velocities (non-relativistic velocities):

The net work done on an object is equal to the change in its kinetic energy.

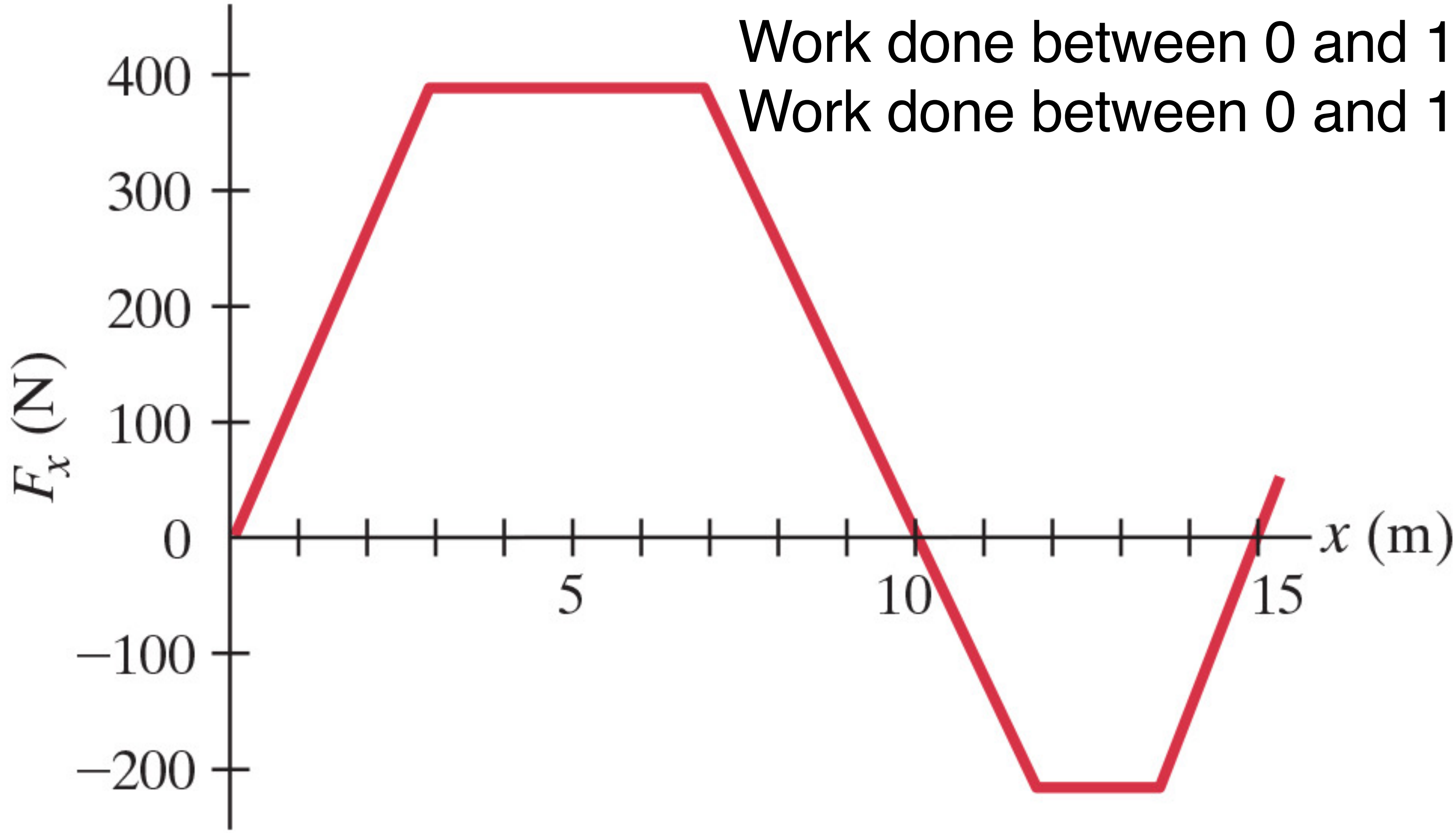
- In the case of the bus: $F_{net}d = \Delta E = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

Chapter 7.
Example problem.



To calculate the work done by the forces, you only need to determine the displacement in the direction of the force.

Chapter 7.
Example problem.



Work done between 0 and 10 m.
Work done between 0 and 15 m.

Chapter 8

Conservation of Energy

Excluded Sections:
8.10

Chapter 8.
Conservation of mechanical energy

- Consider what would happen if we define the mechanical energy of a system to be equal to the sum of the kinetic energy K and the potential energy U:

$$E = K + U$$

- If the total mechanical energy is constant, we must require that $\Delta E = 0$, or

$$\Delta K + \Delta U = 0$$

- We conclude any change in the kinetic energy ΔK must be accompanied by an equal but opposite change in the potential energy ΔU.

Chapter 8.
Potential energy in one dimension.

- Per definition, the change in potential energy is related to the work done by the force:

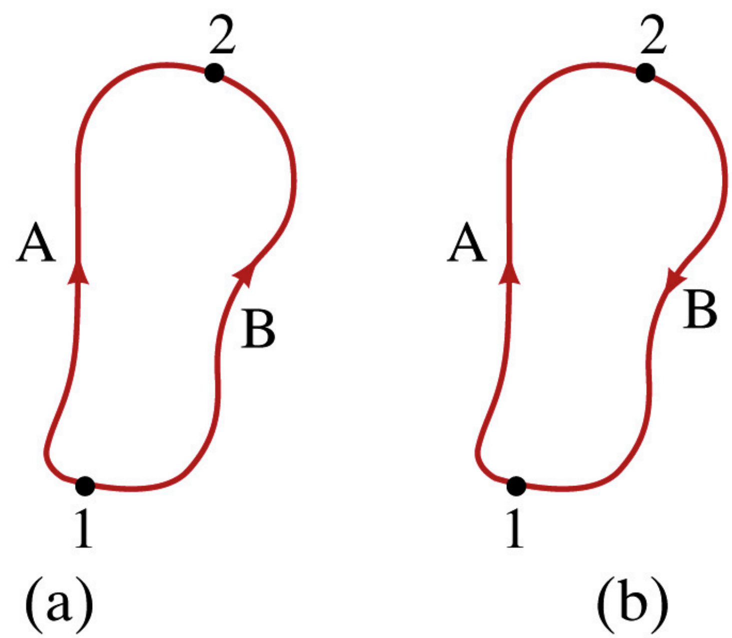
$$\Delta U = -W = -\int_{x_0}^{x} F(x')dx'$$

