Physics 121.
Lecture 17.

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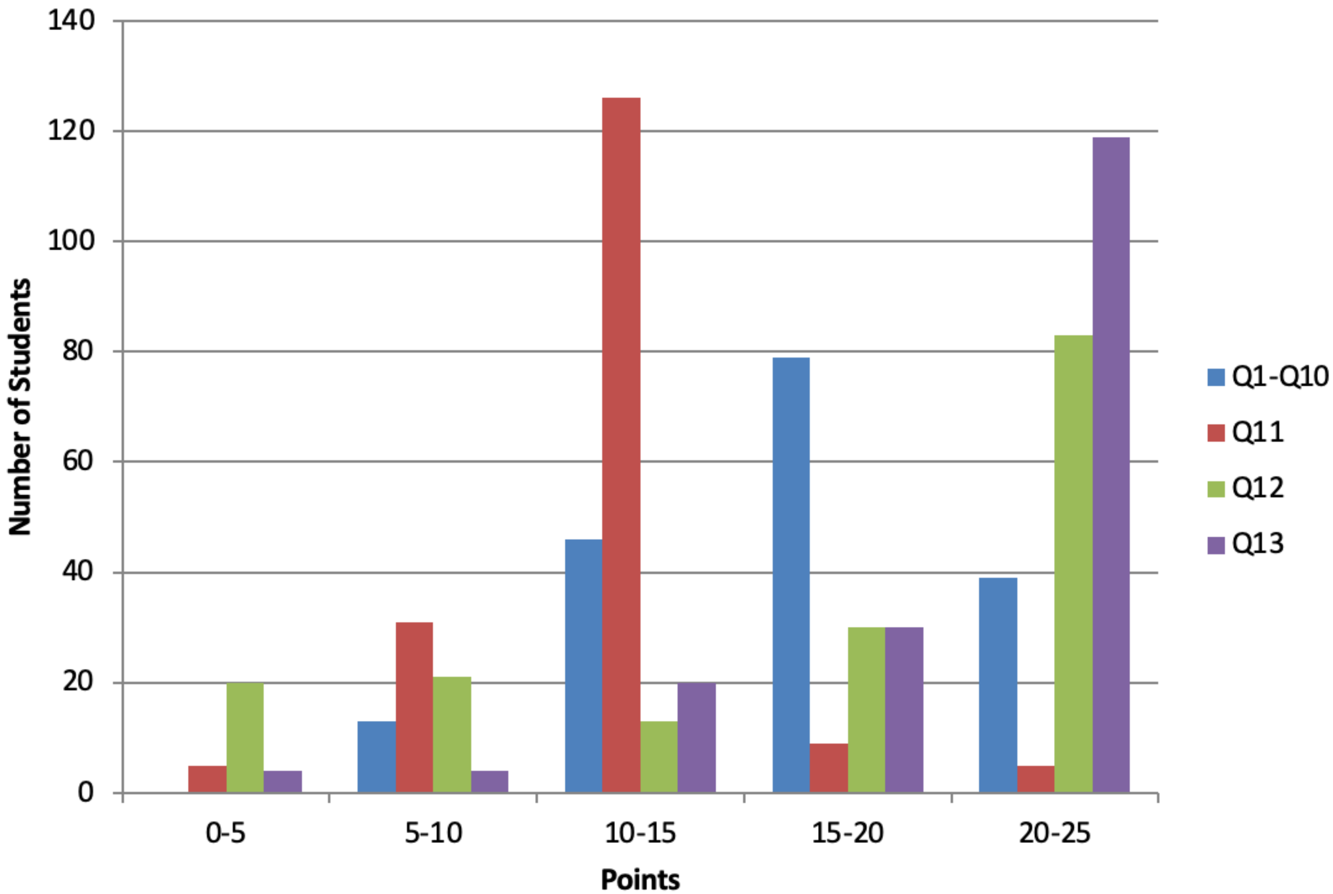
- Course Information
- Topics to be discussed today:
  - Review of Rotational Motion
  - Rolling Motion
  - Angular Momentum
  - Conservation of Angular Momentum

