

Physics 121.
Lecture 21.



Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 1

1

Physics 121.
Lecture 21.

- Course Information
- Topics to be discussed today:
 - Simple Harmonic Motion (Review).
 - Simple Harmonic Motion: Example Systems.
 - Damped Harmonic Motion
 - Driven Harmonic Motion

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 2

2

Physics 121.
Lecture 21.

- Homework set # 8 is due on Saturday morning, April 18, at 8.30 am.
- Homework set # 9 will be available on Saturday morning at 8.30 am, and will be due on Saturday morning, April 25, at 8.30 am.
- Exam # 3 is scheduled for Tuesday April 28 between 8 am in 9.20 am in Hubbell.

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 3

3

And now for something completely different.
A preview of the ideal gas law.



Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 4

4

Sorry, fun is over!

The Flying Dutchman is brought to you by Mike, Joe,
Jimmy, and Tony!

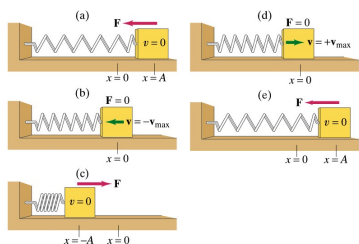
I would not be able to have so much fun teaching if
they did not provide fantastic demo support.

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 5

5

Harmonic motion (a quick review).
Motion that repeats itself at regular intervals.



Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 6

6

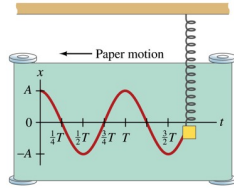
Simple Harmonic Motion (a quick review).

Phase Constant

Amplitude

$$x(t) = A \cos(\omega t + \phi)$$

Angular Frequency



Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 7

7

Simple Harmonic Motion (a quick review).

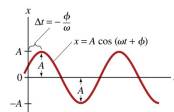
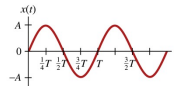
- Other variables frequently used to describe simple harmonic motion:

- The period T : the time required to complete one oscillation. The period T is equal to $2\pi/\omega$.

- The frequency of the oscillation is the number of oscillations carried out per second:

$$\nu = \frac{1}{T}$$

The unit of frequency is the Hertz (Hz). Per definition, $1 \text{ Hz} = 1 \text{ s}^{-1}$.



Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 8

8

Simple Harmonic Motion (a quick review). What forces are required?

- Using Newton's second law we can determine the force responsible for the harmonic motion:

$$F = ma = -m\omega^2 x$$

- The total mechanical energy of a system carrying out simple harmonic motion is constant.
- A good example of a force that produces simple harmonic motion is the spring force: $F = -kx$. The angular frequency depends on both the spring constant k and the mass m :

$$\omega = \sqrt{\frac{k}{m}}$$

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 9

9

Simple Harmonic Motion (SHM). The equation of motion.

- All examples of SHM were derived from the following equation of motion:

$$\frac{d^2 x}{dt^2} = -\omega^2 x$$

- The most general solution to the equation is

$$x(t) = A\cos(\omega t + \alpha) + B\sin(\omega t + \beta)$$

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 10

10

Simple Harmonic Motion (SHM). The equation of motion.

- If $A = B$

$$\begin{aligned} x(t) &= A\cos(\omega t + \alpha) + A\sin(\omega t + \beta) = \\ &= A \left(\sin\left(\frac{1}{2}\pi - \omega t - \alpha\right) + \sin(\omega t + \beta) \right) = \\ &= 2A\sin\left(\frac{1}{4}\pi + \frac{\beta}{2} - \frac{\alpha}{2}\right) \cos\left(\frac{1}{4}\pi - \omega t - \frac{\beta}{2} - \frac{\alpha}{2}\right) \end{aligned}$$

which is SHM.

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 11

11

Simple Harmonic Motion (SHM). The equation of motion.

- If $A = 0$:

$$x(t) = B\sin(\omega t + \beta): \text{SHM motion.}$$

- If $B = 0$:

$$x(t) = A\cos(\omega t + \alpha): \text{SHM motion}$$

- If $A \neq B \neq 0$: no SHM motion.

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 12

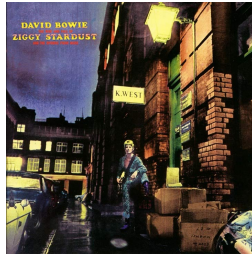
12



4 Minute 13 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 4 minute 13 second intermission.

- You can:
 - Stretch out.
 - Talk to your neighbors.
 - Ask me a quick question.
 - Enjoy the fantastic music.



Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 13

13

Damped Harmonic Motion.

- Consider what happens when in addition to the restoring force a damping force (such as the drag force) is acting on the system:

$$F = -kx - b \frac{dx}{dt}$$

- The equation of motion is now given by:

$$\frac{d^2x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \frac{k}{m}x = 0$$

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 14

14

Damped Harmonic Motion.

