Physics 121.
Lecture 21.

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- Course Information
- Topics to be discussed today:
  - Simple Harmonic Motion (Review).
  - Simple Harmonic Motion: Example Systems.
  - Damped Harmonic Motion
  - Driven Harmonic Motion

Physics 121.
Lecture 21.

- Homework set # 8 is due on Saturday morning, April 18, at 8.30 am.
- Homework set # 9 will be available on Saturday morning at 8.30 am, and will be due on Saturday morning, April 25, at 8.30 am.
- Exam # 3 is scheduled for Tuesday April 28 between 8 am in 9.20 am in Hubbell.

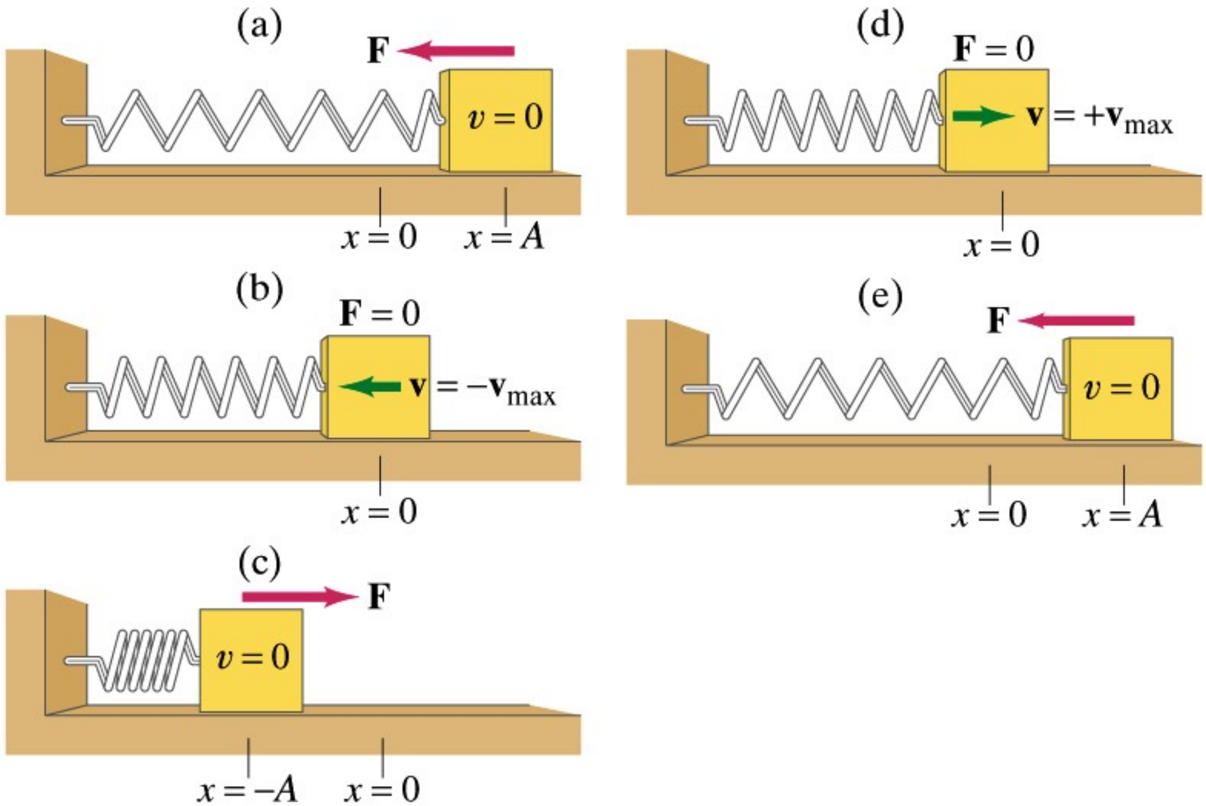
And now for something completely different. A preview of the ideal has law.

Sorry, fun is over!

The Flying Dutchman is brought to you by Mike, Joe, Jimmy, and Tony!

I would not be able to have so much fun teaching if they did not provide fantastic demo support.

Harmonic motion (a quick review).
Motion that repeats itself at regular intervals.



