Physics 121.
Lecture 23.

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Physics 121.
Lecture 23.

- Course information
- Topics to be discussed today:
  - Temperature and pressure (review).
  - Microscopic view of linear expansion,
  - The universal gas law.

Physics 121.
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- Homework set # 9 is now available and is due on Saturday morning, April 19, at 8.30 am.
- Midterm Exam # 3 will take place on Tuesday April 28 between 8.00 am and 9.20 am in Hubbell. The material to be covered is the material contained in Chapters 10, 11, 12, and 14.
- I will review the material covered on Exam # 3 on Thursday.
- There will be extra office hours on Sunday and Monday. Details will be announced via email and during the review of the material covered on Exam # 3 on Thursday April 23.

Temperature (a quick review). Measuring temperature.

- In order to measure temperature we must:
  - Agree on a standard reference point to which we assign a certain temperature.
  - Agree on a unit.
  - Agree on a standard thermometer against which all other thermometers can be calibrated.
- The unit of temperature will be the Kelvin (K).
- The standard reference point is the triple point of water (T = 273.16 K).

http://www.fluke.fr/common/prod_pages/pages/hart/products/tpw.htm

Thermodynamic variables (a quick review). The constant volume thermometer.

- The difference in height between the levels (h) is measured.
- The difference in height can be used to determine the pressure of the gas:

$$p = p_0 + \rho g h$$

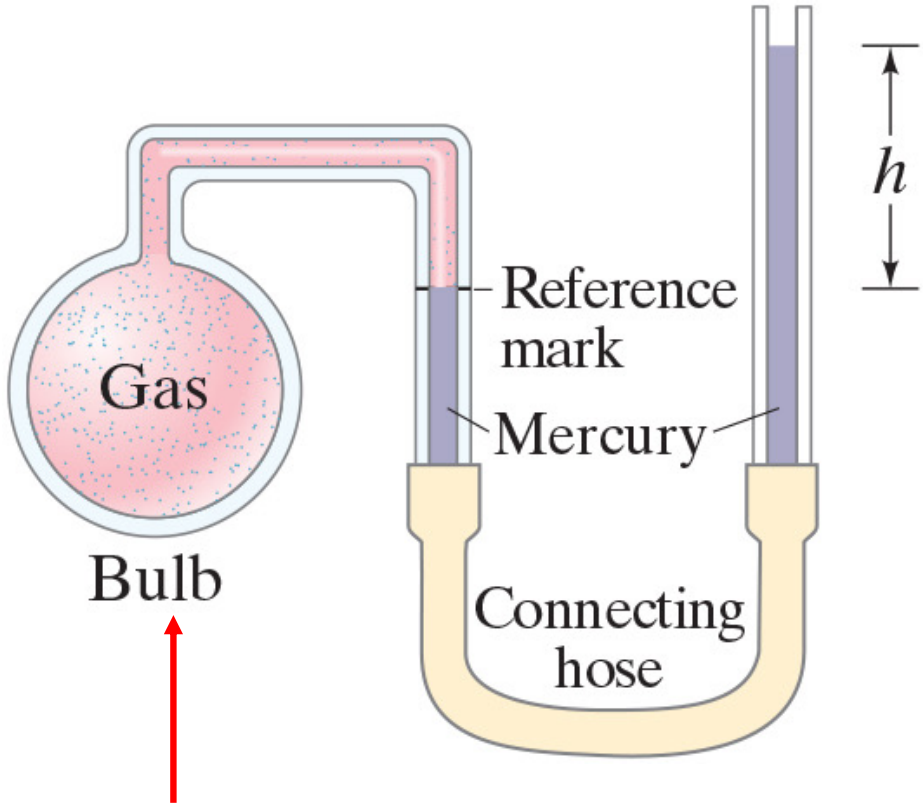
where p_0 is the atmospheric pressure and ρ is the density of the mercury.

- The temperature of the body is defined in terms of the pressure p:

$$T = Cp$$

where

$$C = 273.16/p_3$$

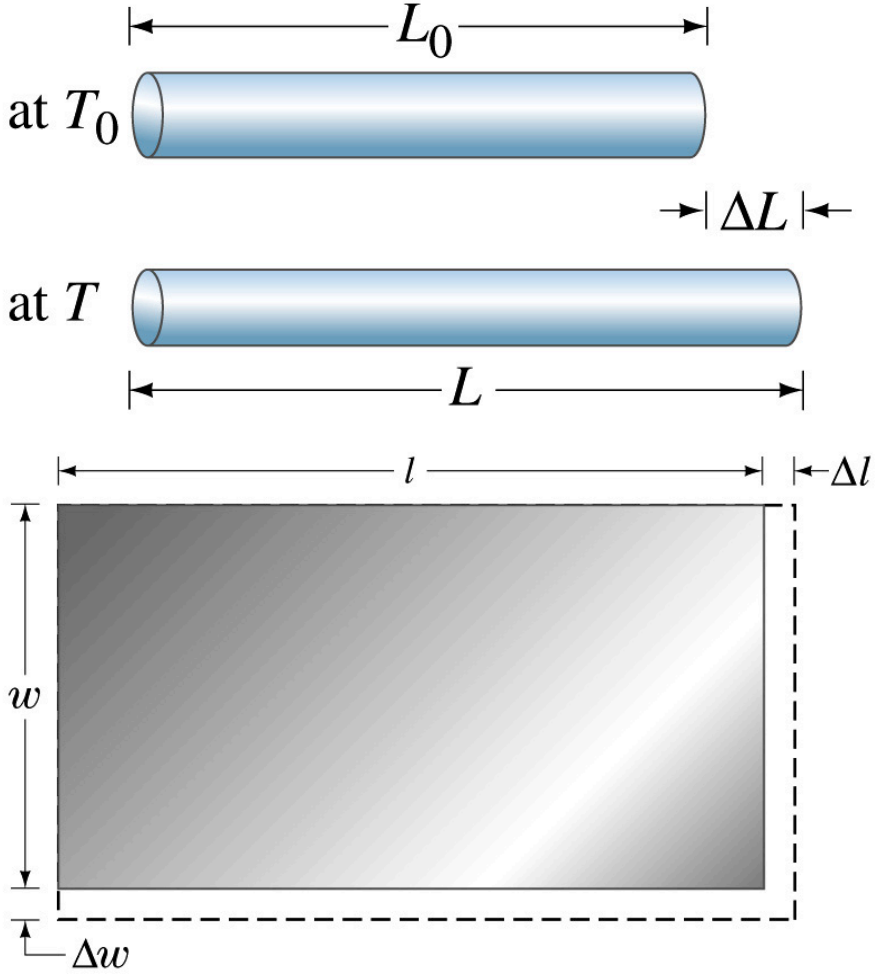


Temperature (a quick review). Linear expansion.

