Physics 121.
Lecture 18.

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- Course Information
- Topics to be discussed today:
  - Review of Angular Momentum
  - Conservation of Angular Momentum
  - Precession

Physics 121.
March 27, 2008.

- Homework set # 6 is now available and is due on Saturday April 4 at 8.30 am.

- Homework set # 7 is also available and is due on Saturday April 11 at 8.30 am.

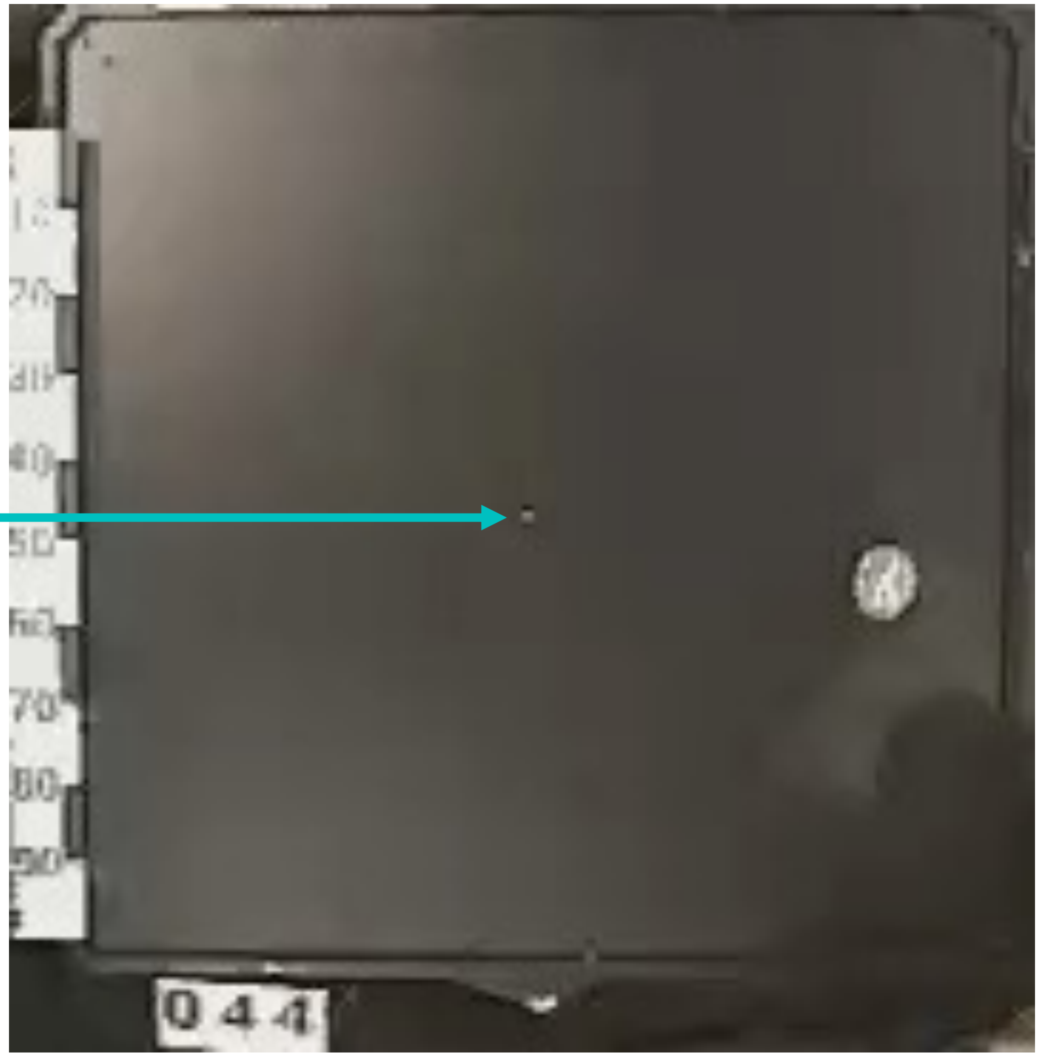
- Homework set # 7 has two components:
  - WeBWork (75%)
  - Video analysis (25%)

- Make sure you pick up the results of exam # 2 during workshop this week.

Video analysis.
Is angular momentum conserved?

Ruler

Origin



Mass puck is 33.6 g.

Concept test.

- Let us practice what we have learned.

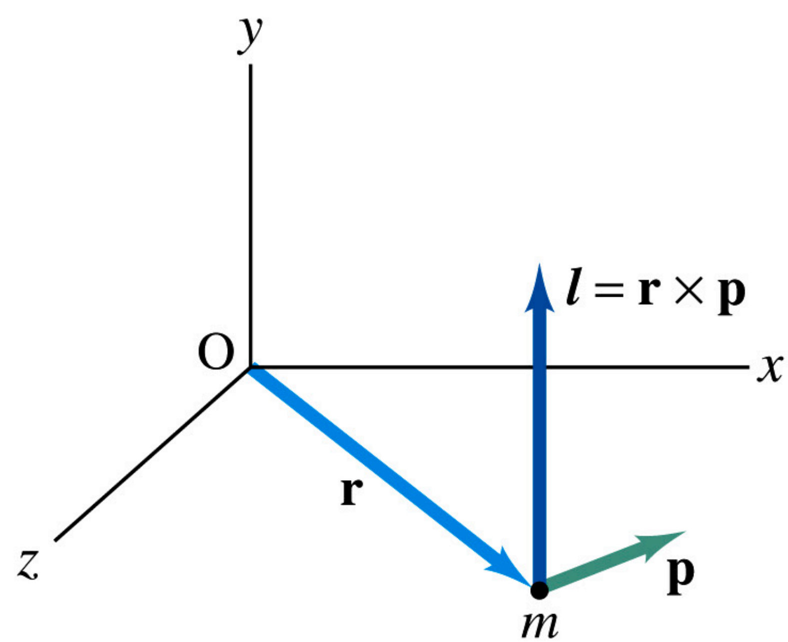
Angular momentum.
A quick review.