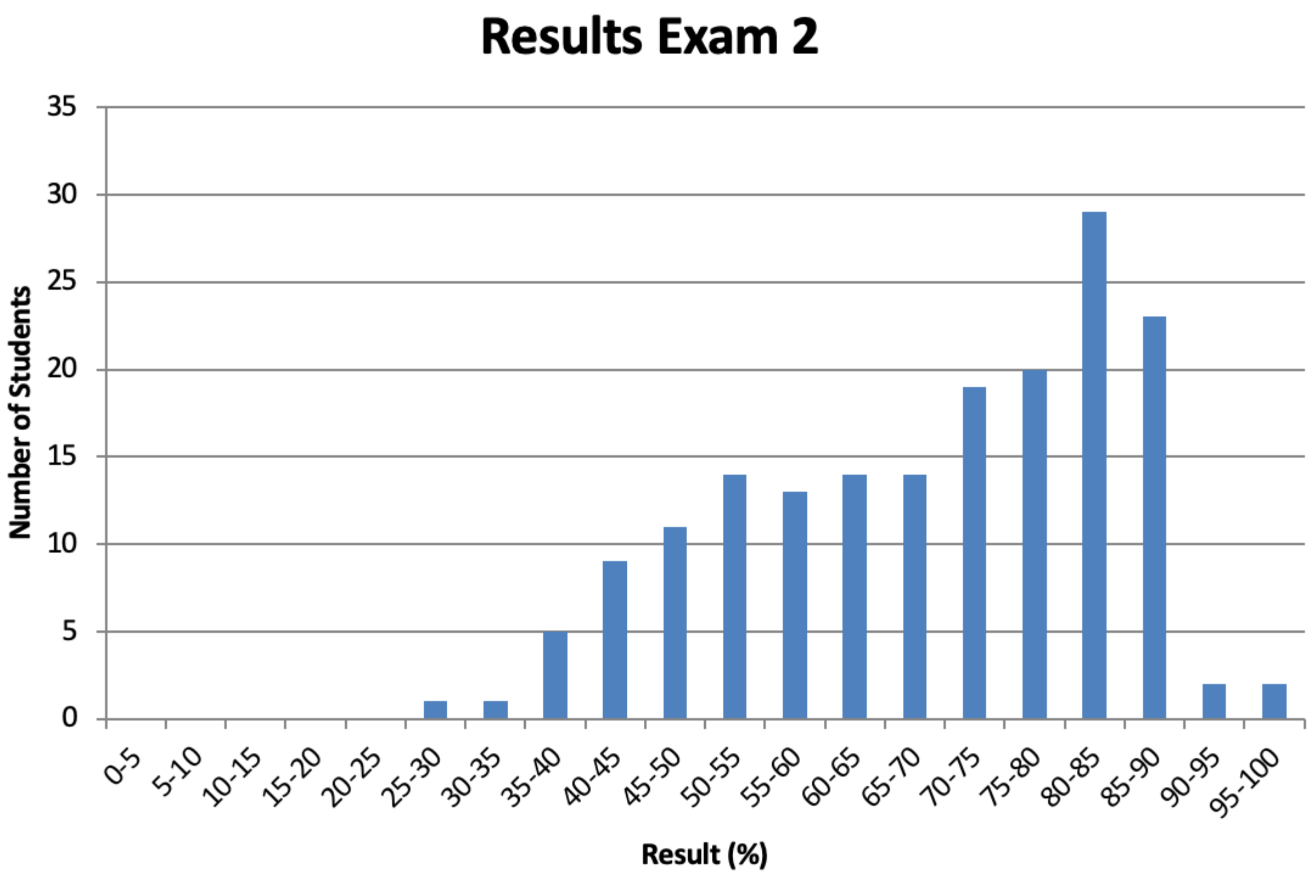
Physics 121.
Lecture 17.

- Homework set # 6 is now available and is due on Saturday April 4 at 8.30 am.
- The grades for exam # 2 were distributed via email on Saturday March 28.
- You should pick up your exam in workshop this week. Please check it carefully for any errors.
- If you are not happy with the grading of your exam, you need to write down why you feel you deserve more points on a particular question, attach the note to your blue booklet(s), and return them to me before the end of lecture 20 (April 9).

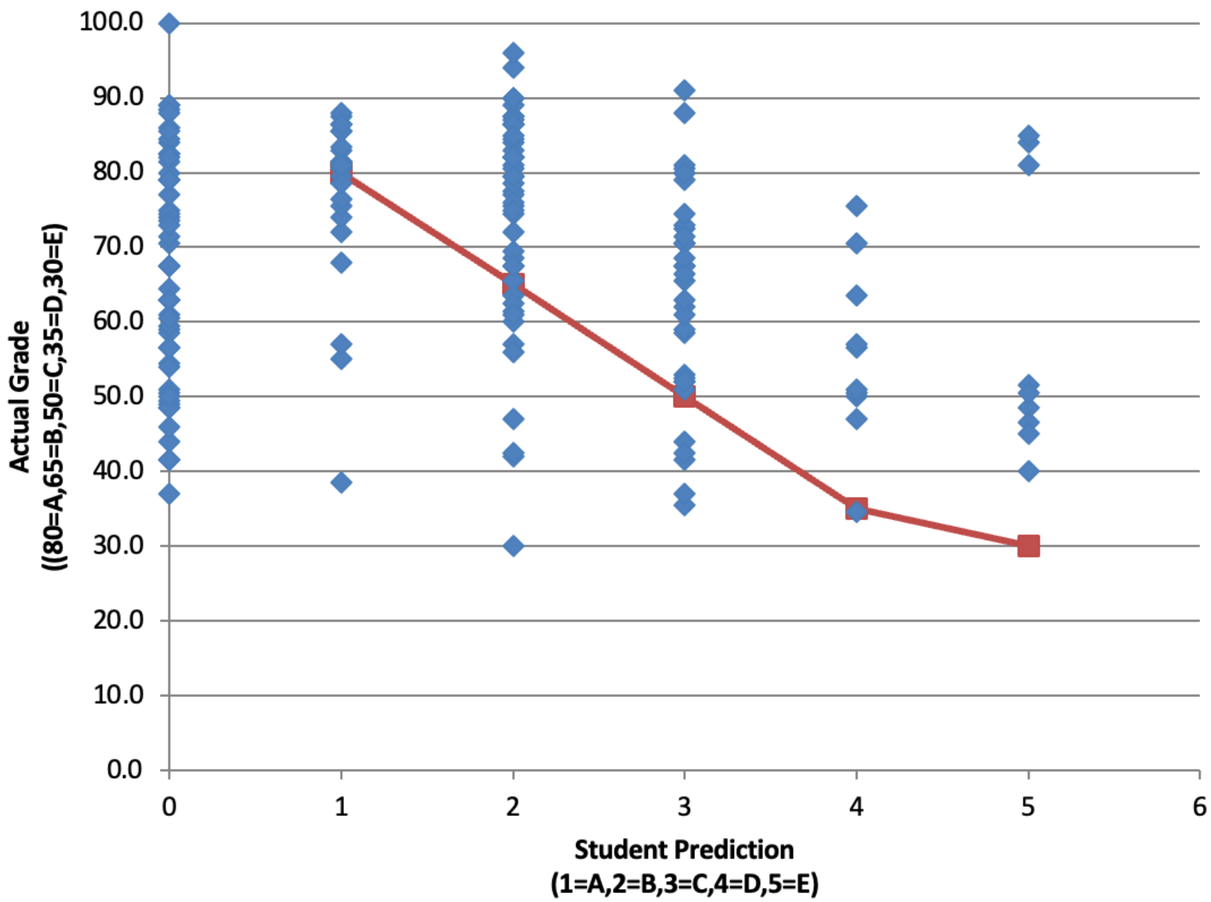
Results of individual questions on Exam # 2.