- When the temperature of a material increases, its length will increase:

$$\Delta L = \alpha L \Delta T$$

- The coefficient α is the coefficient of linear expansion. Typical values are 0.5 x 10^{-6} K^{-1} and 10 x 10^{-6} K^{-1} at room temperature.
- Note: a solid will expand in every direction!!!!!



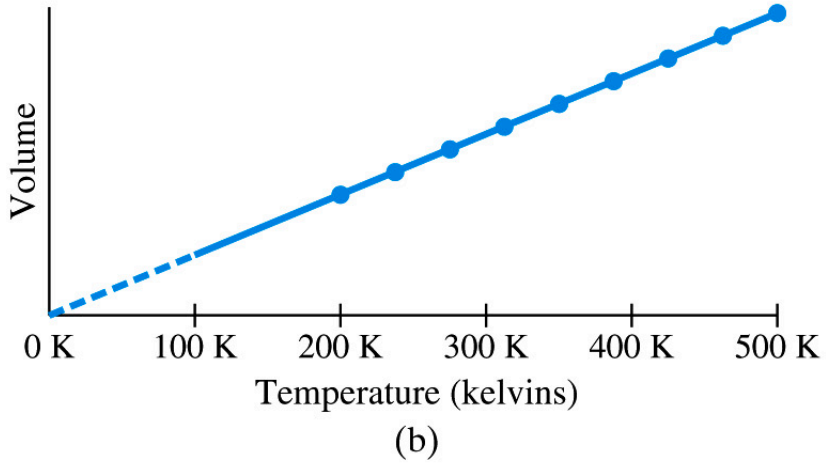
Temperature (a quick review). Volume expansion.

- When we deal with liquids we usually talk about volume expansion:

$$\Delta V = \beta V \Delta T$$

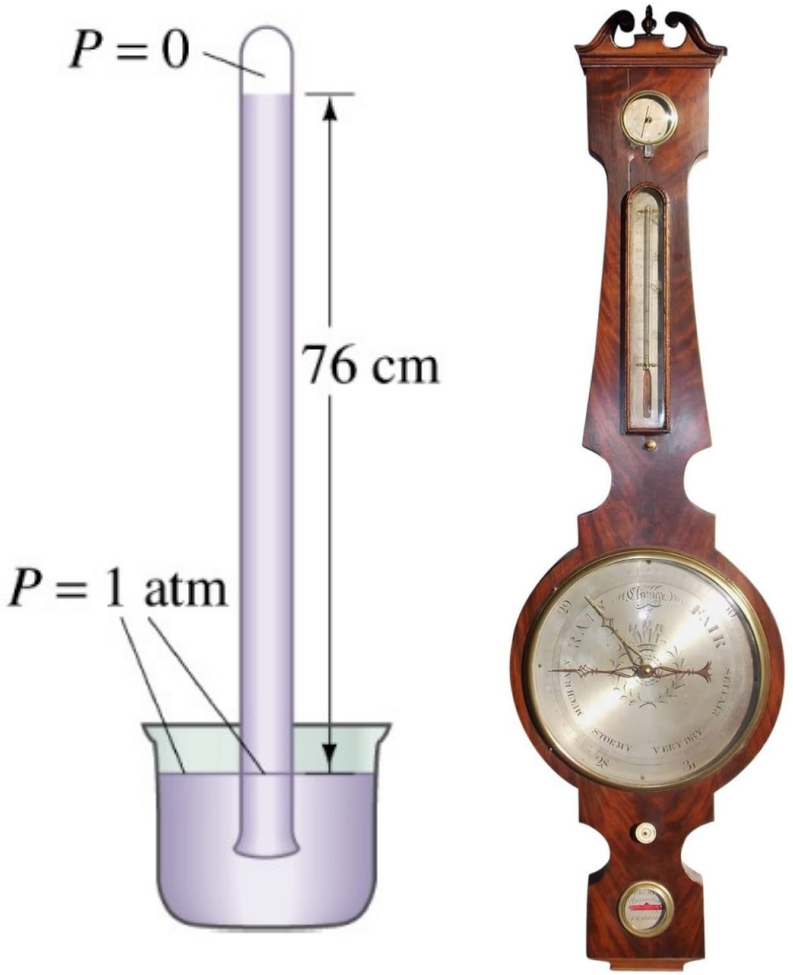
- The coefficient β is the volume expansion coefficient.

- The coefficient of volume expansion β is related to the coefficient of linear expansion α : $\beta = 3\alpha$.



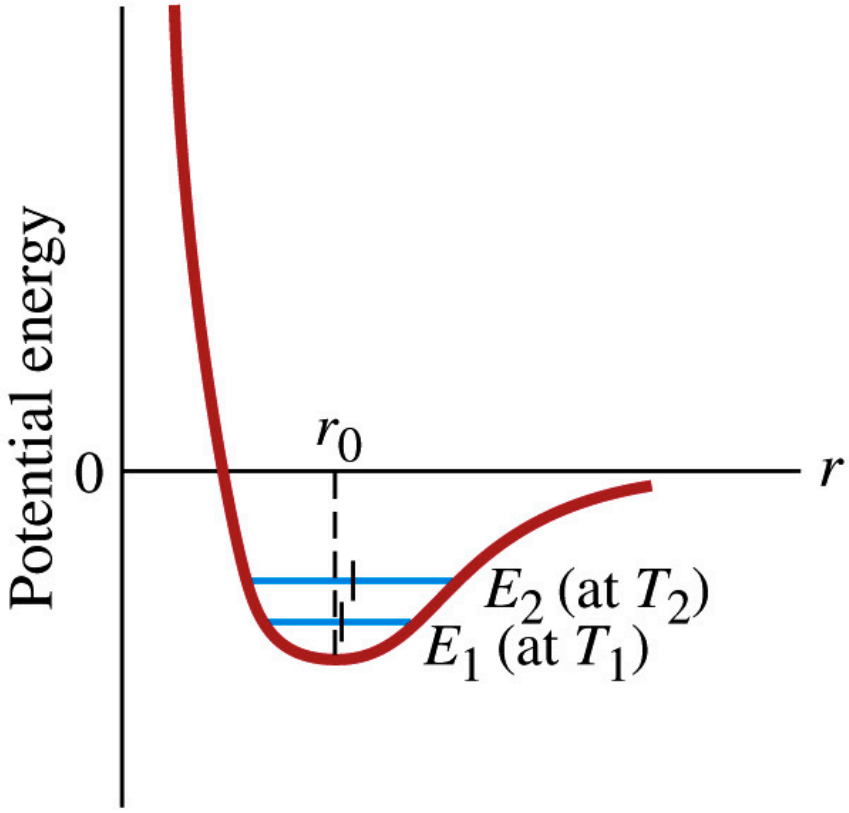
Thermodynamic variables (a quick review). Pressure.