- The potential energy at x can thus be related to the potential energy at a point x_0:

$$U(x) = U(x_0) + \Delta U = U(x_0) - \int_{x_0}^{x} F(x')dx'$$

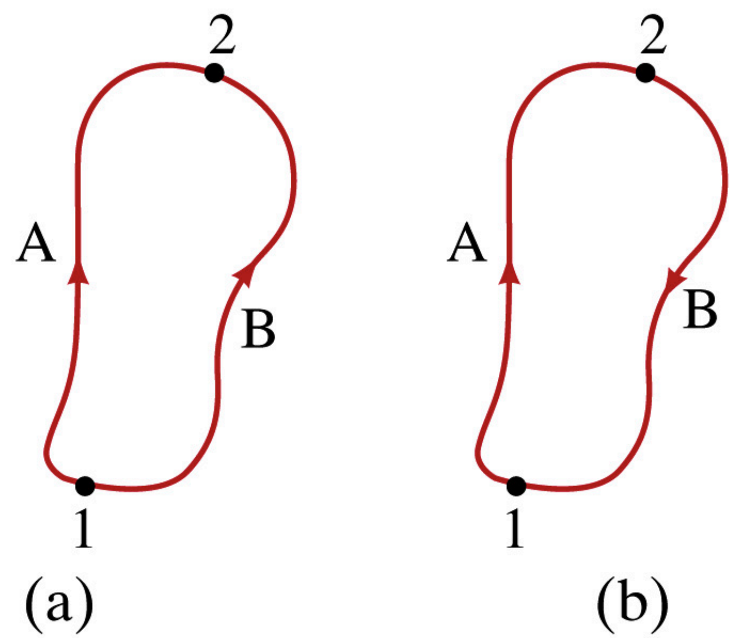
Chapter 8.
Potential energy and path dependence.

- The difference between the potential energy at (2) and at (1) depends on the work done by the force F along the path between (1) and (2).
- But there are many roads that lead from (1) to (2). The potential at (2) is uniquely defined only if the work done is path independent.
- This is not true for all forces. For example, the work done by the friction force is always negative, and the work is path dependent.



Chapter 8.
Conservative and non-conservative forces.

- If the work is independent of the path, the work around a closed path will be equal to 0 J.
- A force for which the work is independent of the path is called a **conservative force**. Examples: spring force, gravitational force.
- A force for which the work depends on the path is called a **non-conservative force**. Examples: friction force, drag force.



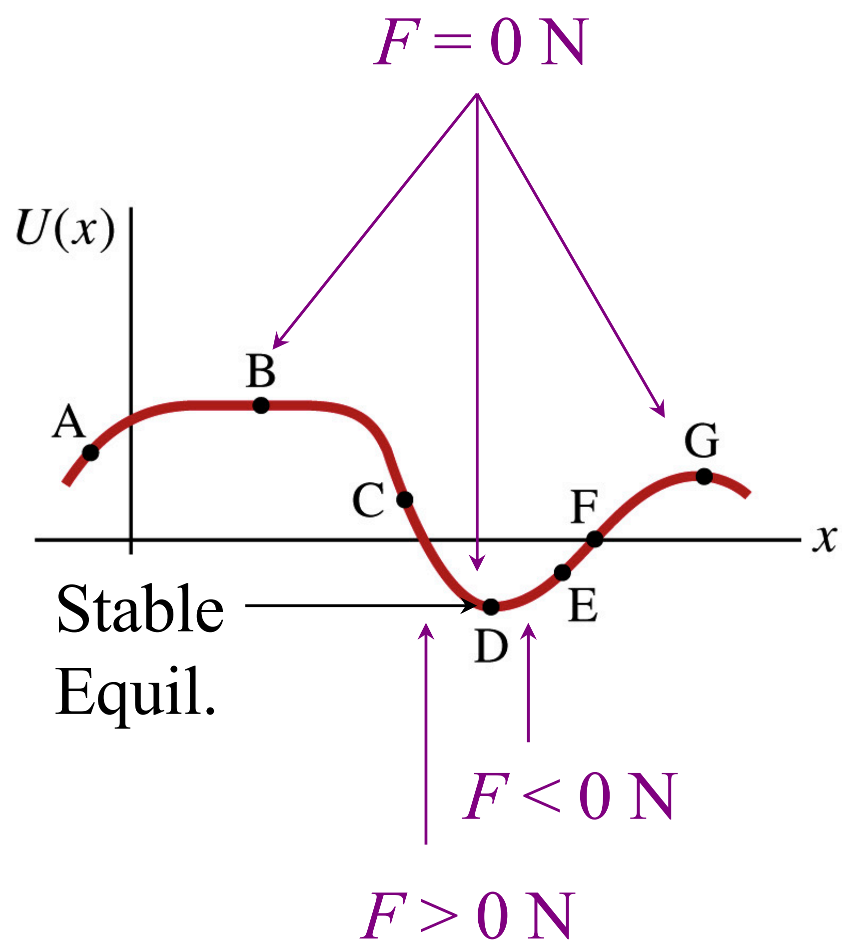
Chapter 8.
Potential energy in one dimension.

- The potential energy is directly related to the force acting on the object.
  - If we know the force, we can calculate the change ΔU:

$$\Delta U = -W = -\int_{x_0}^{x} F(x')dx'$$

  - If we know the change dU, we can calculate the force F:

$$F(x) = -\frac{dU}{dx}$$



Chapter 8.
Conservation of energy and dissipative forces.