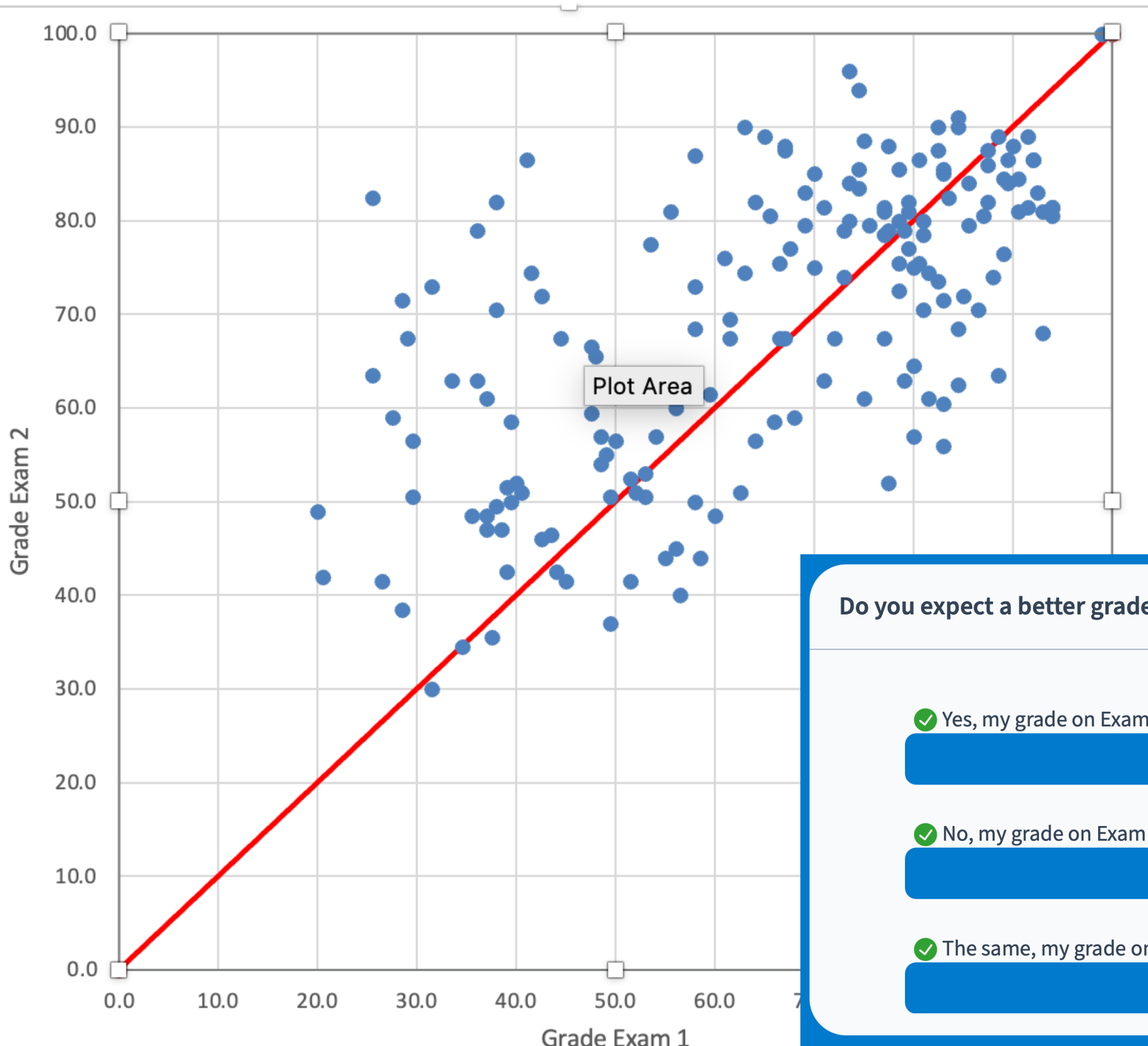
Grade Distribution Exam # 2.

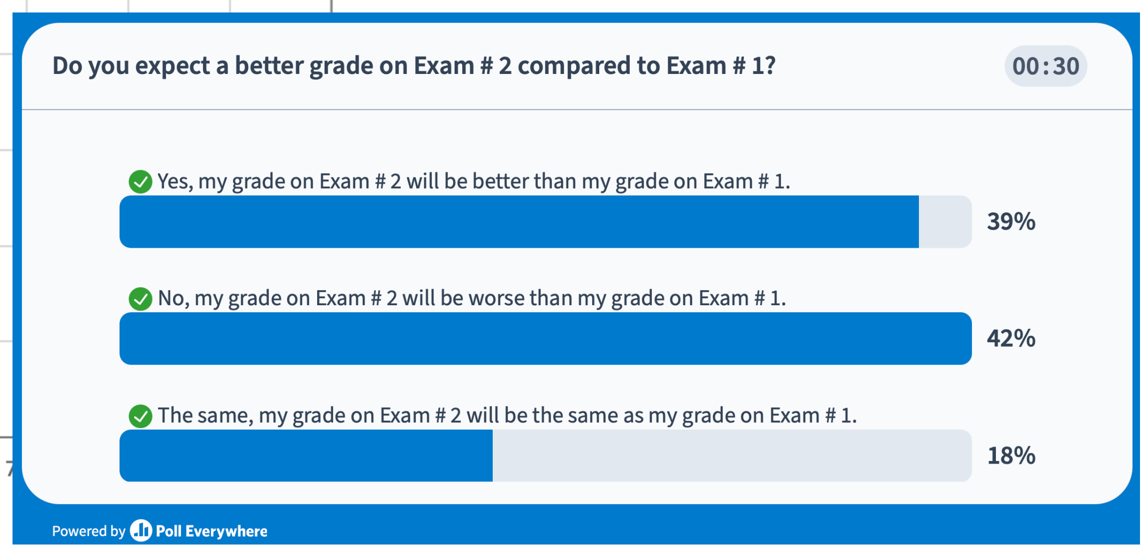


Your prediction versus reality.
Reality is better than your prediction.