- Pressure is an important thermodynamic variable.
- Pressure is defined as the force per unit area.
- The SI unit is pressure is the Pascal: 1 Pa = 1 N/m^2. Another common unit is the atm (atmospheric pressure) which is the pressure exerted by the atmosphere on us (1 atm = 1.013 x 10^5 N/m^2).
- A pressure of 1 atm will push a mercury column up by 76 cm.



Volume expansion.
A microscopic view.

- The atoms in a solid are held together in a three-dimensional periodic lattice by spring-like interaction forces. The potential energy for a pair of neighboring atoms depends on their separation r, and has a minimum at $r = r_0$. The distance r_0 is the lattice spacing of a solid when the temperature approaches zero. The potential energy curve rises more steeply when the atoms are pushed together ($r < r_0$) than when they are pulled apart ($r > r_0$). The average separation distance at a temperature above the absolute zero will therefore be larger than r_0.
- **A solid with a symmetric potential energy curve would not expand.**

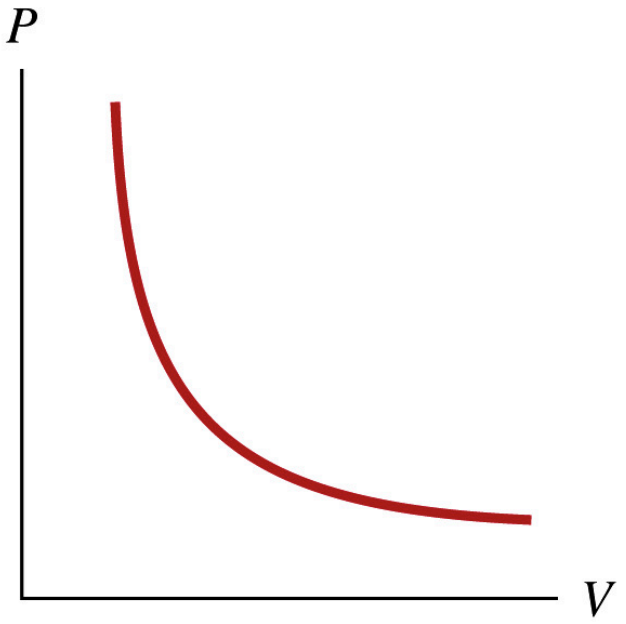


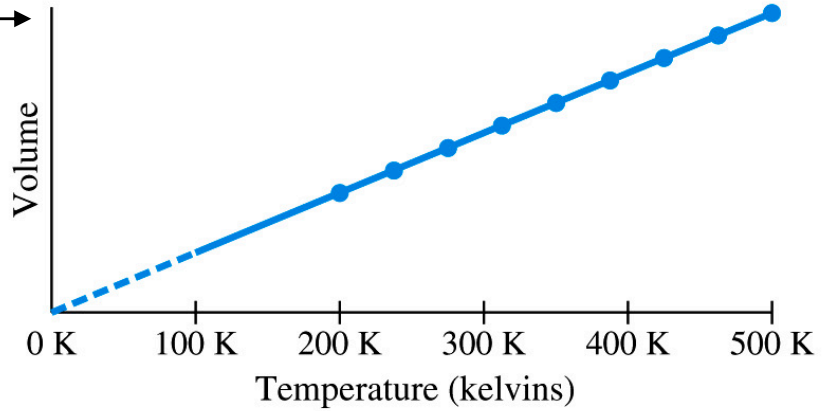
The equation of state of a gas.

- Thermal expansion of a gas is more complicated then thermal expansion of solids or liquids.
- The volume taken up by a gas is usually equal to the volume that is available.
- The volume expansion theory we just discussed applies only to a gas if its pressure is kept constant.
- In order to specify state of a gas, we need to specify its temperature, its volume, and its pressure. The relation between these variables and the mass of the gas is called **the equation of state**.

The equation of state of a gas.

- The equation of state of a gas was initially obtained on the basis of observations.
  - Boyle's Law (1627 - 1691): ***PV*** = **constant** for gases maintained at constant temperature.
  - Charle's Law (1746 - 1823): ***V/T*** = **constant** for gases maintained at constant pressure.
  - Gay-Lussac's Law (1778 - 1850): ***P/T*** = **constant** for gases maintained at constant volume





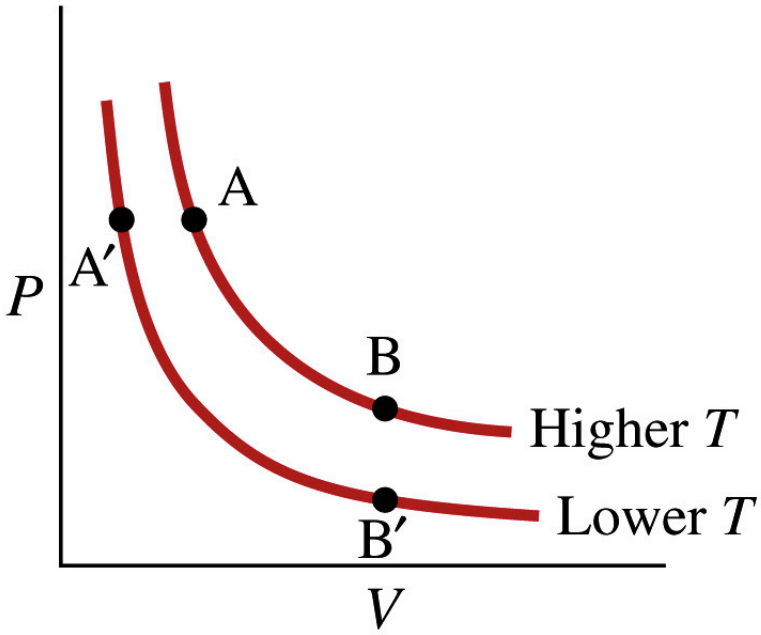
The equation of state of a gas.

- Combining the various gas laws, we can obtain a single more general relation between pressure, temperature, and volume:

$$pV \propto T$$

- Another observation that needs to be included is the dependence on the amount of gas: if pressure and temperature are kept constant, the volume is proportional to the mass m:

$$pV \propto mT$$



The equation of state of a gas.

- The equation of state of a gas can be written as

$$pV = nRT$$

where

- p = pressure (in Pa).
- V = volume (in m^3).
- n = number of moles of gas (1 mole = 6.02 x 10^{23} molecules or atoms). Note the number of molecules in a mole is also known as Avogadro's number N_A.
- R = the universal gas constant (R = 8.315 J/(mol K).
- T = temperature (in K).