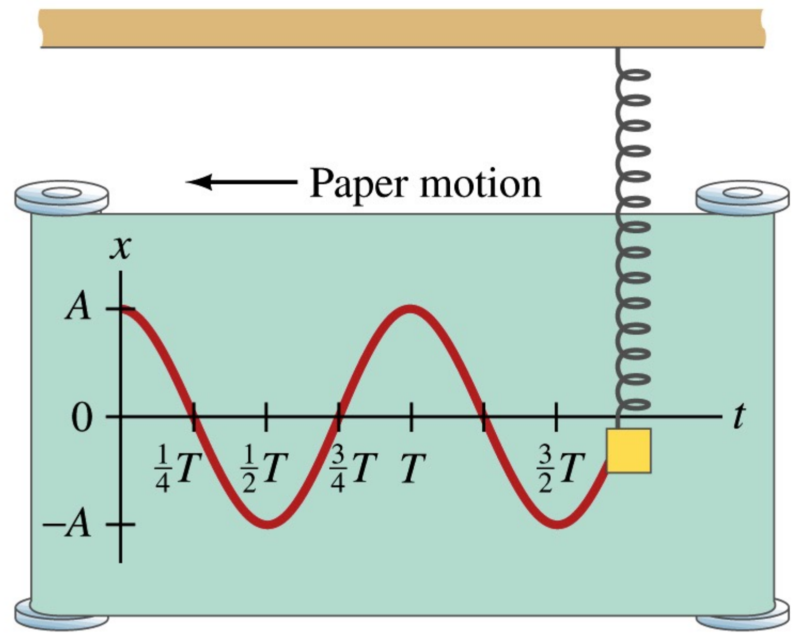
Simple Harmonic Motion (a quick review).

Phase Constant

Amplitude

$$x(t) = A\cos(\omega t + \phi)$$

Angular Frequency

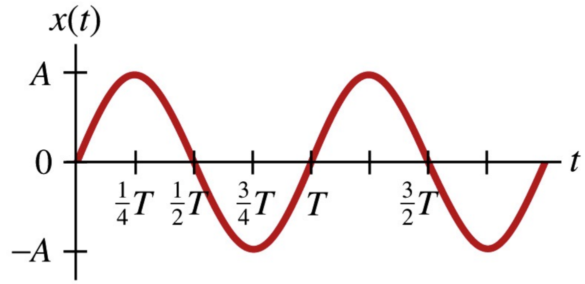


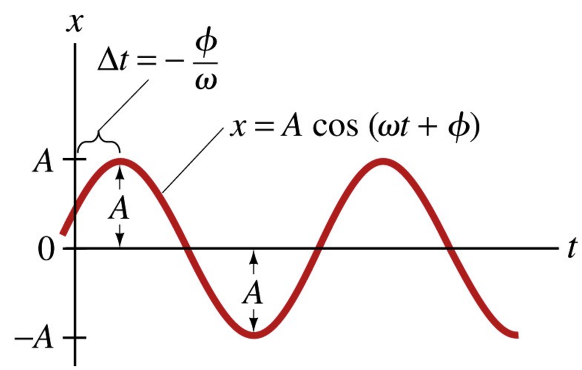
Simple Harmonic Motion (a quick review).

- Other variables frequently used to describe simple harmonic motion:
  - The period T: the time required to complete one oscillation. The period T is equal to $2\pi/\omega$.
  - The frequency of the oscillation is the number of oscillations carried out per second:

    $$\nu = \frac{1}{T}$$

    The unit of frequency is the Hertz (Hz). Per definition, 1 Hz = 1 s^{-1}.





Simple Harmonic Motion (a quick review). What forces are required?

- Using Newton's second law we can determine the force responsible for the harmonic motion:

$$F = ma = -m\omega^2 x$$

- The total mechanical energy of a system carrying out simple harmonic motion is constant.

- A good example of a force that produces simple harmonic motion is the spring force: $F = -kx$. The angular frequency depends on both the spring constant k and the mass m:

$$\omega = \sqrt{\frac{k}{m}}$$

Simple Harmonic Motion (SHM).
The equation of motion.

- All examples of SHM were derived from he following equation of motion:

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

- The most general solution to the equation is

$$x(t) = A\cos(\omega t + \alpha) + B\sin(\omega t + \beta)$$

Simple Harmonic Motion (SHM). The equation of motion.

- If $A = B$

$$x(t) = A\cos(\omega t + \alpha) + A\sin(\omega t + \beta) =$$

$$= A\left(\sin\left(\frac{1}{2}x - \omega t - \alpha\right) + \sin(\omega t + \beta)\right) =$$

$$= 2A\sin\left(\frac{1}{4}\pi + \frac{\beta}{2} - \frac{\alpha}{2}\right)\cos(\frac{1}{4}\pi - \omega t - \frac{\beta}{2} - \frac{\alpha}{2})$$

which is SHM.