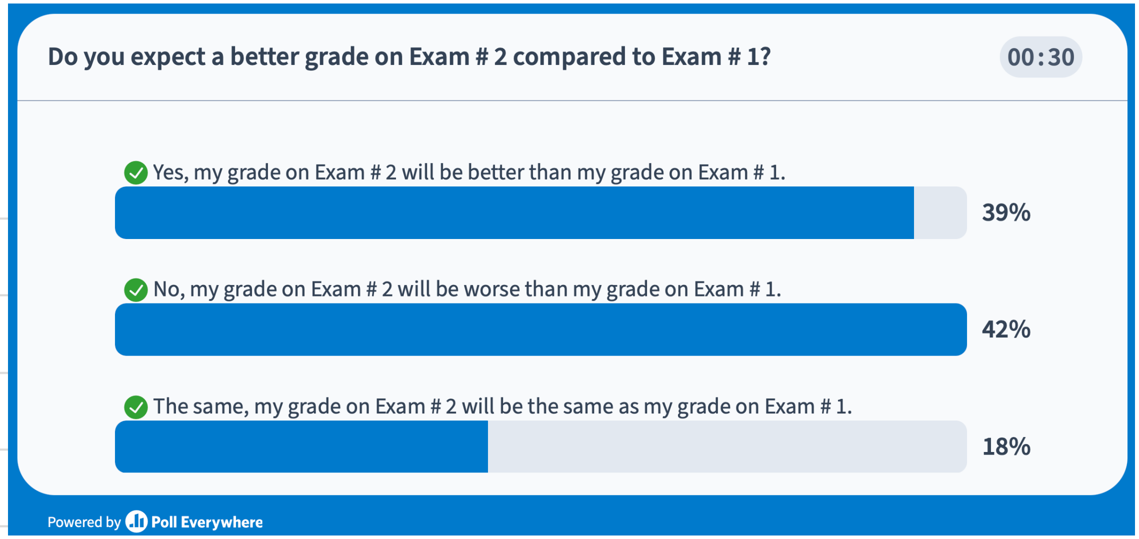


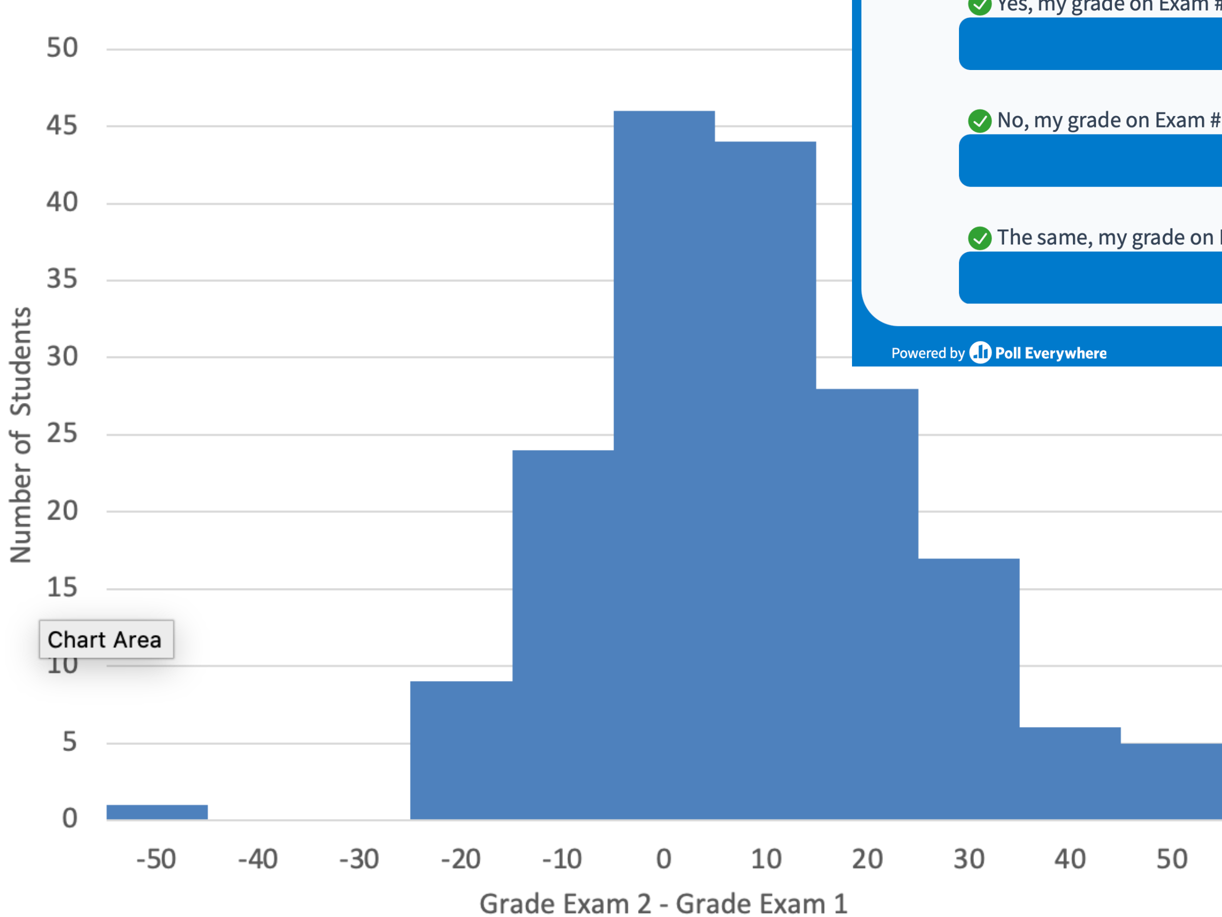
Most of you did better on Exam # 2.