- Note: the equation of state is the equation of state of an ideal gas. Gases at very high pressure and/or close to the freezing point show deviations from the ideal gas law.

The equation of state of a gas. Example problem.

- A cylinder contains oxygen at 20°C and a pressure of 15 atm at a volume of 12 l. The temperature is raised to 35°C , and the volume is reduced to 8.5 l. What is the final pressure of the gas?

- Since the amount of gas does not change, we can rewrite the ideal gas law in the following way: pV/T = constant. Since we know the initial state, we can determine the missing information about the final state:

$$p_i V_i / T_i = p_f V_f / T_f$$

The equation of state of a gas. Example problem.

- The final pressure of the gas is equal to

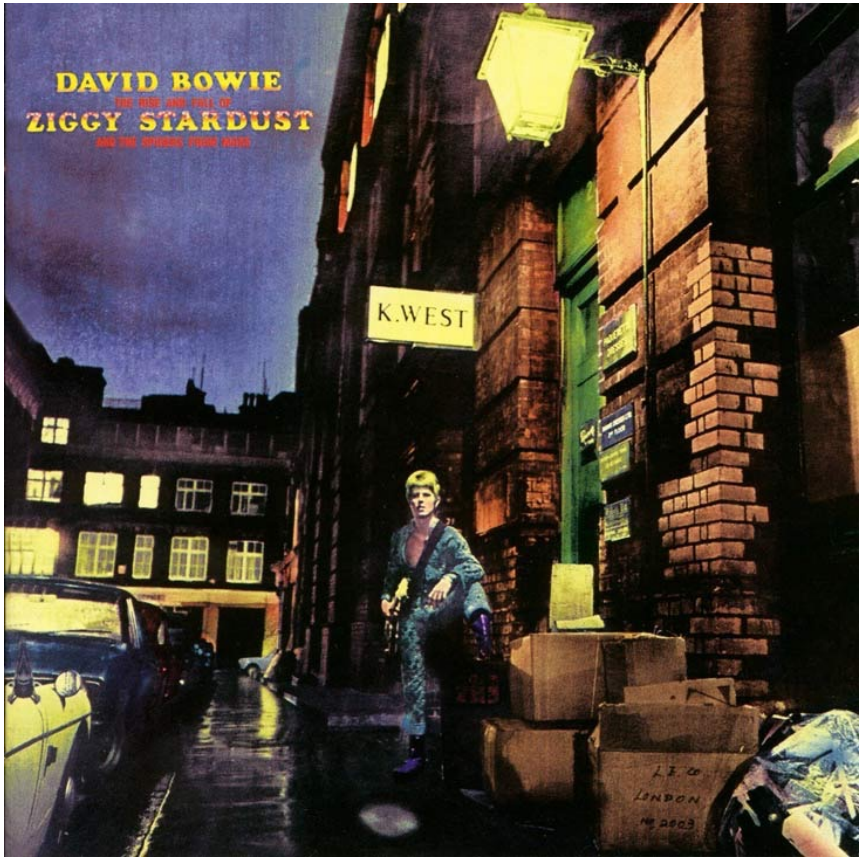
$$p_f = p_i (V_i/V_f) / (T_i/T_f)$$

- Note:
  - This relation will preserve the units of pressure.
  - The units of volume cancel, and we can keep the volume in units of liters. Note: for whatever we unit we choose, zero volume in SI units, correspond to zero volume in all other units.
  - The units of temperature must be in Kelvin. The temperature ratio $T_i/T_f = (273.15 + 20)/(273.15 + 35) = 0.951$ when T is expressed in Kelvin. The ratio would be 0.571 when T is expressed in Celsius.
- When we use the correct units, we find that $p_f = 22$ atm.

2 Minute 58 Second Intermission.

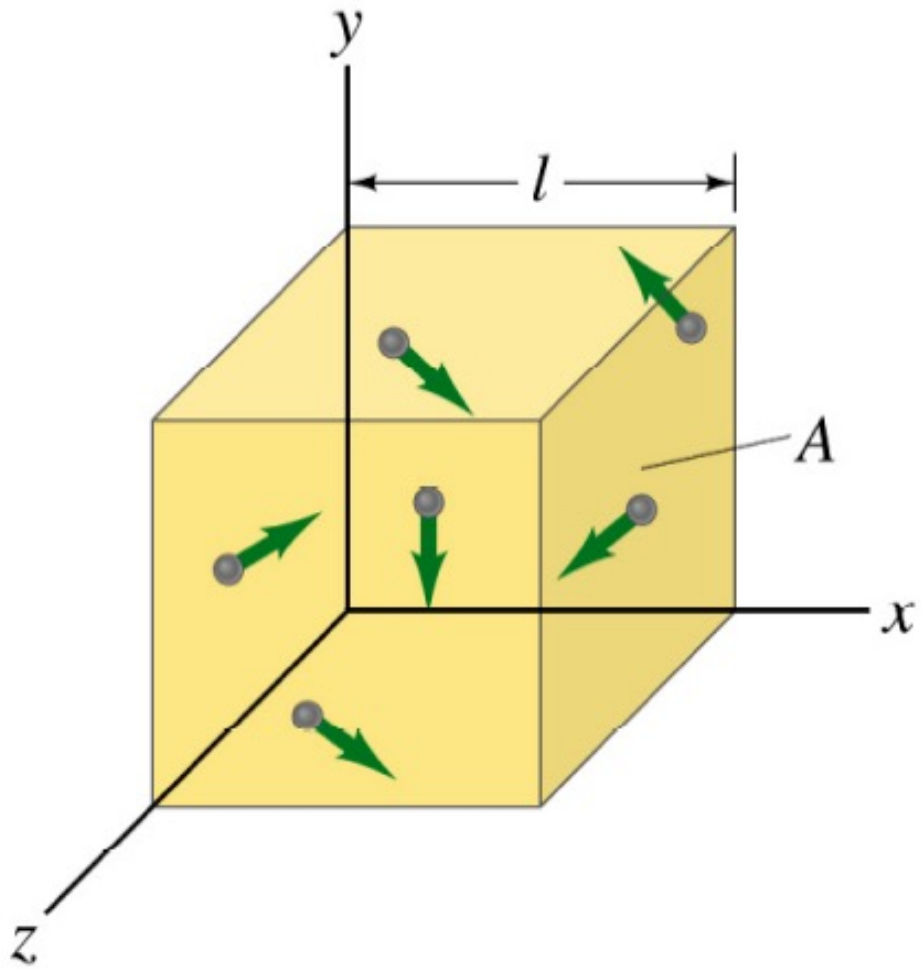
- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 2 minute 58 second intermission.

- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.



The molecular point of view of a gas.

- Consider a gas contained in a container.
- The molecules in the gas will continuously collide with the walls of the vessel.
- Each time a molecule collides with the wall, it will carry out an elastic collision.
- Since the linear momentum of the molecule is changed, the linear momentum of the wall will change too.
- Since force is equal to the change in linear momentum per unit time, the gas will exert a force on the walls.



The molecular point of view of a gas.

- Consider the collision of a single molecule with the left wall.
- In this collision, the linear momentum of the molecule changes by

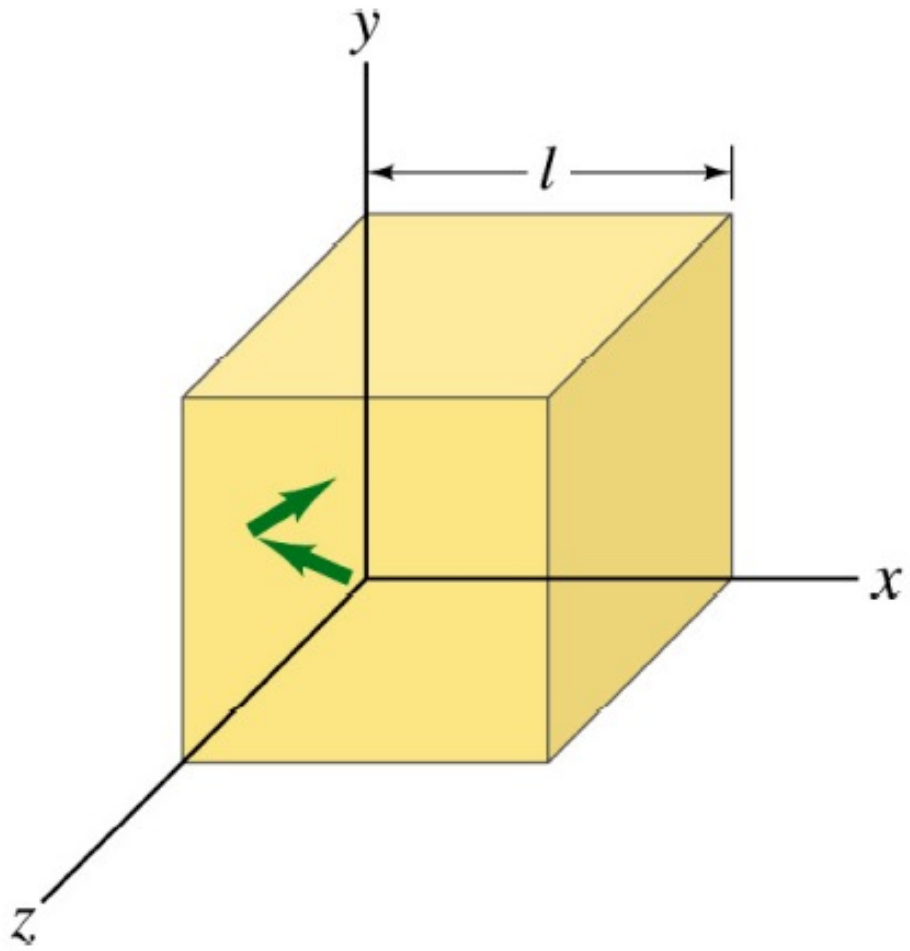
$$mv_x - (-mv_x) = 2mv_x$$

- The same molecule will collide with this wall again after a time

$$\Delta t = 2l/v_x$$

- The force that this single molecule exerts on the left wall is thus equal to

$\Delta p/\Delta t = (2mv_x)/(2l/v_x) = mv_x^2/l$



The molecular point of view of a gas.

- The force that this single molecule exerts on the left wall is thus equal to

$$F_{left} = \frac{mv_x^2}{l}$$

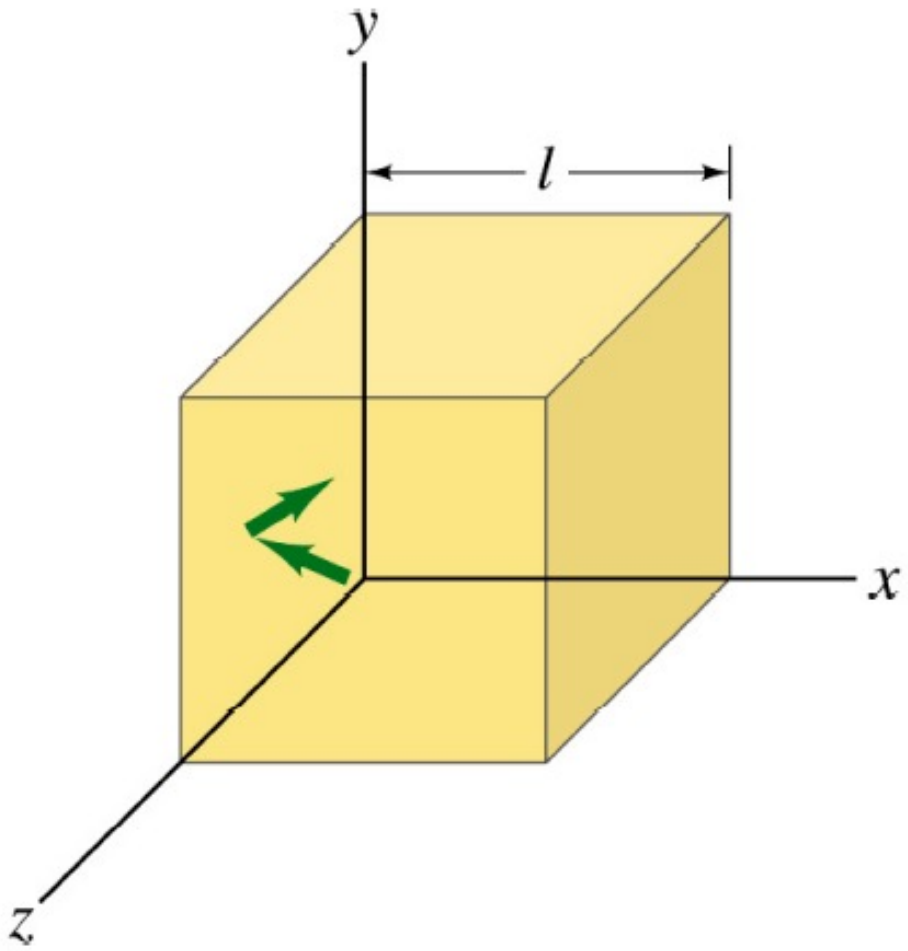
- If the pressure exerted on the left wall by this molecule is equal to

$$p_{left} = \frac{F_{left}}{A} = \frac{mv_x^2}{lA}$$

where A is the area of the left wall.