Simple Harmonic Motion (SHM).
The equation of motion.

- If $A = 0$:

$$x(t) = B\sin(\omega t + \beta)\text{: SHM motion.}$$

- If $B = 0$:

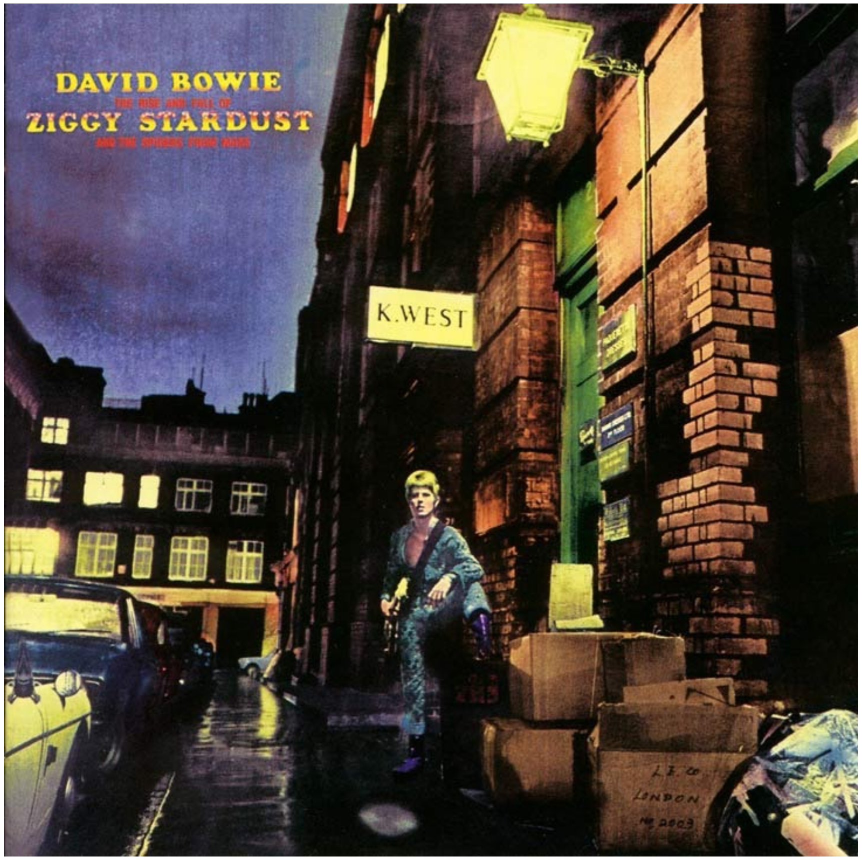
$$x(t) = A\cos(\omega t + \alpha)\text{: SHM motion}$$

- If $A \neq B \neq 0$: no SHM motion.

4 Minute 13 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 4 minute 13 second intermission.

- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.



Damped Harmonic Motion.

- Consider what happens when in addition to the restoring force a damping force (such as the drag force) is acting on the system:

$$F = -kx - b\frac{dx}{dt}$$

- The equation of motion is now given by:

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

Damped Harmonic Motion.

- The general solution of this equation of motion is

$$x(t) = Ae^{i\omega t}$$

- If we substitute this solution in the equation of motion we find

$$-\omega^2 Ae^{i\omega t} + i\omega \frac{b}{m} Ae^{i\omega t} + \frac{k}{m} Ae^{i\omega t} = 0$$

- In order to satisfy the equation of motion, the angular frequency must satisfy the following condition:

$$\omega^2 - i\omega \frac{b}{m} - \frac{k}{m} = 0$$

Damped Harmonic Motion.

- We can solve this equation and determine the two possible values of the angular velocity:

$$\omega = \frac{1}{2}\left(i\frac{b}{m} \pm \sqrt{4\frac{k}{m} - \frac{b^2}{m^2}} \right) \approx \frac{1}{2} i \frac{b}{m} \pm \sqrt{\frac{k}{m}}$$