- When dissipative forces, such as friction forces, are present, mechanical energy is no longer conserved.

- For example, a friction force will reduce the speed of a moving object, thereby dissipating its kinetic energy.

- The amount of energy dissipated by these non-conservative forces can be calculated if we know the magnitude and direction of these forces along the path followed by the object we are studying:

$$\Delta K + \Delta U = W_{NC}$$

where W_{NC} is the work done by the non-conservative forces.

Chapter 8.
Potential energy: the spring force.

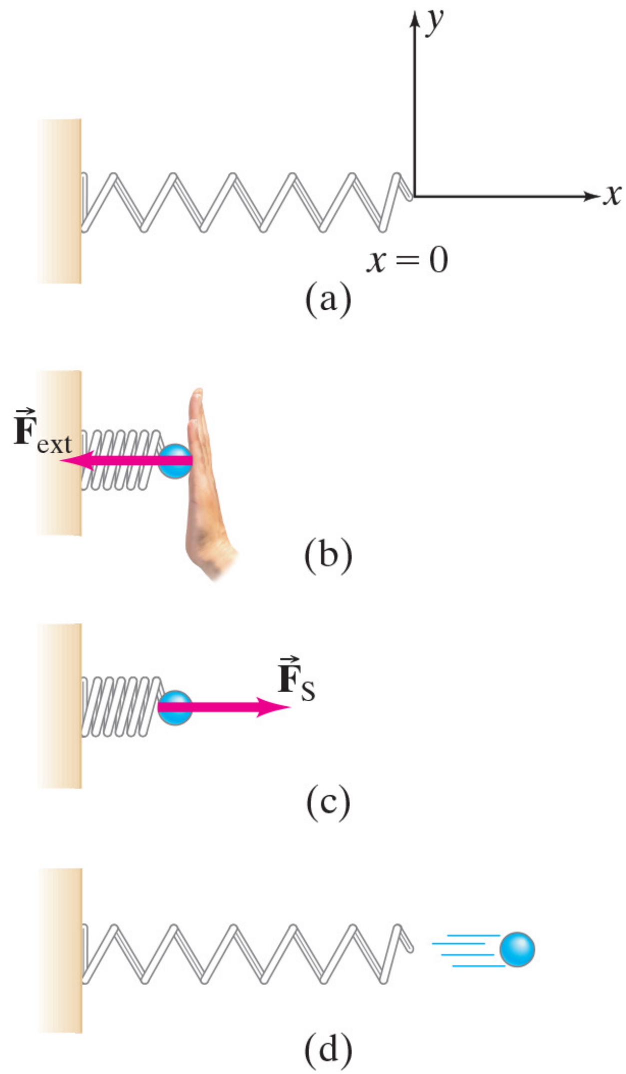
- The force exerted by a spring is given by Hooke's Law:

$$F(x) = -kx$$

- Consider the potential energy at x, taking the rest position as our reference point (where $U = 0$ J):

$$U(x) = -\int_{x_0}^{x} -kx'dx' = \frac{1}{2}kx^2$$

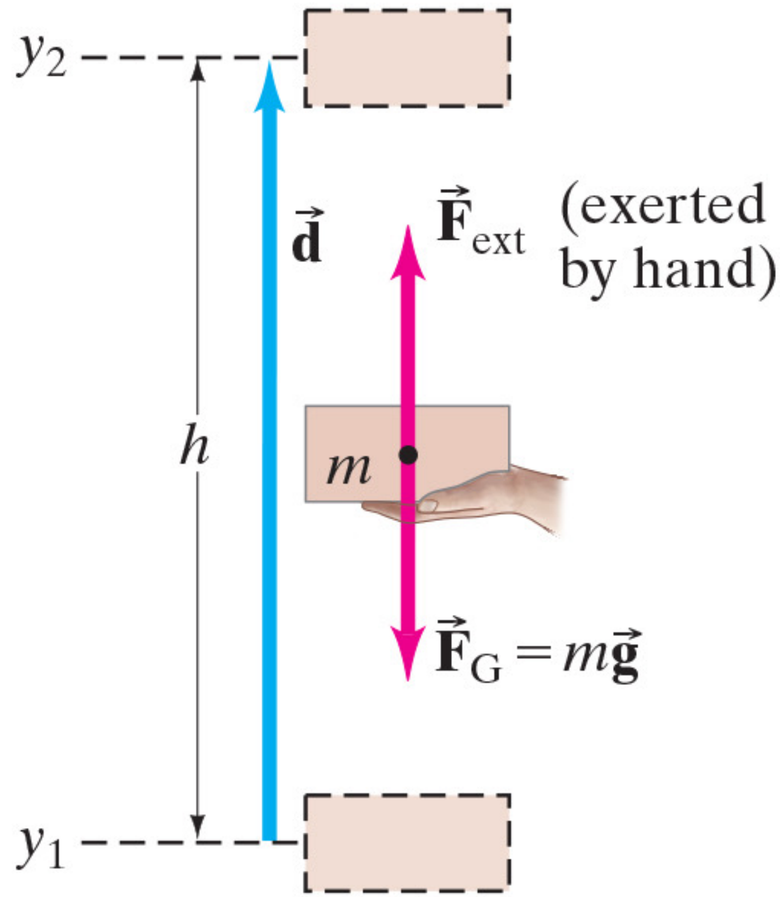
- We see that the potential energy has a minimum when $x = 0$ m.



Chapter 8.
Potential energy: the gravitational force.

- Consider what happens when we increase the height of an object of mass *m* by *h*.
- The work done by the gravitational force is negative (force and displacement point in opposite directions) and equal in magnitude to *mgh*.
- The potential energy due to the gravitational force at position 2 is thus equal to

$$U(y_2) = U(y_1) - W = U(y_1) + mgh$$



Usually, the potential energy on the surface is taken to be 0 J.