- We have seen many similarities between the way in which we describe linear and rotational motion.
- Our treatment of these types of motion are similar if we recognize the following equivalence:

linear	rotational
mass m	moment I
force $\vec{F}$	torque $\vec{\tau} = \vec{r} \times \vec{F}$

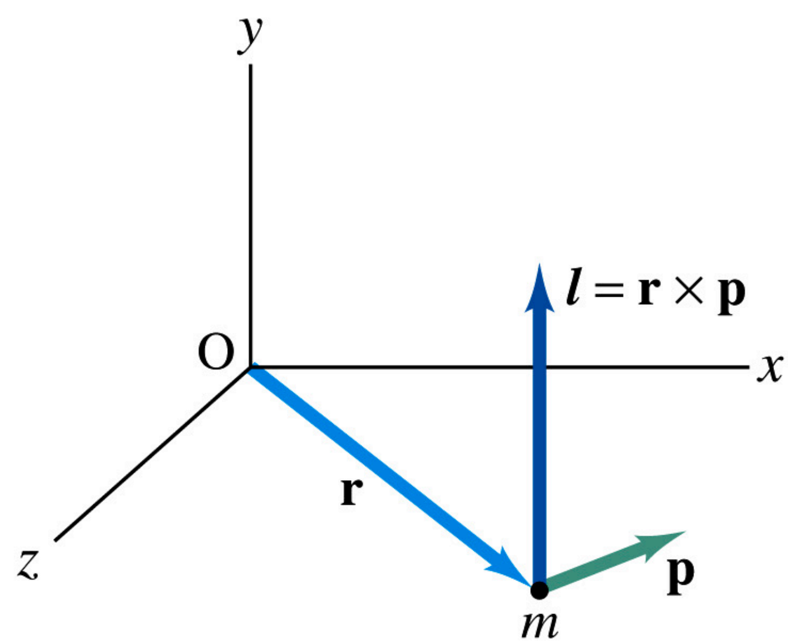
- What is the equivalent to linear momentum?

  Answer: **angular momentum**.



Angular momentum. A quick review.

- The angular momentum is defined as the vector product between the position vector and the linear momentum.
- Note:
  - Compare this definition with the definition of the torque.
  - Angular momentum is a vector.
  - The unit of angular momentum is kg m^2/s.
  - The angular momentum depends on both the magnitude and the direction of the position and linear momentum vectors.
  - Under certain circumstances the angular momentum of a system is conserved!

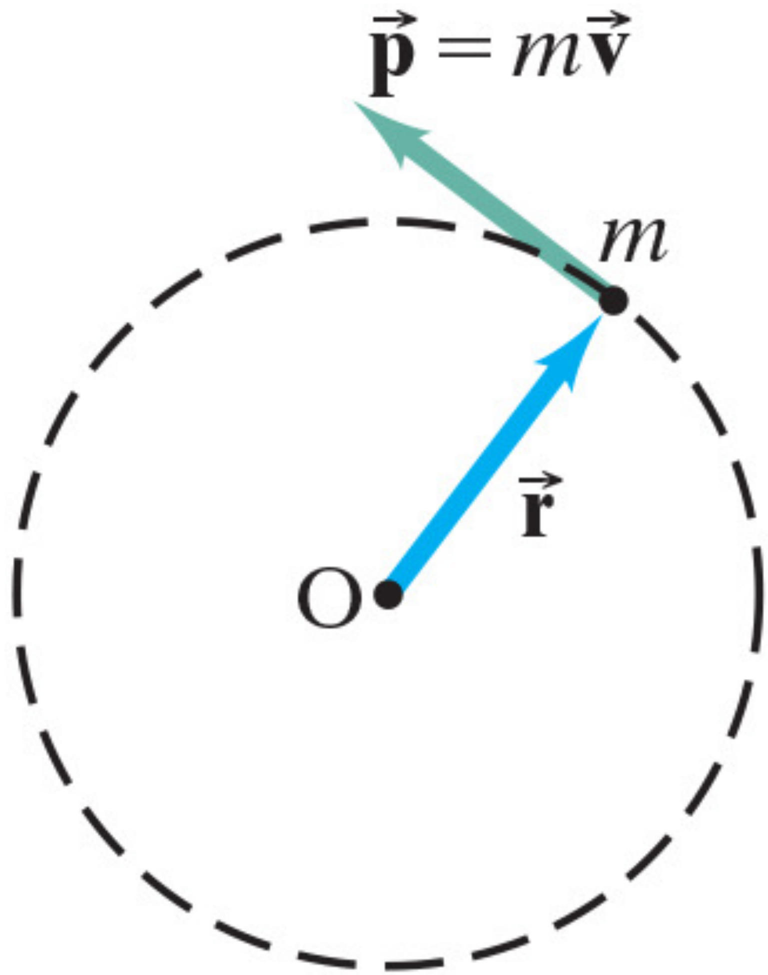


Angular momentum.
A quick review.