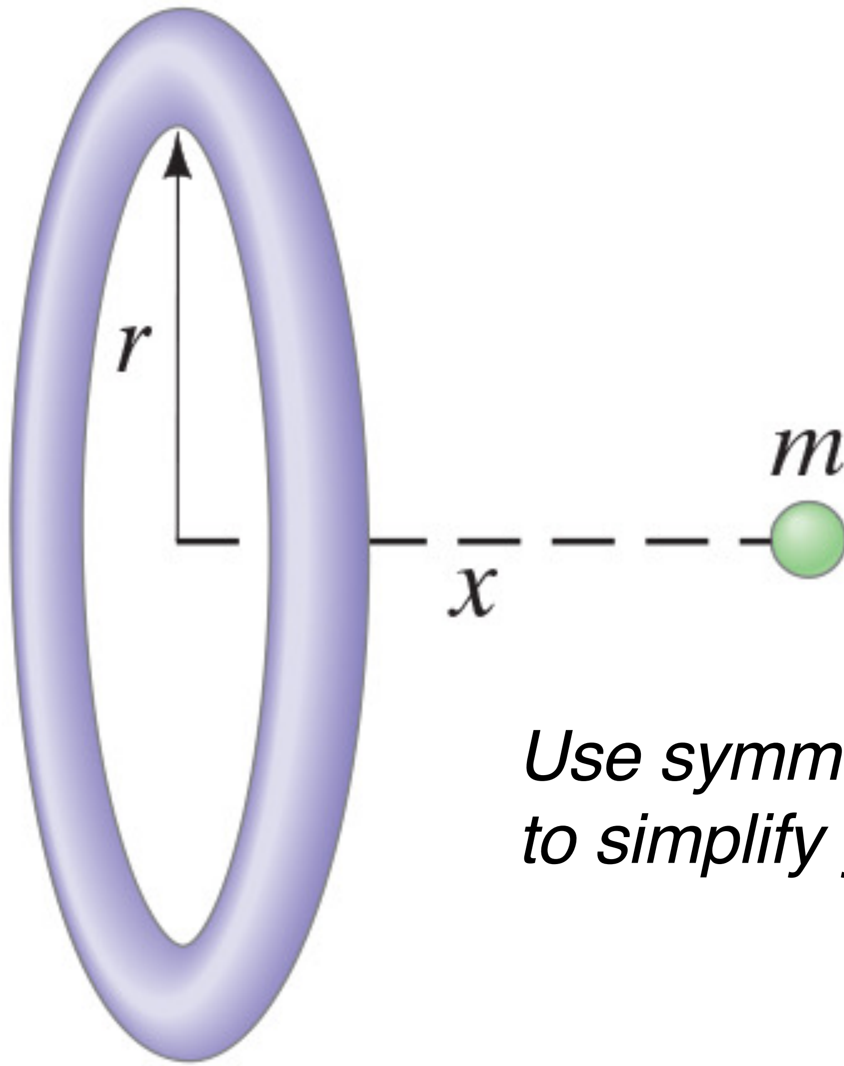
Grade change between Exam # 2 and Exam # 1.





25% of the Phy 121 students obtained a similar grade on Exam 2.
56% earned a better grade.

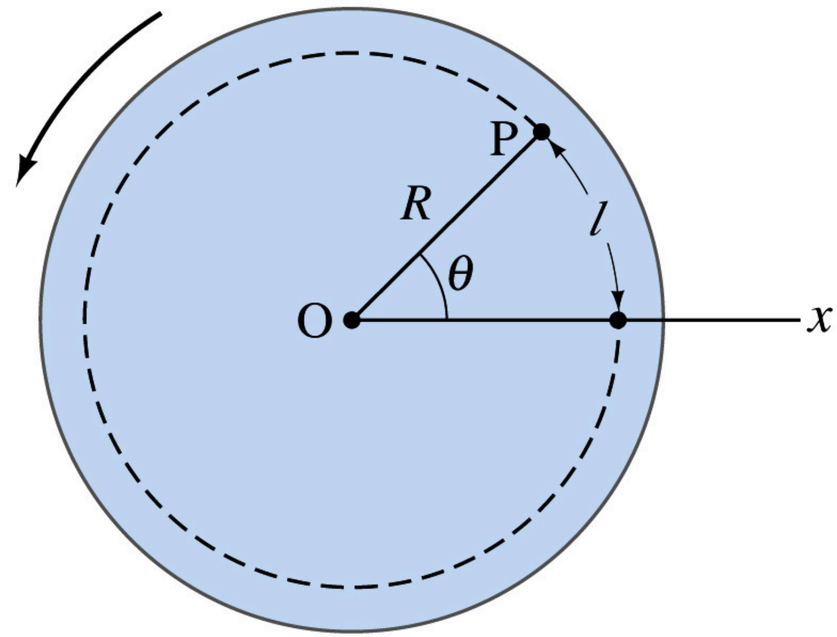
Chapter 6.
Calculating the gravitational force.



Use symmetry arguments to simplify your calculations.

Rotational variables.
A quick review.