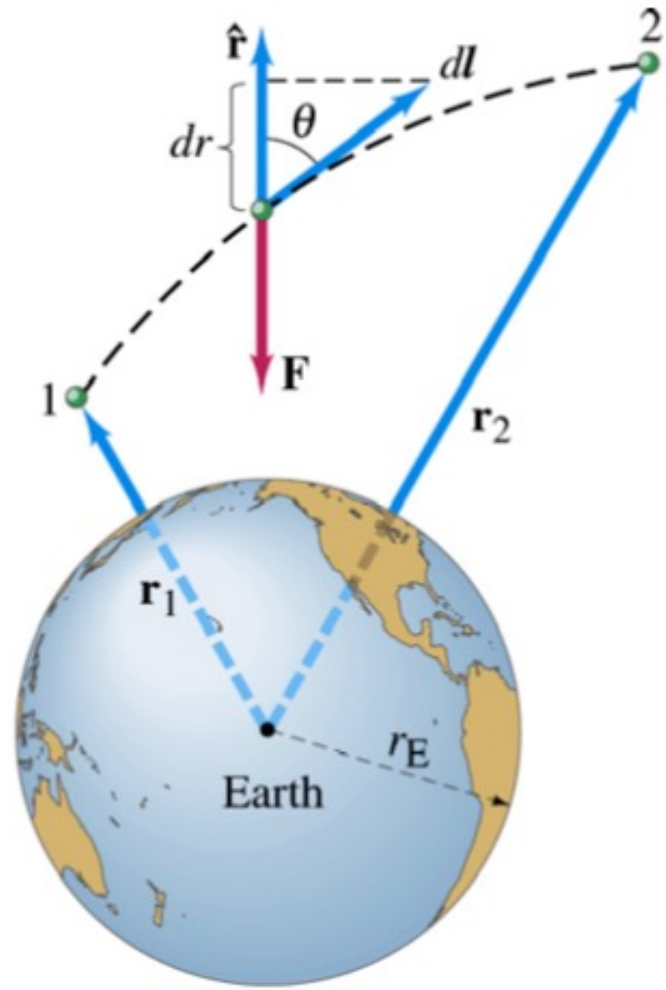
Chapter 8.
The gravitational potential energy.

- The potential energy difference between position (1) and (2) is thus equal to

$$U_2 - U_1 = -W = GmM_E\left(-\frac{1}{r_2} + \frac{1}{r_1}\right)$$

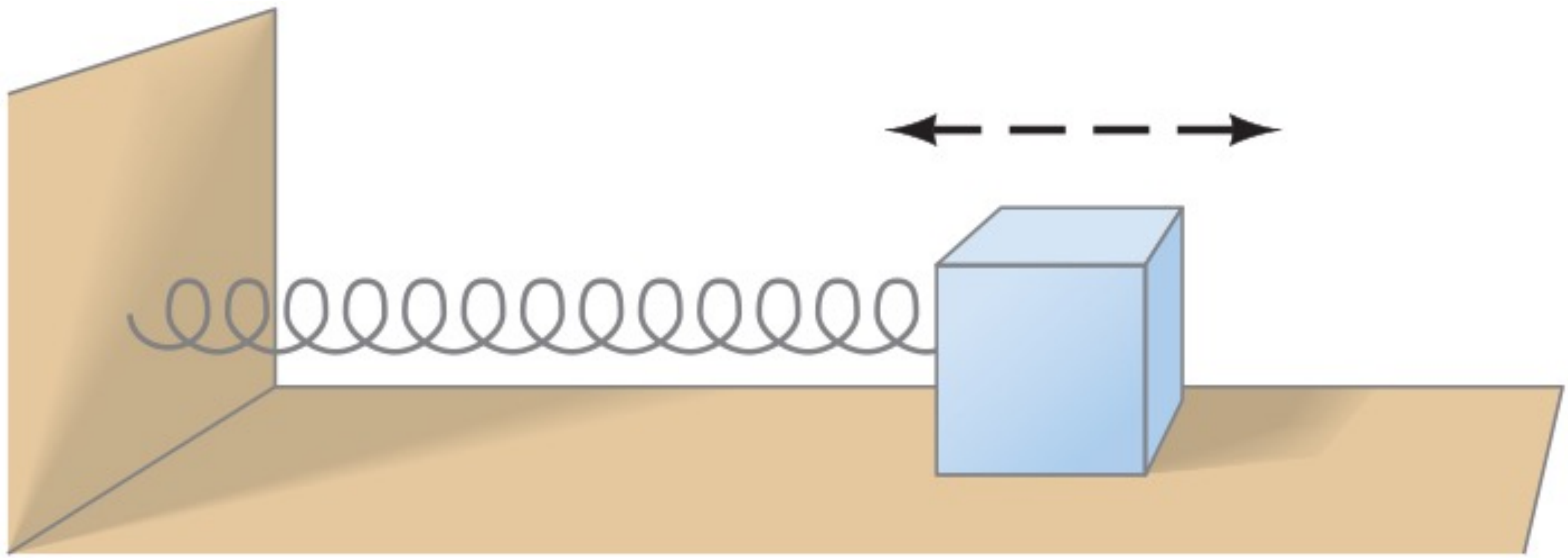
- It is common to choose the gravitational potential energy to be equal to 0 J at infinity. With the choice, the potential energy at (1) will be equal to