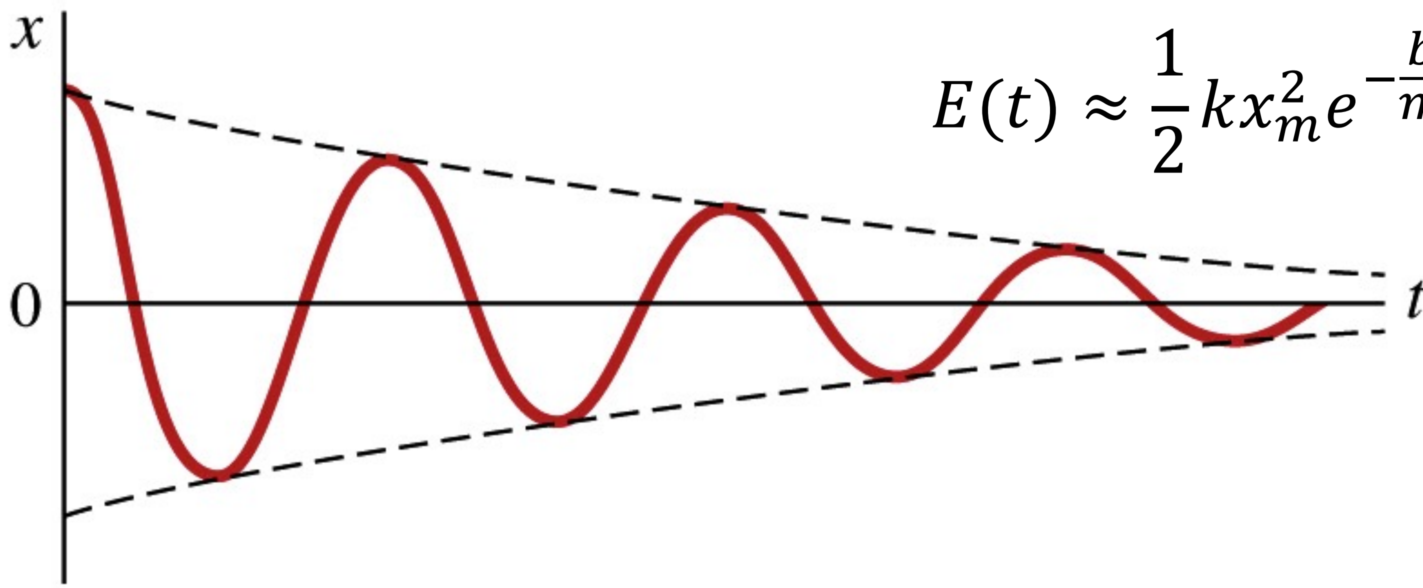
- The solution to the equation of motion is thus given by

$$x(t) \approx x_m \underbrace{e^{-\frac{b}{2m}t}}_{\text{Damping Term}} \underbrace{e^{it\sqrt{\frac{k}{m}}}}_{\text{SHM Term}}$$

Damped Harmonic Motion.

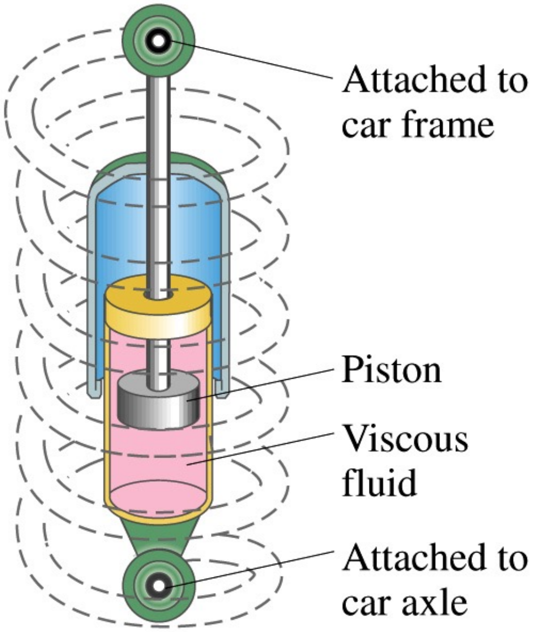
$$x(t) \approx x_m e^{-\frac{b}{2m}t} e^{it\sqrt{\frac{k}{m}}}$$

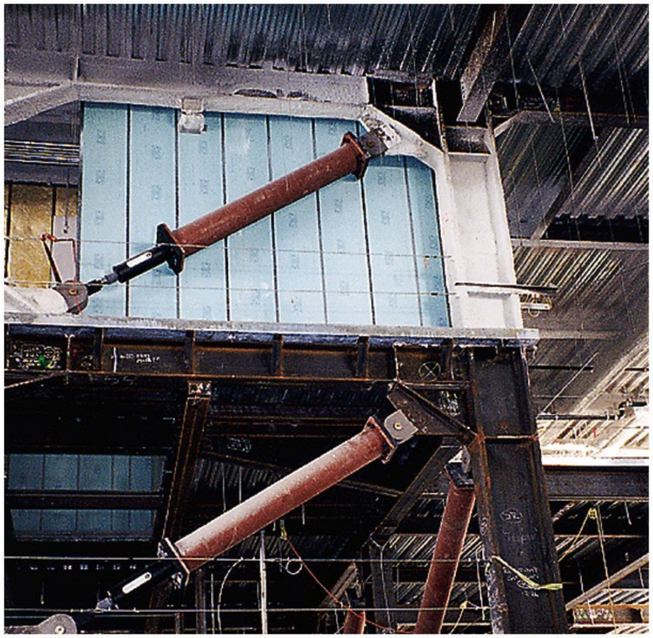
$$E(t) \approx \frac{1}{2} k x_m^2 e^{-\frac{b}{m}t}$$