- The general solution of this equation of motion is

$$x(t) = Ae^{i\omega t}$$

- If we substitute this solution in the equation of motion we find

$$-\omega^2 Ae^{i\omega t} + i\omega \frac{b}{m} Ae^{i\omega t} + \frac{k}{m} Ae^{i\omega t} = 0$$

- In order to satisfy the equation of motion, the angular frequency must satisfy the following condition:

$$\omega^2 - i\omega \frac{b}{m} - \frac{k}{m} = 0$$

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 15

15

Damped Harmonic Motion.

- We can solve this equation and determine the two possible values of the angular velocity:

$$\omega = \frac{1}{2} \left(i \frac{b}{m} \pm \sqrt{4 \frac{k}{m} - \frac{b^2}{m^2}} \right) \approx \frac{1}{2} i \frac{b}{m} \pm \sqrt{\frac{k}{m}}$$

- The solution to the equation of motion is thus given by

$$x(t) \approx x_m e^{-\frac{b}{2m}t} e^{it\sqrt{\frac{k}{m}}}$$

Damping Term
SHM Term

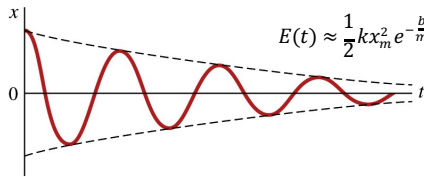
Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 16

16

Damped Harmonic Motion.

$$x(t) \approx x_m e^{-\frac{b}{2m}t} e^{it\sqrt{\frac{k}{m}}}$$



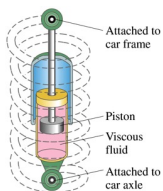
The general solution contains a SHM term, with an amplitude that decreases as function of time

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 17

17

Damped Harmonic Motion has many practical applications.



Damping is not always a curse.

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 18

18

Driven Harmonic Motion.

- Consider what happens when we apply a time-dependent force $F(t)$ to a system that normally would carry out SHM with an angular frequency ω_0 .

- Assume the external force $F(t) = mF_0\sin(\omega t)$. The equation of motion can now be written as

$$\frac{d^2 x}{dt^2} = -\omega_0^2 x + F_0 \sin(\omega t)$$

- The steady state motion of this system will be harmonic motion with an angular frequency equal to the angular frequency of the driving force.

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 19

19

Driven Harmonic Motion.

- Consider the general solution

$$x(t) = A \cos(\omega t + \phi)$$

- The parameters in this solution must be chosen such that the equation of motion is satisfied. This requires that

$$-\omega^2 A \cos(\omega t + \phi) + \omega_0^2 A \cos(\omega t + \phi) - F_0 \sin(\omega t) = 0$$

- This equation can be rewritten as

$$(\omega_0^2 - \omega^2)A(\cos(\omega t)\cos(\phi) - \sin(\omega t)\sin(\phi)) - F_0 \sin(\omega t) = 0$$

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 20

20

Driven Harmonic Motion.

- Our general solution must thus satisfy the following condition:

$$(\omega_0^2 - \omega^2)A \cos(\omega t) \cos \phi - ((\omega_0^2 - \omega^2)A \sin \phi - F_0) \sin(\omega t) = 0$$

- Since this equation must be satisfied at all time, we must require that the coefficients of $\cos(\omega t)$ and $\sin(\omega t)$ are 0. This requires that

$$(\omega_0^2 - \omega^2)A \cos \phi = 0$$

and

$$(\omega_0^2 - \omega^2)A \sin \phi - F_0 = 0$$

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 21

21

Driven Harmonic Motion.

- The interesting solutions are solutions where $A \neq 0$ and $\omega \neq \omega_0$. In this case, our general solution can only satisfy the equation of motion if

$$\cos\phi = 0$$

and

$$(\omega_0^2 - \omega^2)A \sin\phi - F_0 = (\omega_0^2 - \omega^2)A - F_0 = 0$$

- The amplitude of the motion is thus equal to

$$A = \frac{F_0}{(\omega_0^2 - \omega^2)}$$

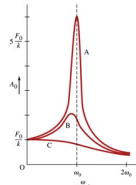
Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 22

22

Driven Harmonic Motion.

- If the driving force has a frequency close to the natural frequency of the system, the resulting amplitudes can be very large even for small driving amplitudes. The system is said to be in resonance.
- In realistic systems, there will also be a damping force. Whether or not resonance behavior will be observed will depend on the strength of the damping term.

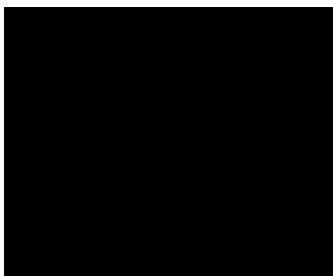


Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 23

23

Driven Harmonic Motion.



Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 24

24

Done for today!
Thursday: Temperature and Heat!



Unusually Strong Cyclone Off the Brazilian Coast: A lot of Rotational Motion!

Credit: [Jacques Descloîtres](#), [MODIS Land Rapid Response Team](#), [GSEC](#), [NASA](#).

Frank L. H. Wolfs

Department of Physics and Astronomy, University of Rochester, Slide 25