- Consider an object carrying out circular motion.
- For this type of motion, the position vector will be perpendicular to the momentum vector.
- The magnitude of the angular momentum is equal to the product of the magnitude of the radius $\vec{r}$ and the linear momentum $\vec{p}$:

$$L = mvr = mr^2\frac{v}{r} = I\omega$$

- Note: compare this with the linear momentum $p = mv$!



Conservation of angular momentum. A quick review.

- Consider the change in the angular momentum of a particle:

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r}\times\vec{p}) = m\left(\vec{r}\times\frac{d\vec{v}}{dt} + \frac{d\vec{r}}{dt}\times\vec{v}\right) = m(\vec{r}\times\vec{a} + \vec{v}\times\vec{v})$$

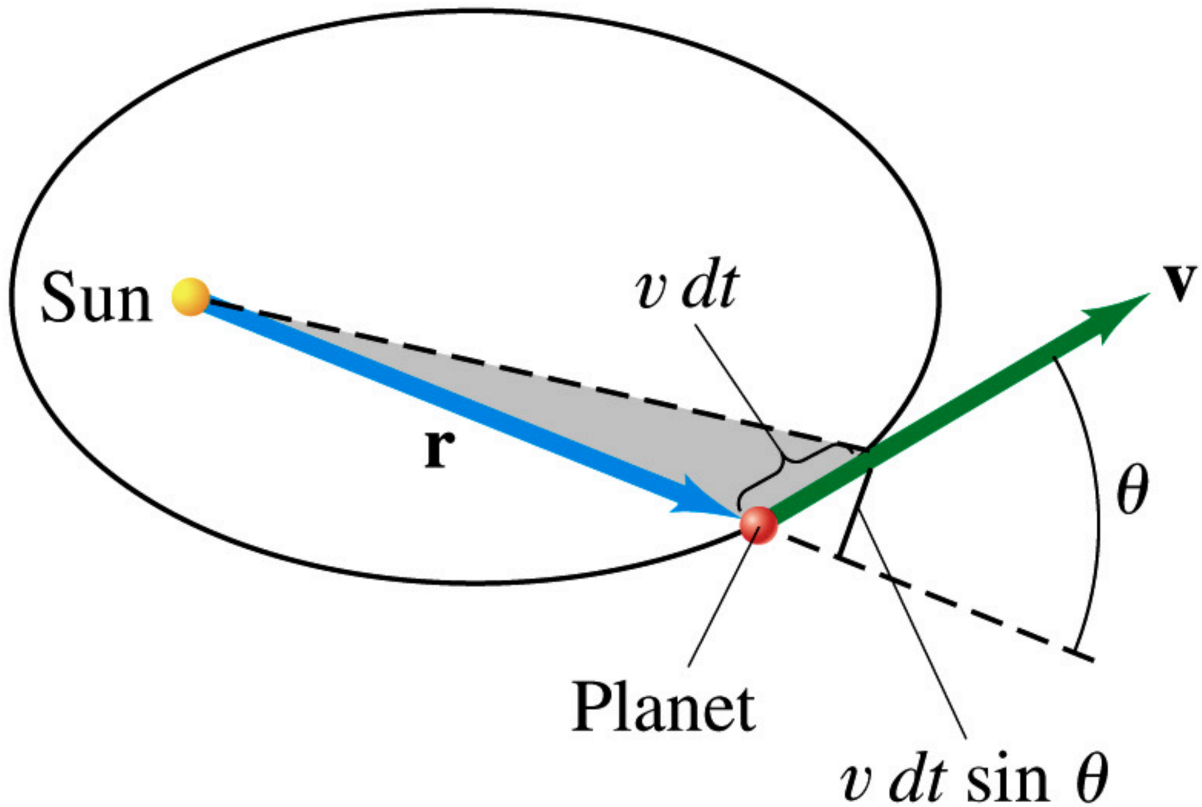
$$\frac{d\vec{L}}{dt} = \vec{r}\times m\vec{a} = \vec{r}\times\sum\vec{F} = \sum\vec{\tau}$$

- Consider what happens when the net torque is equal to 0:

$$\frac{d\vec{L}}{dt} = 0 \implies L = \text{constant}$$

- When we take the sum of all torques, the torques due to the internal forces cancel and the sum is equal to torque due to all external forces.
- Note: notice again the similarities between linear and rotational motion.

Conservation of angular momentum. Consequences.