- The variables that are used to describe rotational motion are:
  - Angular position θ
  - Angular velocity $\omega = d\theta/dt$
  - Angular acceleration $\alpha = d\omega/dt$
- The rotational variables are related to the linear variables:
  - Linear position $l = R\theta$
  - Linear velocity $v = R\omega$
  - Linear acceleration $a = R\alpha$



Rotational kinetic energy. A quick review.

- Since the components of a rotating object have a non-zero (linear) velocity we can associate a kinetic energy with the rotational motion:

$$K = \sum_i \frac{1}{2} m_i v_i^2 = \sum_i \frac{1}{2} m_i (\omega r_i)^2 = \frac{1}{2} \left(\sum_i m_i r_i^2\right) \omega^2$$

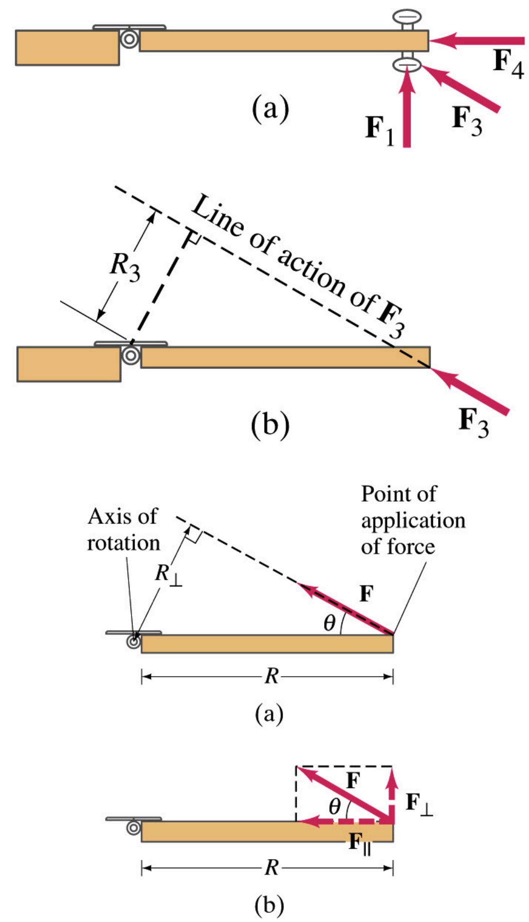
- The kinetic energy is proportional to the square of the rotational velocity ω. Note: the equation is similar to the translational kinetic energy (1/2 mv^2) except that instead of being proportional to the the mass m of the object , the rotational kinetic energy is proportional to the **moment of inertia *I*** of the object:

$$I = \sum_i m_i r_i^2 \quad \Longrightarrow \quad K = \frac{1}{2} I \omega^2$$

Note: units of I: kg m^2

Torque.
A quick review.

- In general, the torque associated with a force F is equal to
$$|\vec{\tau}| = rF\sin\phi = |\vec{r}\times\vec{F}|$$
- The arm of the force (also called the moment arm) is defined as $r\sin\phi$. The arm of the force is the perpendicular distance of the axis of rotation from the line of action of the force.
- If the arm of the force is 0, the torque is 0, and there will be no rotation.
- The maximum torque is achieved when the angle ϕ is 90°.



Torque.
A quick review.

- The torque τ of the force F is related to the angular acceleration α:

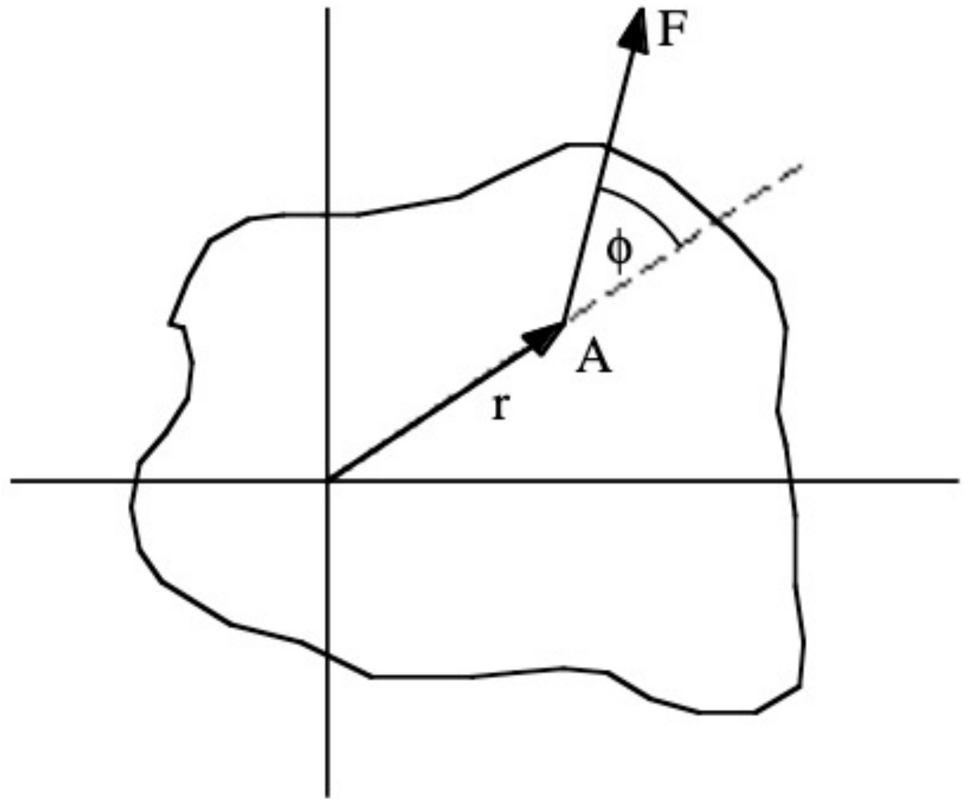
$$\vec{\tau} = I\vec{\alpha}$$

- This equation looks similar to Newton's second law for linear motion:

$$\vec{F} = m\vec{a}$$

- Corresponding variables for linear and rotation al motion:

linear	rotational
mass m	moment I
force $\vec{F}$	torque $\vec{\tau}$



The direction of the torque defines the direction of the angular acceleration.