The general solution contains a SHM term,
with an amplitude that decreases as function of time

Damped Harmonic Motion has many practical applications.





Damping is not always a curse.

Driven Harmonic Motion.

- Consider what happens when we apply a time-dependent force $F(t)$ to a system that normally would carry out SHM with an angular frequency ω_0.

- Assume the external force $F(t) = mF_0\sin(\omega t)$. The equation of motion can now be written as

$$\frac{d^2x}{dt^2} = -\omega_0^2 x + F_0\sin(\omega t)$$

- The steady state motion of this system will be harmonic motion with an angular frequency equal to the angular frequency of the driving force.

Driven Harmonic Motion.

- Consider the general solution

$$x(t) = A\cos(\omega t + \phi)$$

- The parameters in this solution must be chosen such that the equation of motion is satisfied. This requires that

$$-\omega^2 A\cos(\omega t + \phi) + \omega_0^2 A\cos(\omega t + \phi) - F_0 \sin(\omega t) = 0$$

- This equation can be rewritten as

$$(\omega_0^2 - \omega^2) A\big(\cos(\omega t)\cos(\phi) - \sin(\omega t)\sin(\phi)\big) - F_0 \sin(\omega t) = 0$$

Driven Harmonic Motion.

- Our general solution must thus satisfy the following condition:

$$(\omega_0^2 - \omega^2)A\cos(\omega t)\cos\phi - ((\omega_0^2 - \omega^2)A\sin\phi - F_0)\sin(\omega t) = 0$$

- Since this equation must be satisfied at all time, we must require that the coefficients of $\cos(\omega t)$ and $\sin(\omega t)$ are 0. This requires that

$$(\omega_0^2 - \omega^2)A\cos\phi = 0$$

and

$$(\omega_0^2 - \omega^2)A\sin\phi - F_0 = 0$$

Driven Harmonic Motion.

- The interesting solutions are solutions where $A \neq 0$ and $\omega \neq \omega_0$. In this case, our general solution can only satisfy the equation of motion if

$$\cos\phi = 0$$

and

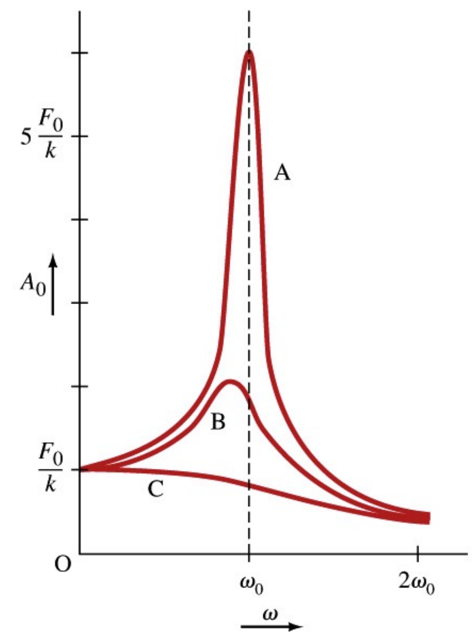
$$(\omega_0^2 - \omega^2)A\sin\phi - F_0 = (\omega_0^2 - \omega^2)A - F_0 = 0$$

- The amplitude of the motion is thus equal to

$$A = \frac{F_0}{(\omega_0^2 - \omega^2)}$$

Driven Harmonic Motion.

- If the driving force has a frequency close to the natural frequency of the system, the resulting amplitudes can be very large even for small driving amplitudes. The system is said to be in resonance.
- In realistic systems, there will also be a damping force. Whether or not resonance behavior will be observed will depend on the strength of the damping term.



Driven Harmonic Motion.

Done for today!
Thursday: Temperature and Heat!

Unusually Strong Cyclone Off the Brazilian Coast: A lot of Rotational Motion!
Credit: Jacques Descloitres, MODIS Land Rapid Response Team, GSFC, NASA