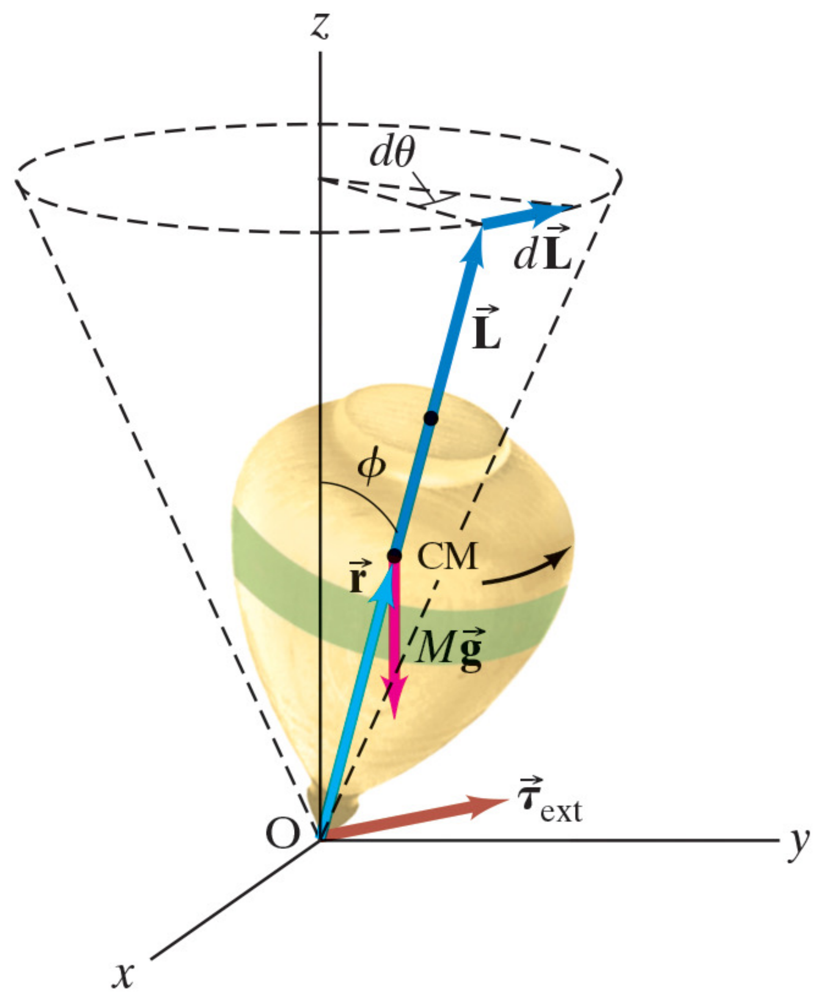
$$\left|\vec{L}\right| = rmv\sin\theta = m\frac{rvdt\sin\theta}{dt} = 2m\frac{dA}{dt} = \text{constant}$$

Conservation of angular momentum. A quick review.

- The connection between the angular momentum $\vec{L}$ and the torque $\vec{\tau}$

$$\sum\vec{\tau} = \frac{d\vec{L}}{dt}$$

  is only true if $\vec{L}$ and $\vec{\tau}$ are calculated with respect to the same reference point (which is at rest in an inertial reference frame).
- The relation is also true if $\vec{L}$ and $\vec{\tau}$ are calculated with respect to the center of mass of the object (note: center of mass can accelerate).

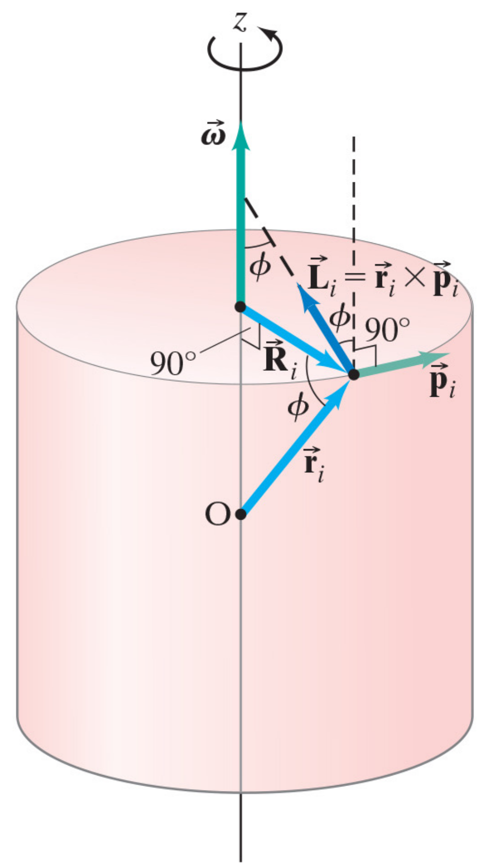


The angular momentum of rotating rigid objects.

- Consider a rigid object rotating around the z axis.
- The magnitude of the angular momentum of a part of a small section of the object is equal to

$$l_i = r_i(m_i v_i)$$

- Due to the symmetry of the object, we expect that the angular momentum of the object will be directed on the z axis. Thus, we only need to consider the z component of this angular momentum.



Note the direction of l_i !!!! (perpendicular to r_i and p_i)

The angular momentum of rotating rigid objects.