- The volume of the gas is equal to lA and we can thus rewrite the pressure on the left wall:

$$p_{left} = \frac{mv_x^2}{V}$$



The molecular point of view of a gas.

- The pressure that many molecules exerts on the left wall is equal to

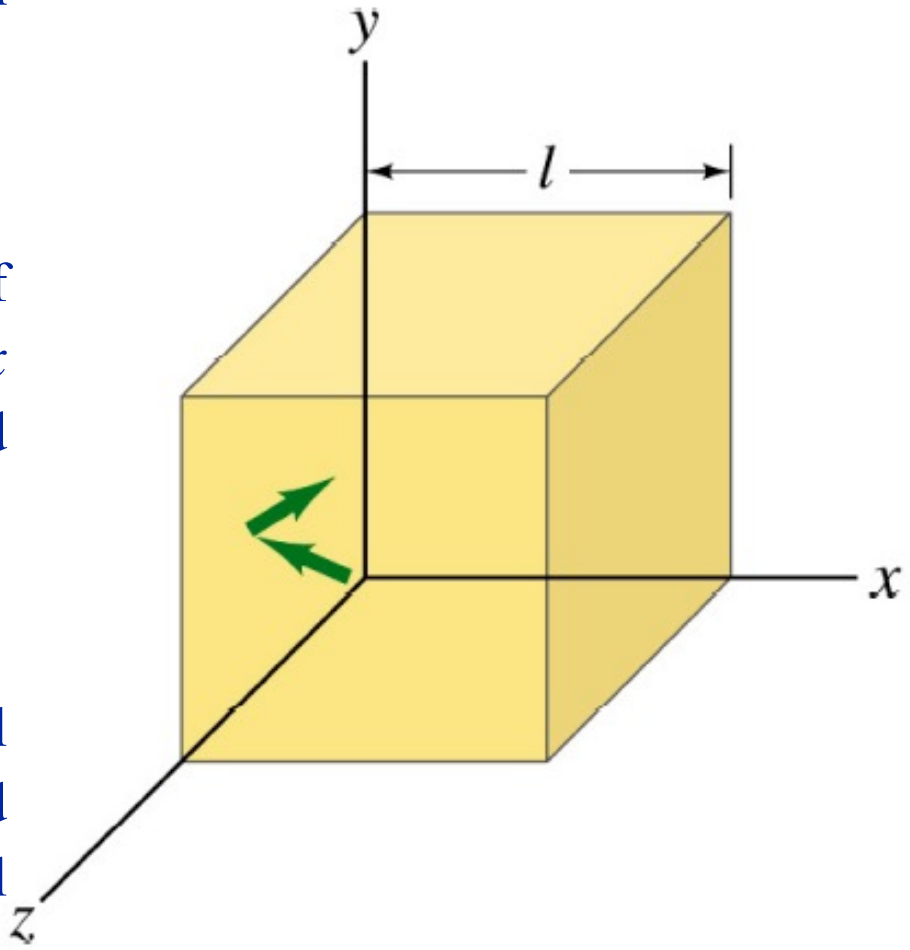
$$p_{left} = m(v_{1x}^2 + v_{2x}^2 + \cdots)/V$$

- This equation can be rewritten in terms of the average of the square of the x component of the molecular velocity and the number of molecules (N):

$$p_{left} = mN(v_x^2)_{average}/V$$

- Assuming that there is no preferential direction, the average square of the x, y, and z components of the molecular velocity will be the same:

$$(v_x^2)_{average} = (v_y^2)_{average} = (v_z^2)_{average}$$



The molecular point of view of a gas.

- The force on the left wall can be rewritten in terms of the average squared velocity

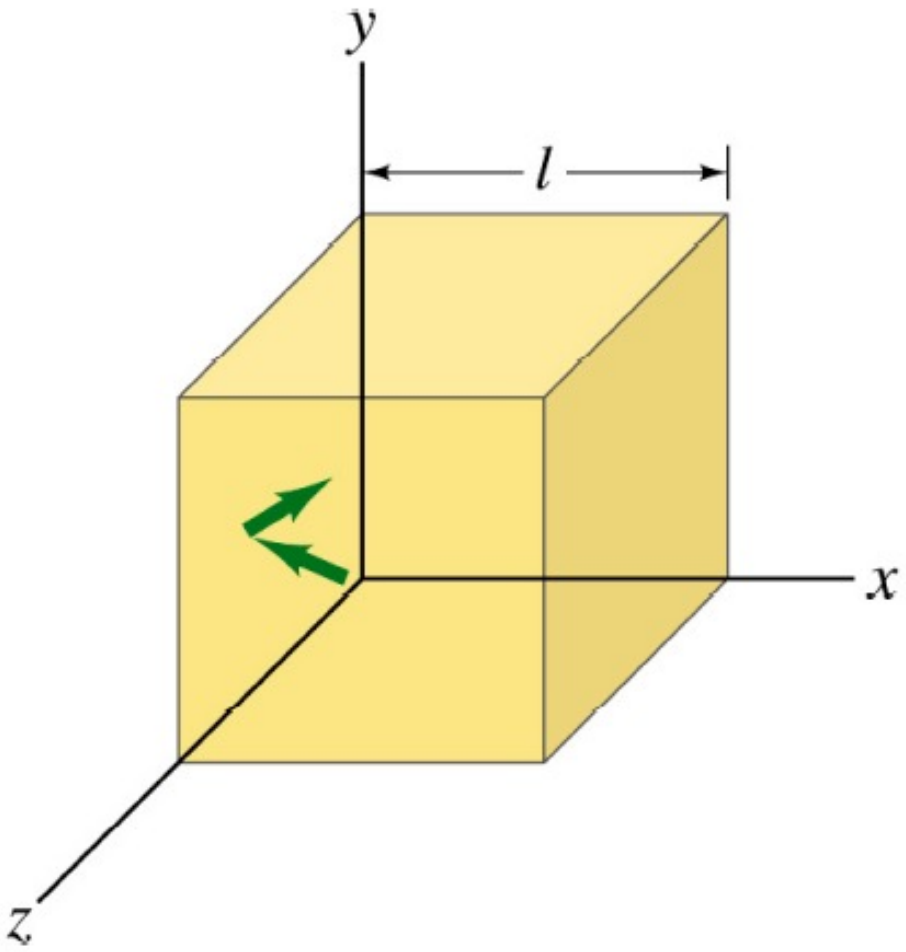
$$p_{left} = mN(v^2)_{average}/3V$$

- Assuming there is no preferential direction of motion of the molecules, the pressure on all walls will be the same and we thus conclude:

$$pV = mN(v^2)_{average}/3$$

- Compare this to the ideal gas law:

$$pV = NkT$$



The molecular point of view of a gas.