$$U_1 = -G\frac{mM_E}{r_1}$$



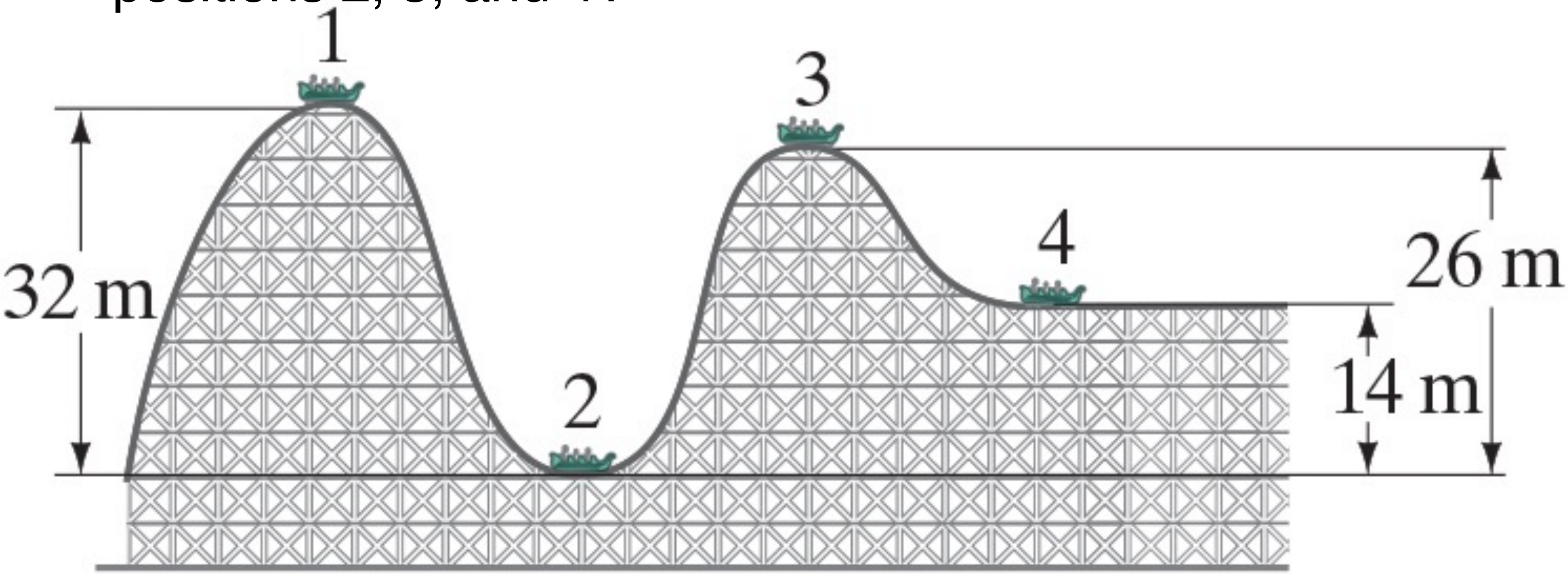
Chapter 8.
Example problem.

A block of mass M is attached to a spring with spring constant k. When the spring is compressed a distance d, the block is released. It moves to a distance p beyond the equilibrium position before returning back. What is the coefficient of kinetic friction?



Chapter 8.
Example problem.

If released from rest at position 1, what is the speed at positions 2, 3, and 4?



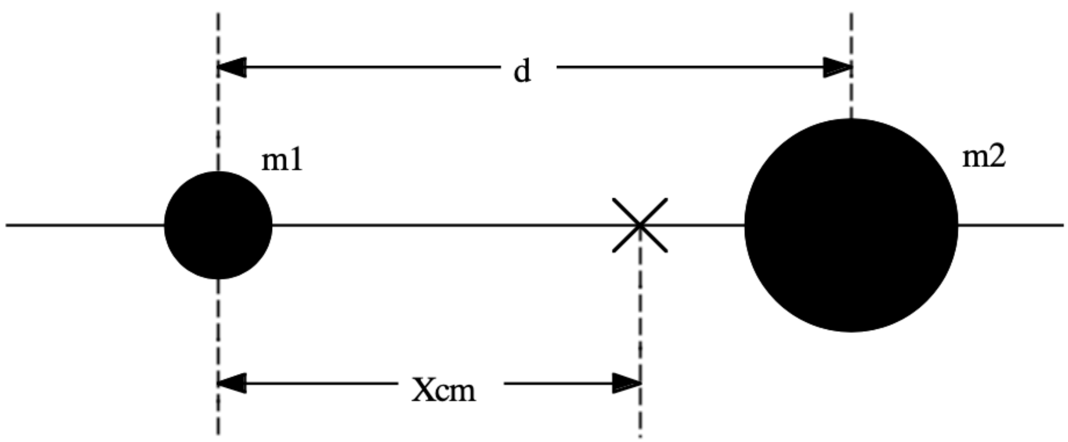
Chapter 9

Linear momentum

Chapter 9.
The center of mass.

We will start with considering one-dimensional objects. For an object consisting out of two point masses, the center of mass is defined as

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{1}{M} \sum_i m_i x_i$$

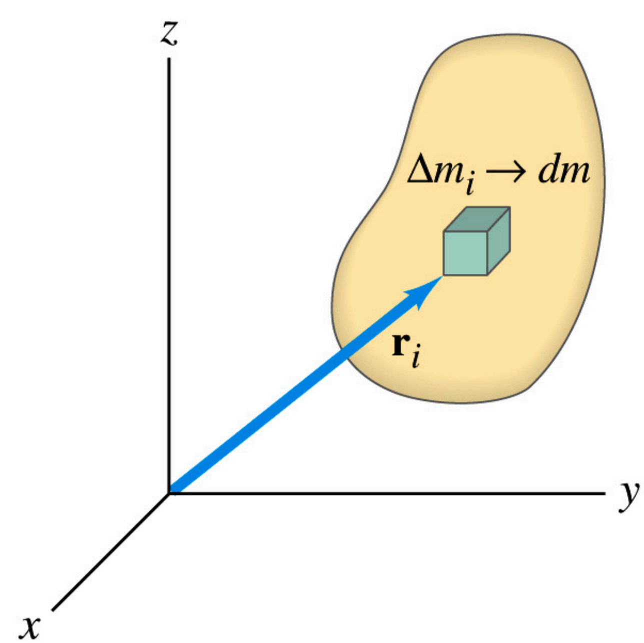


Chapter 9.
The center of mass.

- In two or three dimensions the calculation of the center of mass is very similar, except that we need to use vectors.