- The z component of the angular momentum is

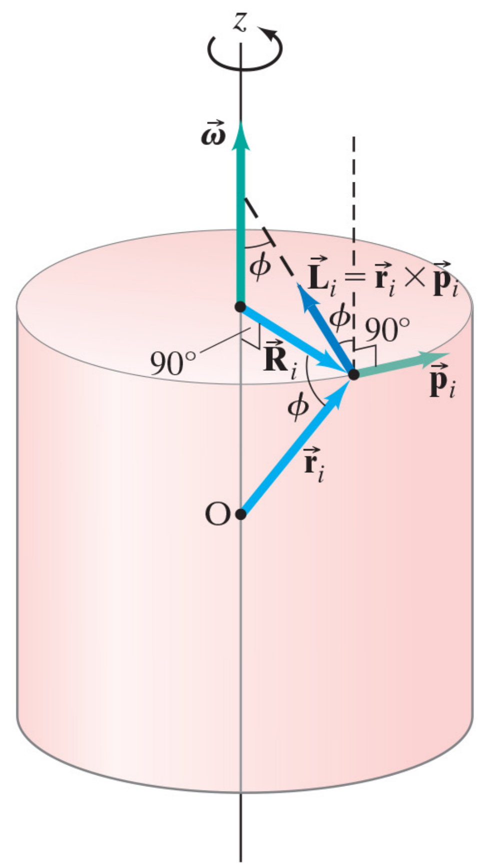
$$L_{i,z} = l_i \sin\theta = (r_i \sin\theta) m_i v_i = R_i m_i v_i$$

- The total angular momentum of the rotating object is the sum of the angular momenta of the individual components:

$$L_z = \sum L_{i,z} = \sum R_i (m_i \omega R_i) = \omega \sum m_i R_i^2$$

- The total angular momentum is thus equal to

$$L_z = I\omega$$



Note the direction of l_i !!!! (perpendicular to r_i and p_i)

Conservation of angular momentum. Sample problem.

- The moment inertia of a collapsing spinning star changes to one-third of its initial value. What is the ratio of the new rotational kinetic energy to the initial rotational kinetic energy?

- When the star collapses, it is compressed, and its moment of inertia decreases. In this particle case, the reduction is a factor of 3:

$$I_f = \frac{1}{3} I_i$$

- The forces responsible for the collapse are internal forces, and angular momentum is conserved.

Conservation of angular momentum. Sample problem.

- The initial kinetic energy of the star can be expressed in terms of its initial angular momentum:

$$K_i = \frac{1}{2} I_i \omega_i^2 = \frac{1}{2} \frac{L_i^2}{I_i}$$

- The final kinetic energy of the star ca also be expressed in terms of its angular momentum:

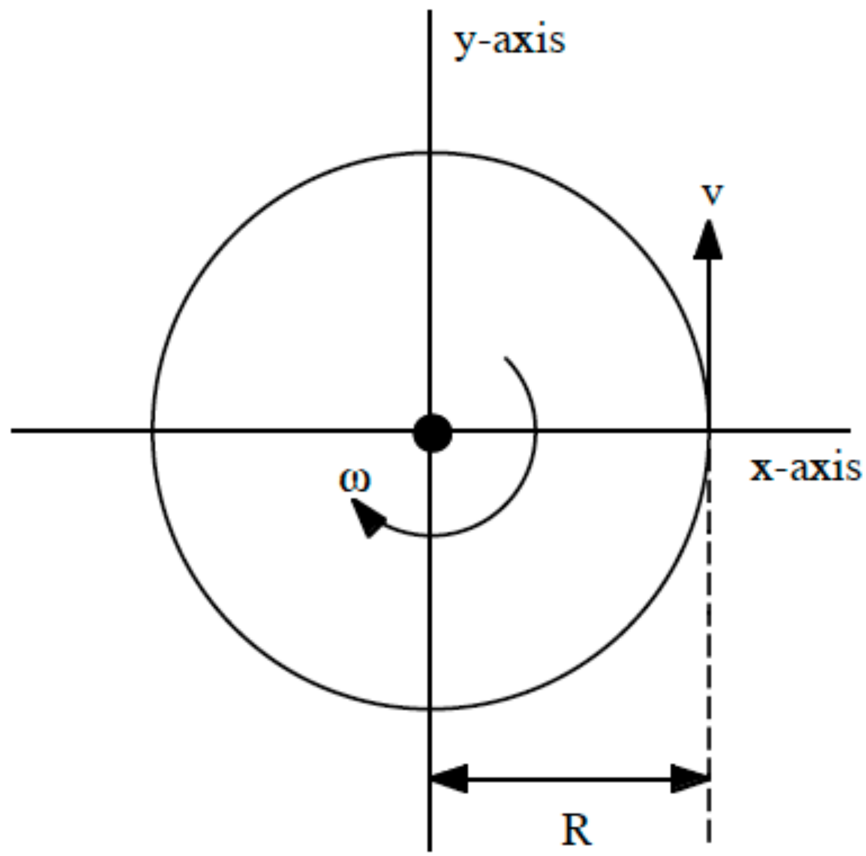
$$K_f = \frac{1}{2} \frac{L_f^2}{I_f} = \frac{1}{2} \frac{L_i^2}{\frac{1}{3} I_i} = 3K_i$$

- Note: **the kinetic energy increased**! Where does this energy come from?

Conservation of angular momentum. Sample problem.