- Comparing

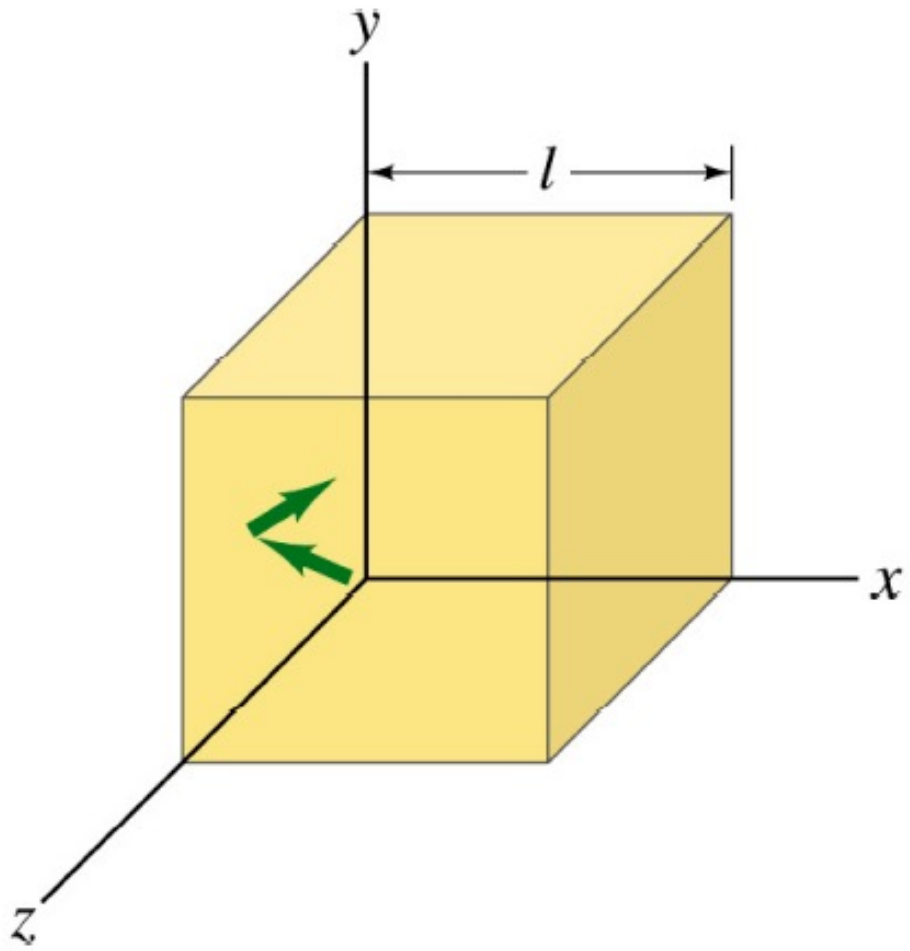
$$pV = mN(v^2)_{average}/3$$

and the ideal gas law

$$pV = NkT$$

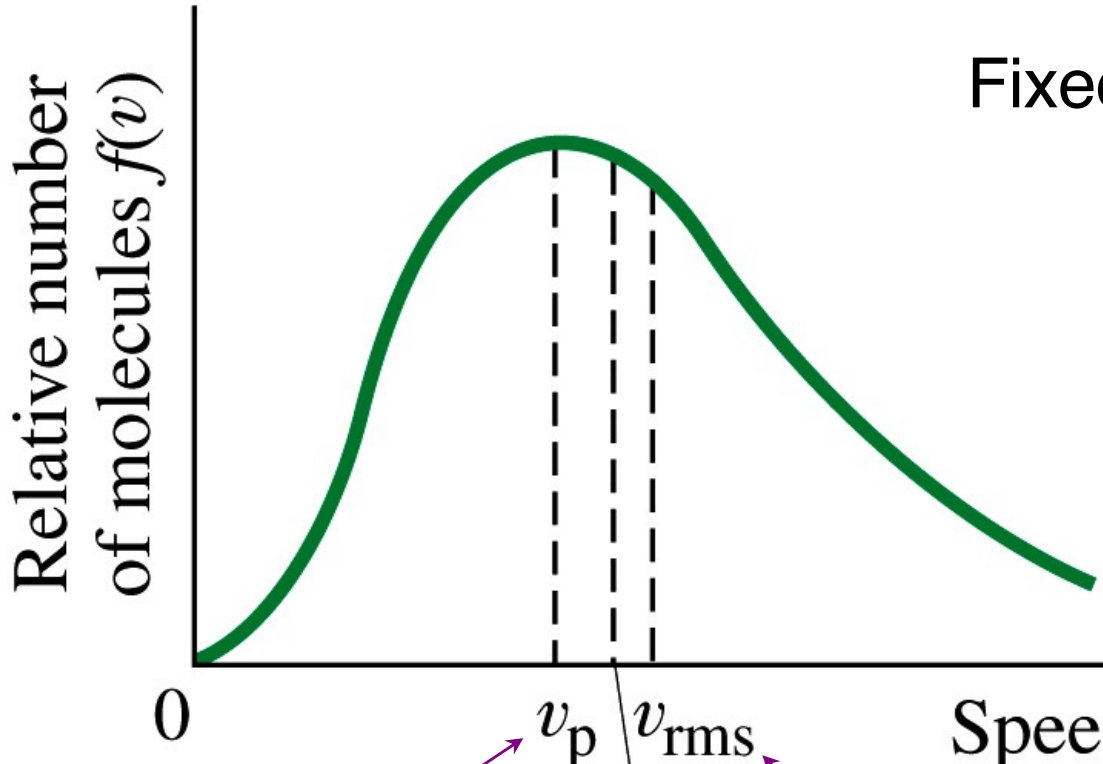
we find that

$$kT = \frac{mN(v^2)_{average}}{3} = \frac{2}{3}K_{average}$$



- We thus conclude that:
  - $K_{average}$ is proportional to the T.
  - When $T \rightarrow 0$ K $K_{average} \rightarrow 0$ J.

Distribution of molecular speeds. The Maxwell distribution.