- If we are not dealing with discreet point masses, we need to replace the sum with an integral:

$$\vec{r}_{cm} = \frac{1}{M} \int_V \vec{r}\, dm$$



Chapter 9.
Motion of the center of mass.

- To examine the motion of the center of mass we start with its position and then determine its velocity and acceleration:

$$M\vec{r}_{cm} = \sum_i m_i \vec{r}_i$$

$$M\vec{v}_{cm} = \sum_i m_i \vec{v}_i$$

$$M\vec{a}_{cm} = \sum_i m_i \vec{a}_i$$

Chapter 9.
Motion of the center of mass.

- The expression for $M\vec{a}_{cm}$ can be rewritten in terms of the forces on the individual components:

$$M\vec{a}_{cm} = \frac{d}{dt}(M\vec{v}_{cm}) = \frac{d\vec{P}_{cm}}{dt} = \sum_i m_i\vec{a}_i = \sum_i \vec{F}_i = \vec{F}_{net,ext}$$

- We conclude that the motion of the center of mass is only determined by the external forces. Forces exerted by one part of the system on other parts of the system are called internal forces. According to Newton's third law, the sum of all internal forces cancel out (for each interaction there are two forces acting on two parts: they are equal in magnitude but pointing in an opposite direction and cancel if we take the vector sum of all internal forces).

Chapter 9.
Linear momentum.

- Now consider the special case where there are no external forces acting on the system. In this case, $Ma_{cm,x} = Ma_{cm,y} = Ma_{cm,z} = 0$ N.

- In this case, $Mv_{cm,x}$, $Mv_{cm,y}$, and $Mv_{cm,z}$ are constant.

- The product of the mass and velocity of an object is called the linear momentum $\vec{P}$ of that object.

- In the case of an extended object, we find the total linear momentum by adding the linear momenta of all of its components:

$$\vec{P}_{tot} = \sum_i \vec{p}_i = \sum_i m_i \vec{v}_i = M\vec{v}_{cm}$$

Chapter 9.
Systems with variable mass.

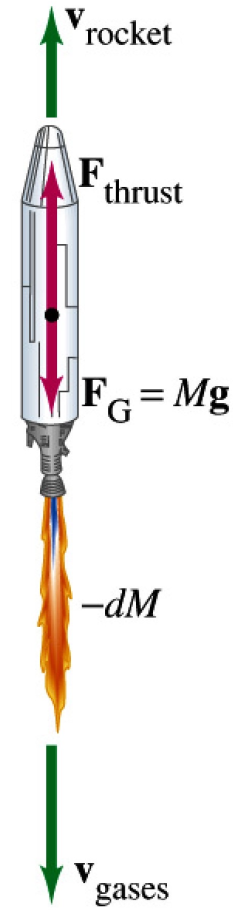
- The **first Rocket equation**:

$$RU_0 = Ma_{rocket}$$

where
  - R is the rate of fuel consumption.
  - U_0 = -u where u is the (positive) velocity of the exhaust gasses relative to the rocket.

- The **second rocket equation:**

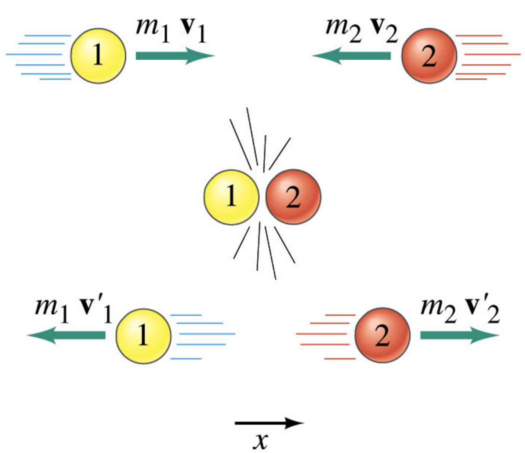
$$v_f = v_i + u \ln \frac{M_i}{M_f}$$

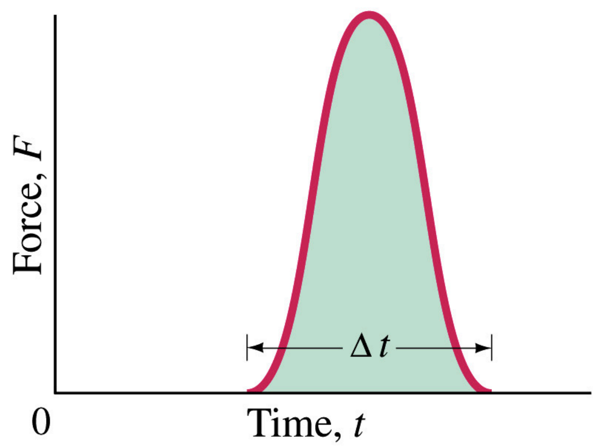


Chapter 9.
The collisions force.

- During a collision, a strong force is exerted on the colliding objects for a short period of time.
- The **collision force** is usually much stronger than any external force.
- The result of the collision force is a change in the linear momentum of the colliding objects.
- The change in the momentum of one of the objects is equal to

$$\vec{p}_f - \vec{p}_i = \int_{\vec{p}_i}^{\vec{p}_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)dt$$

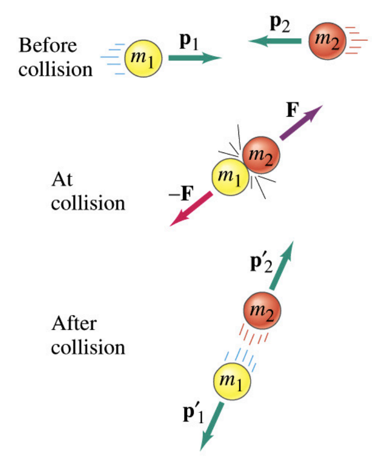


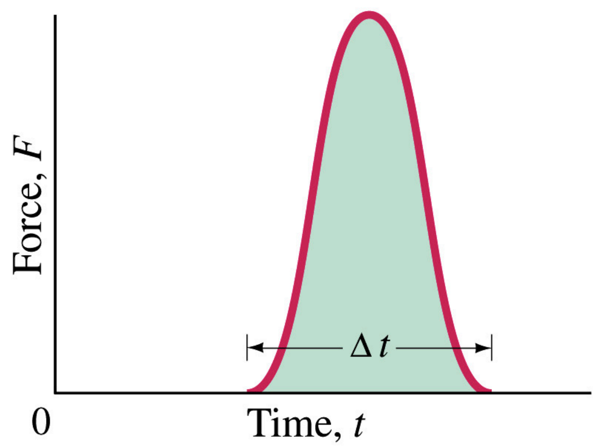


Chapter 9.
Collisions.