A cockroach with mass m runs counterclockwise around the rim of a lazy Susan (a circular dish mounted on a vertical axle) of radius R and rotational inertia I with frictionless bearings. The cockroach's speed (with respect to the earth) is v, whereas the lazy Susan turns clockwise with angular speed ω ($\omega < 0$). The cockroach finds a bread crumb on the rim and, of course, stops.

(a) What is the angular speed of the lazy Susan after the cockroach stops?
(b) (b) Is mechanical energy conserved?



Conservation of angular momentum. Sample problem.

- The initial angular momentum of the cockroach is

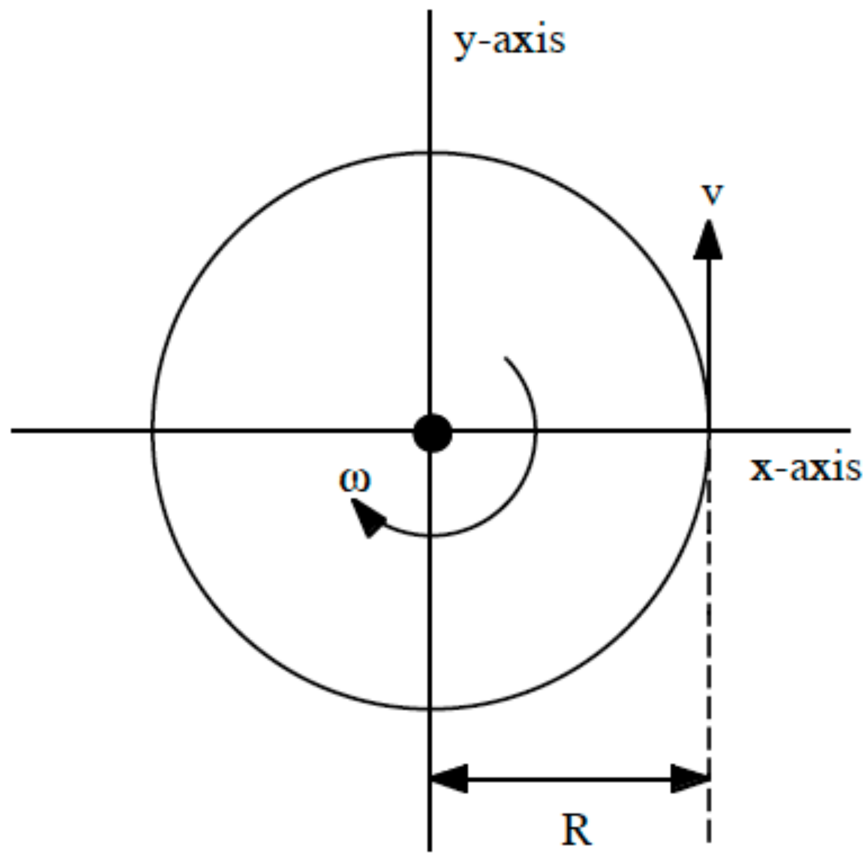
$$\vec{L}_c = \vec{r} \times \vec{p} = Rmv\ \hat{z}$$

- The initial angular momentum of the lazy Susan is

$$\vec{L}_d = I\omega\ \hat{z}$$

- The total initial angular momentum is thus equal to

$$\vec{L} = \vec{L}_c + \vec{L}_d = (Rmv + I\omega)\ \hat{z}$$



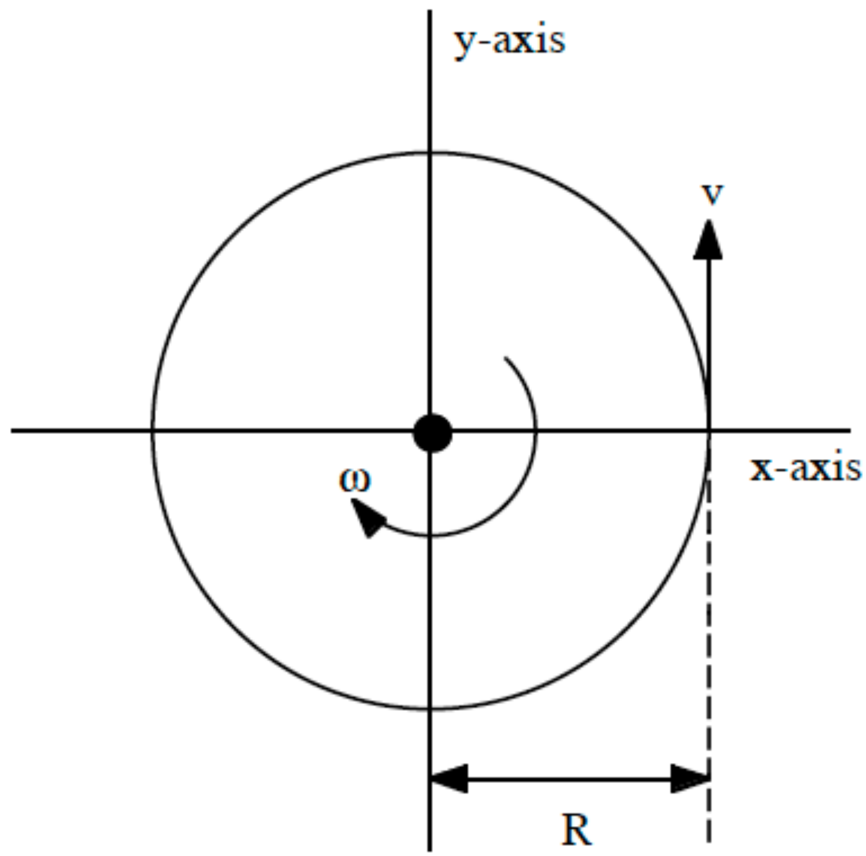
Conservation of angular momentum. Sample problem.