Rolling motion.
A quick review.

- Rolling motion is a combination of translational and rotational motion.

- The kinetic energy of rolling motion has thus two contributions:

  - Translational kinetic energy:

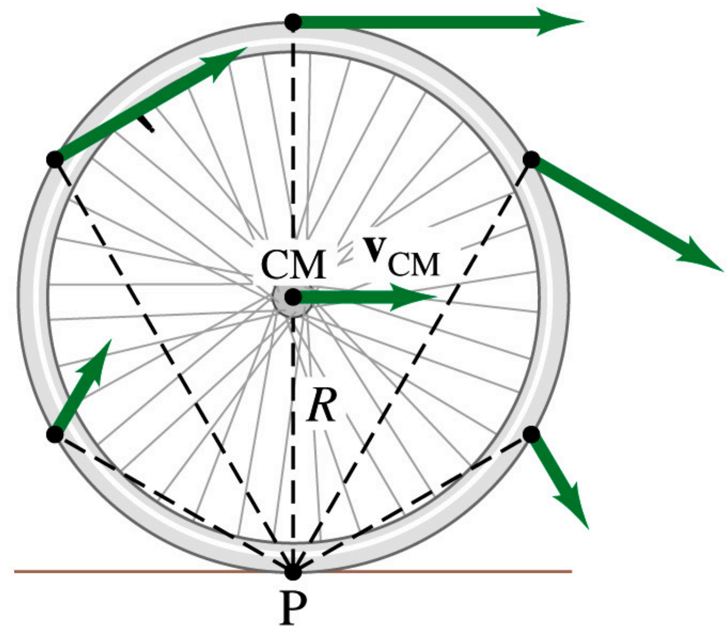
    $$\frac{1}{2}Mv_{cm}^2$$

  - Rotational kinetic energy:

    $$\frac{1}{2}I_{cm}\omega^2$$

- Assuming the wheel does not slip:

$$\omega = \frac{v}{R}$$



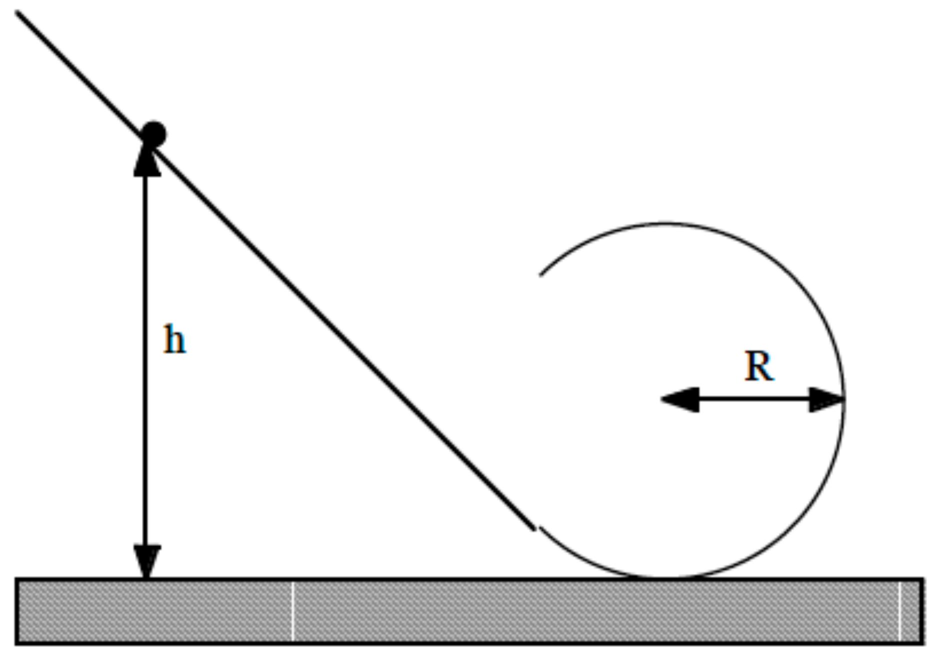
How different is a world with rotational motion? Sample problem.

- Consider the loop-to-loop. What height h is required to make it to the top of the loop?
- First consider the case without rotation:
  - Initial mechanical energy: mgh.
  - Minimum velocity at the top of the loop is determine by requiring that $mv^2/R > mg$ or $v^2 > gR$.
  - The mechanical energy at the top of the loop is thus equal to