- If we consider both colliding object, then the collision force becomes an internal force and the total linear momentum of the system must be conserved if there are no external forces acting on the system.

- Collisions are usually divided into two groups:
  - **Elastic collisions**: kinetic energy is conserved.
  - **Inelastic collisions**: kinetic energy is NOT conserved.





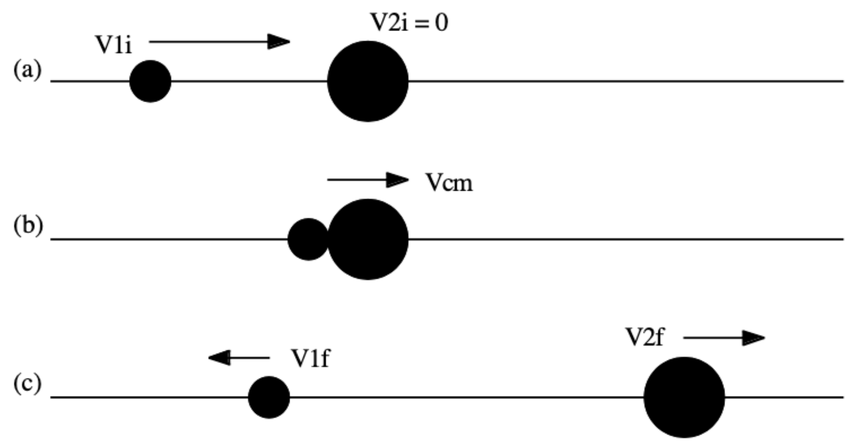
Chapter 9.
Elastic collisions in one dimension.

- The final state of an elastic collision in one dimension is completely defined by the initial conditions.
- We can always use a reference frame where one of the objects is at rest.
- The initial velocity of mass m_1 is v_{1i}. Mass m_2 is at rest.
- The final velocity of mass m_1 is:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

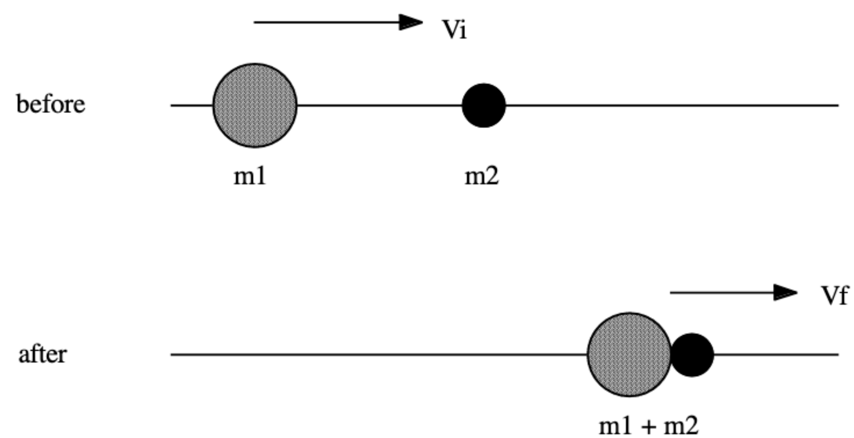
- The final velocity of mass m_2 is:

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$



Chapter 9.
Inelastic collisions in one dimension.

- In inelastic collisions, kinetic energy is not conserved.
- A special type of inelastic collisions are the **completely inelastic collisions**, where the two objects stick together after the collision.
- Conservation of linear momentum in a completely inelastic collision requires that $m_1 v_i = (m_1 + m_2) v_f$.
- We can thus determine the final velocity and kinetic energy of the system:

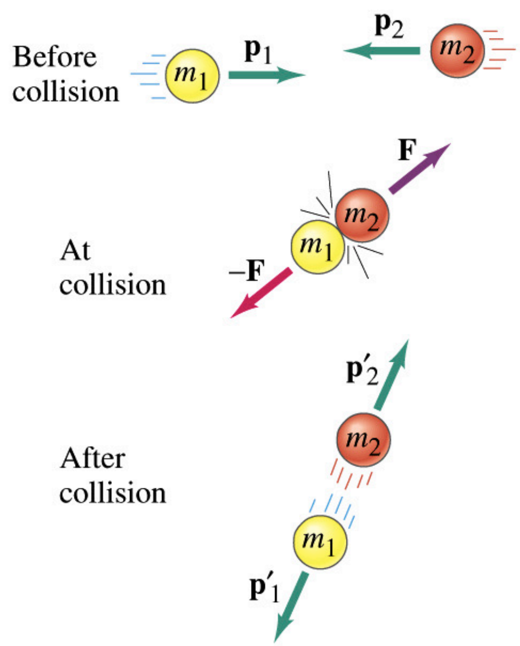


$$v_f = \frac{m_1}{m_1 + m_2} v_i$$

$$K_f = \frac{m_1}{m_1 + m_2} K_i$$

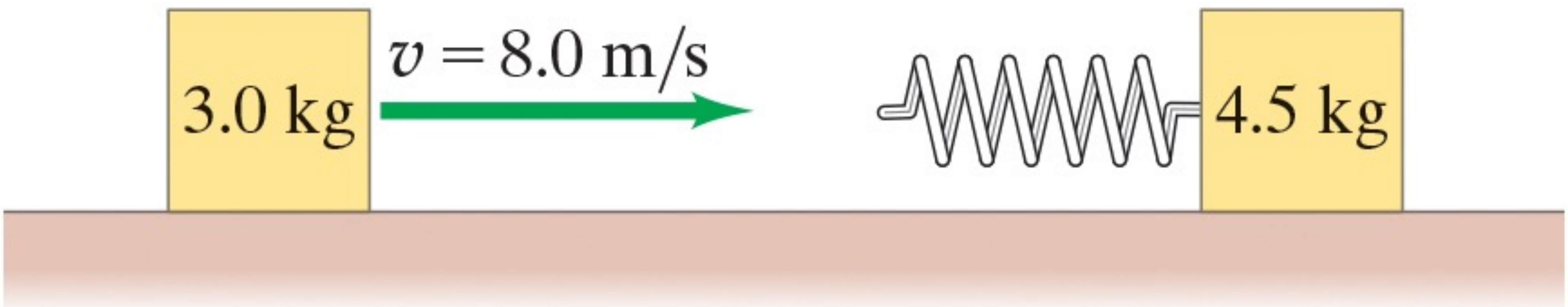
Chapter 9.
Collisions in two or three dimensions.

- Collisions in two or three dimensions are approached in the same way as collisions in one dimension.
- The x, y, and z components of the linear momentum must be conserved if there are no external forces acting on the system.
- The collisions can be elastic or inelastic.
- Even for two-dimensional elastic collisions, the final state is not fully defined by the initial state.



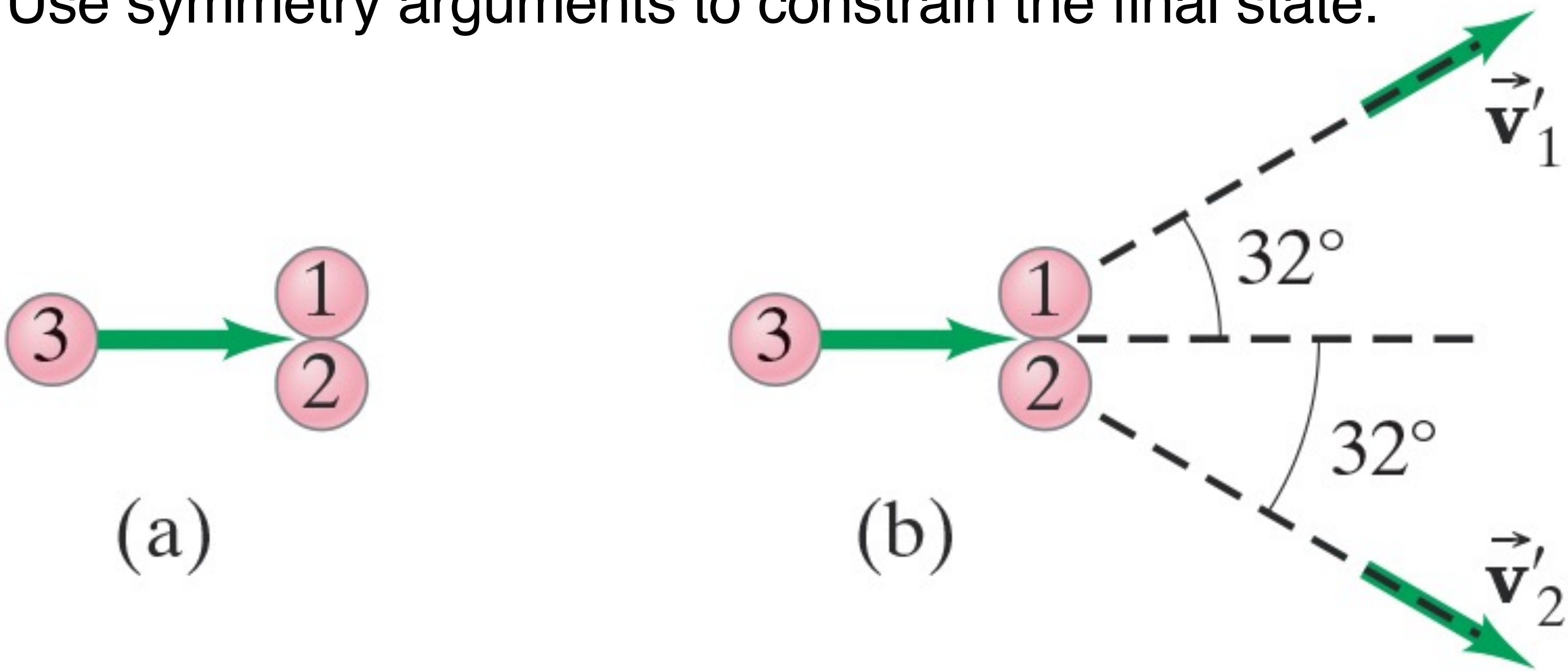
Chapter 9.
Example problem.

a) What will be the maximum compression?
b) What will be the final velocities of the blocks?
c) Is the collision elastic?



Chapter 9.
Example problem.

Use symmetry arguments to constrain the final state.



Final remarks.

- The hardest part of each problem is recognizing the approach to take. Different approaches may lead to the same answer, but can differ greatly in difficulty.

- A suggestion:
  - Look at the end of chapter problems. There is only a limited number of types of question one can ask.
  - But ……. Since the questions are grouped by section, you know already what approach to use based on the section to which the problems are assigned.
  - Some students benefit from copying the questions, cutting them out, writing the chapter/section numbers on the back, mixing them up, and then reading through them and determining what approach you would take if you would see that question on the exam (compare it with the focus of the section to which the problem was assigned).

Final remarks.

- You will only need your pen, a pencil, and an eraser. Being awake might also help!
- You will find a formula sheet attached to the exam. This sheet has been distributed via email before the exam and will also be available from the Physics 121 website,
- The exam will start at 8 am and end at 9.20 am. If you show up late you will just have less time to finish.

Good luck preparing for the exam.