- When the cockroach stops, it will move in the same way as the rim of the lazy Susan. The forces that bring the cockroach to a halt are internal forces, and angular momentum is thus conserved.
- The moment of inertia of the lazy Susan + cockroach is equal to

$$I_f = I + mR^2$$

- The final angular velocity of the system is thus equal to

$$\omega_f = \frac{L_f}{I_f} = \frac{Rmv + I\omega}{I + mR^2}$$



Conservation of angular momentum. Sample problem.

- The initial kinetic energy of the system is equal to

$$K_i = \frac{1}{2}\underbrace{mv^2}_{\text{Cockroach}} + \frac{1}{2}\underbrace{I\omega^2}_{\text{Lazy Susan}}$$

- The final kinetic energy of the system is equal to

$$K_f = \frac{1}{2} I_f \omega_f^2 = \frac{1}{2}(I + mR^2)\left(\frac{Rmv + I\omega}{I + mR^2}\right)^2 = \frac{1}{2}\,\frac{(Rmv + I\omega)^2}{I + mR^2}$$

- The change in the kinetic energy is thus equal to

$$\Delta K = K_f - K_i = -\frac{1}{2}\frac{mI}{I + mR^2}(v - R\omega)^2$$

Loss of Kinetic Energy!!

4 Minute 32 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 4 minute 32 second intermission.

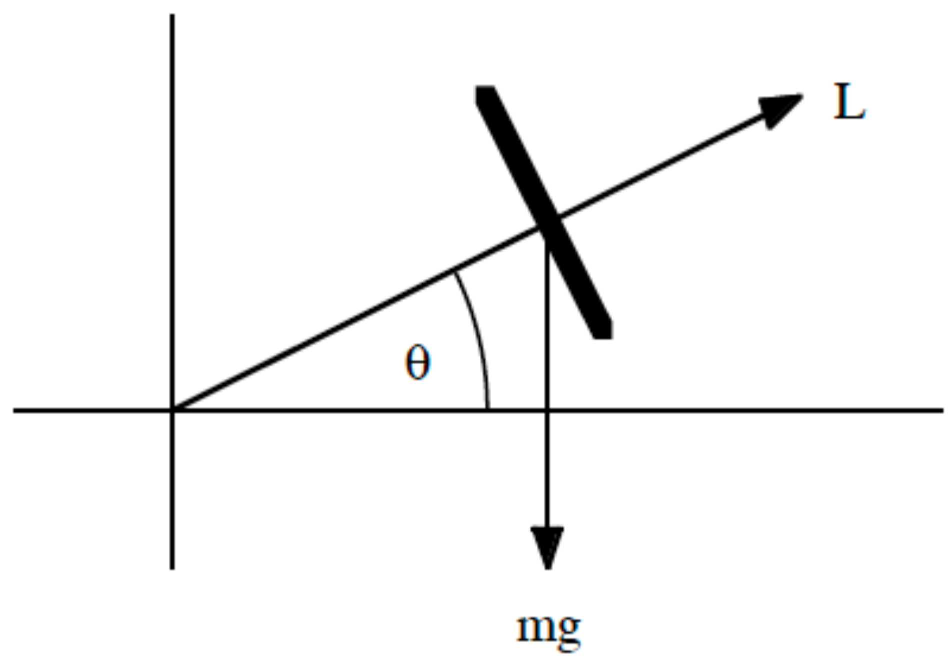
- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.

Quiz.

- Let us practice what we have learned.

Precession.

- Consider a rotating rigid object spinning around its symmetry axis.
- The object carries a certain angular momentum L.
- Consider what will happen when the object is balanced on the tip of its axis (which makes an angle θ with the horizontal plane).
- The gravitational force, which is an external force, will generate a toques with respect of the tip of the axis.



Precession.