Fixed temperature

Integral = *N*

Most probable v

Average of v2

Average v (50% below, 50% above)

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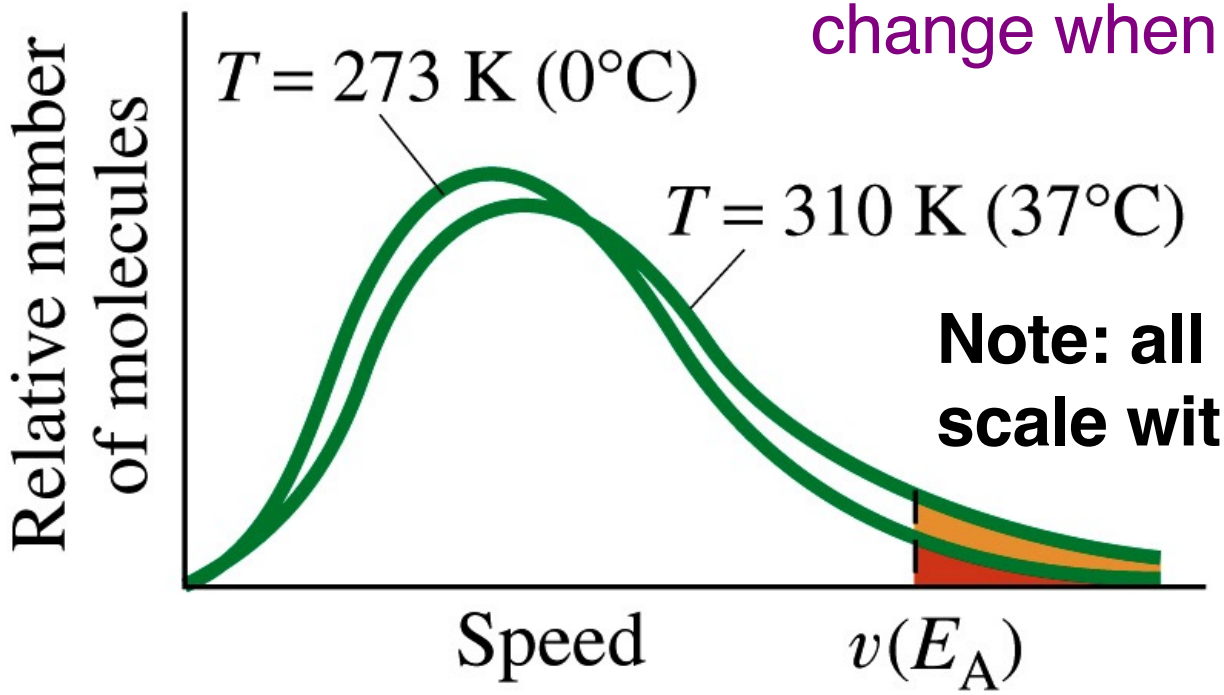
Distribution of molecular speeds. The Maxwell distribution.

Note: the integral does not change when T changes!

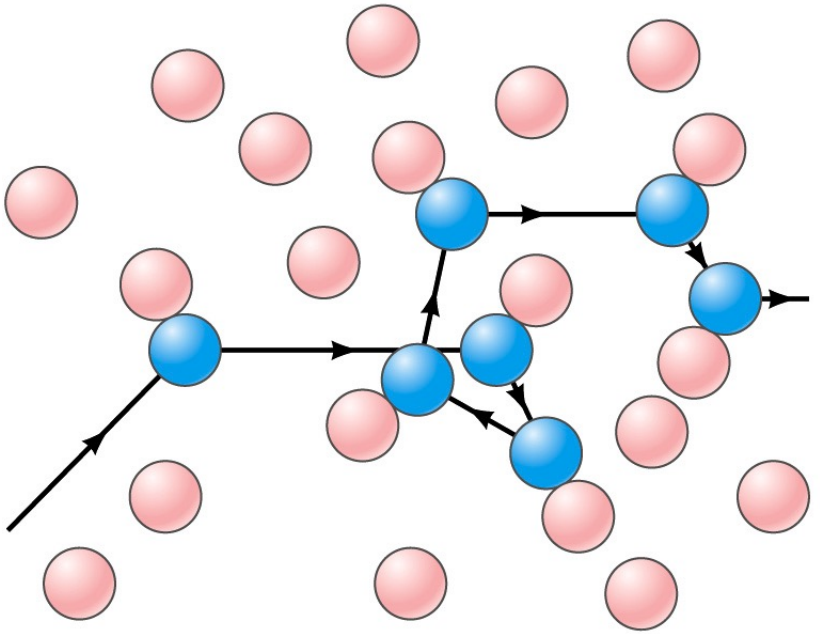


Note: all velocities scale with √T

Increasing T increases the fraction of molecules above a certain velocity.

Molecular speed and the mean-free path.

- The RMS velocities of individual gas molecules are large. For example, for hydrogen at room temperature, the RMS velocity is 1920 m/s.
- Despite the large RMS velocity, the average diffusion velocity is much smaller and is largely determined by the mean-free path of the molecules.
- We expect that the mean-free path is inversely proportional to the cross-sectional area of the molecules and inversely proportional to the density.



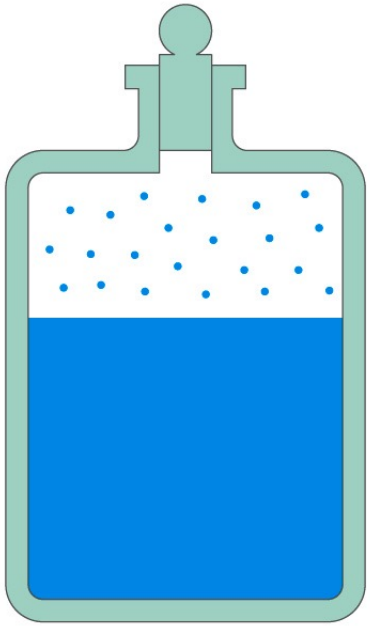
Typical values of the mean-free path are between 10^{-8} and 10^{-7} m

Molecular speed and the propagation of sound.

- Typical values for v_{rms}:
  - For H at 300 K, v_{rms} = 1920 m/s
  - For N at 300 K, v_{rms} = 517 m/s
- The speed of sound in these two gases is 1350 m/s for H and 350 m/s for N.
- Note: The speed of sound in a gas will always be less than v_{rms} since the sound propagates through the gas by disturbing the motion of the molecules. The disturbance is passed on from molecule to molecule by means of collisions; a sound wave can therefore never travel faster than the average speed of the molecules. Since v_{rms} increases with T, we expect v_{sound} to increase with T.

Molecular speed and evaporation.

- Evaporation: transformation from the liquid to the gas phase.
- A microscopic view of evaporation:
  - Molecules with high velocity moving close to the surface can overcome the strong attractive forces between the molecules and escape from the liquid (evaporation).
  - The average velocity of the molecules left behind in the liquid will be lowered.
  - Since the average velocity is proportional to the temperature, the temperature of the liquid is lowered when evaporation takes place.



Done for today!
Next: Using the Kinetic Theory of Gases.

Facing NGC 6946 Credit & Copyright: Robert Gendler