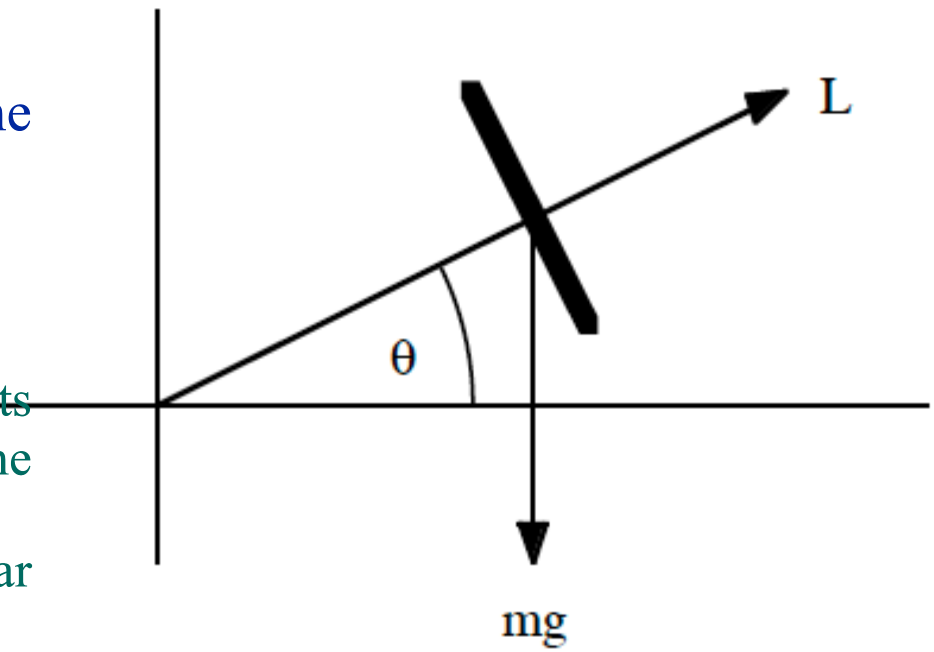
- The external torque is equal to

$$|\vec{\tau}| = \left|\vec{r}\times\vec{F}\right| = rF\sin\left(\frac{\pi}{2}+\theta\right) = mgr\cos\theta$$

- The external torque causes a change in the angular momentum:

$$d\vec{L} = \vec{\tau}dt$$

- Thus:
  - The change in the angular momentum points in the same direction as the direction of the torque.
  - The direction of the torque is perpendicular to the direction of the angular momentum.
  - The torque will thus change the direction of $\vec{L}$ but not its magnitude.

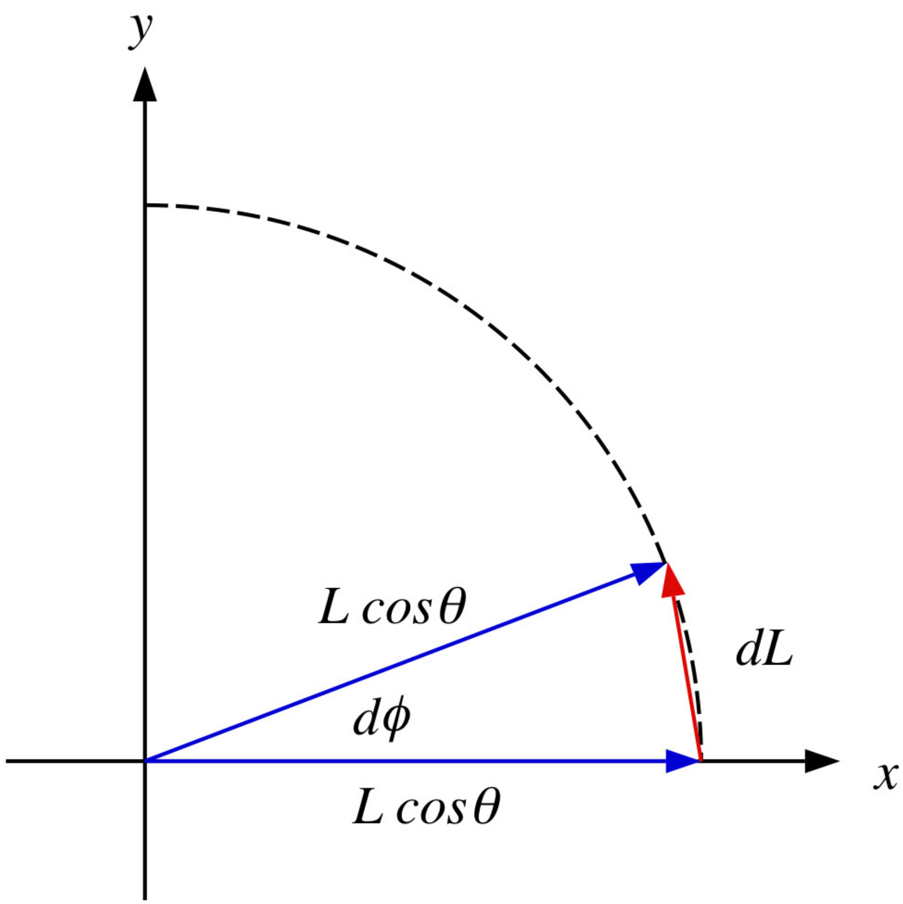


Precession.

- The effect of the torque can be visualized by looking at the motion of the projection of the angular momentum in the *xy* plane.

- The angle of rotation of the projection of the angular momentum vector when the angular momentum changes by dL is equal to

$$d\phi = \frac{dL}{L\cos\theta} = \frac{mgr\cos\theta dt}{L\cos\theta} = \frac{mgrdt}{L}$$



Precession.

- Since the projection of the angular momentum during the time interval dt rotates by an angle $d\phi$, we can calculate the rate of precession:

$$\Omega = \frac{d\phi}{dt} = \frac{mgr}{L} = \frac{mgr}{I\omega}$$

- We conclude the following:
  - The rate of precessions does not depend on the angle θ.
  - The rate of precession decreases when the angular momentum increases.

Done for today! Next week we stop moving and focus on equilibrium.

Spirit Pan from Bonneville Crater's Edge
Credit: Mars Exploration Rover Mission, JPL, NASA