$$\frac{1}{2}mv^2 + 2mgR > \frac{5}{2}mgR$$

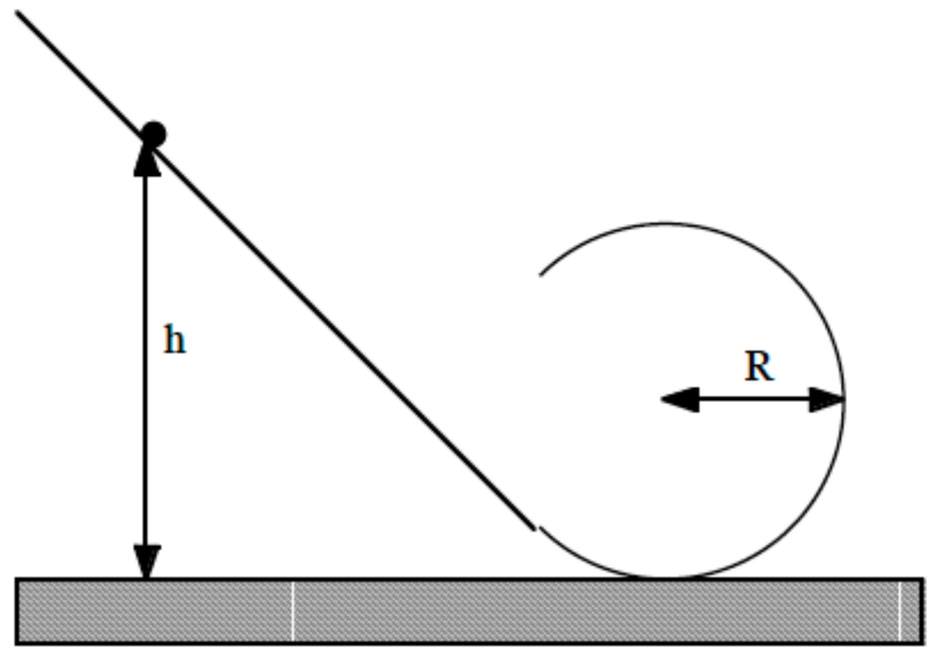
  - Conservation of energy requires

$$h > \frac{5}{2}R$$



How different is a world with rotational motion? Sample problem.

- What changes when the object rotates?
  - The minimum velocity at the top of the loop will not change.
  - The minimum translational kinetic energy at the top of the loop will not change.
  - But in addition to translational kinetic energy, there is now also rotational kinetic energy.
  - The minimum mechanical energy is at the top of the loop has thus increased.
  - The required minimum height must thus have increased.
- OK, let's now calculate by how much the minimum height has increased.



How different is a world with rotational motion? Sample problem.

- The total kinetic energy at the top of the loop is equal to

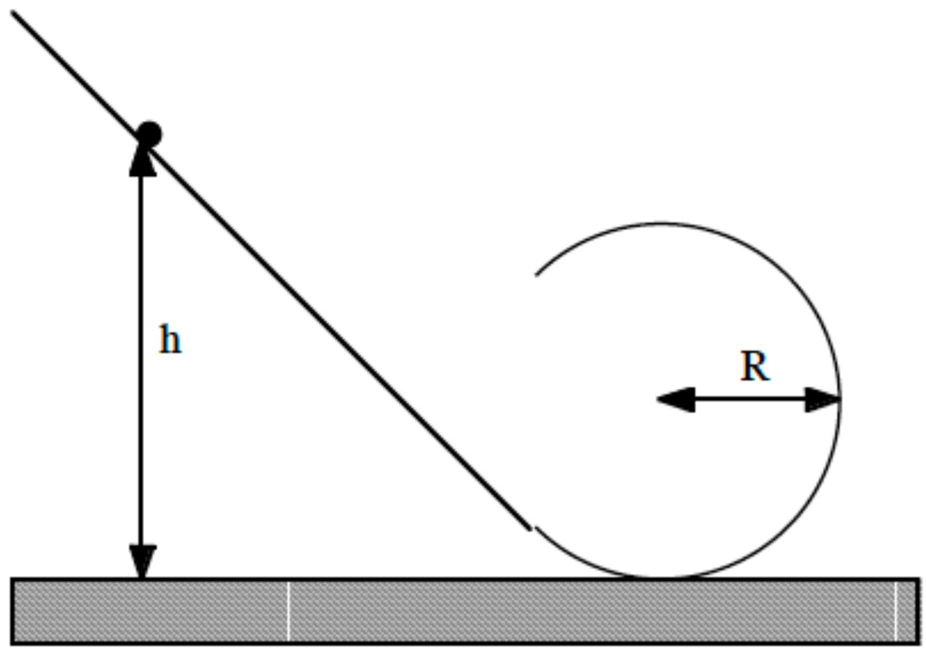
$$K_f = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2 = \frac{1}{2}\left(\frac{I}{r^2} + M\right)v^2$$

- This expression can be rewritten as

$$K_f = \frac{1}{2}\left(\frac{2}{5}M + M\right)v^2 = \frac{7}{10}Mv^2$$

- This is the minimum mechanical energy required to reach this point, and we can determine the minimum height:

$$h \geq \frac{27}{10}R$$



Note: without rotation $h \geq 25/10\ R$!!!

3 Minute 52 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 3 minute 52 second intermission.

- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.

Quiz.

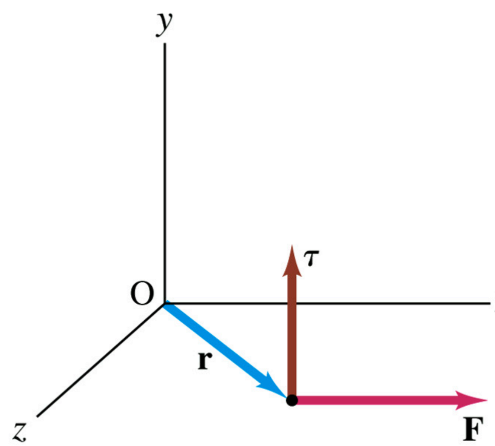
- Let us practice what we have learned.

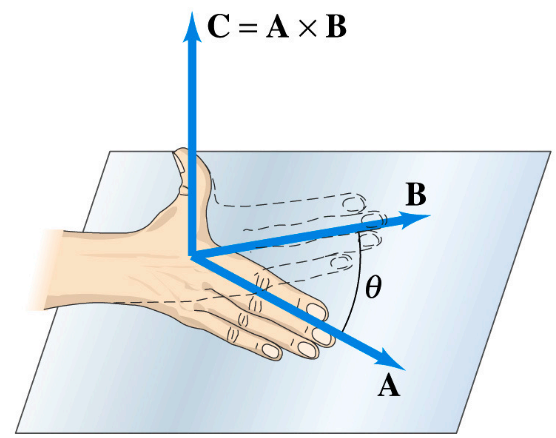
Torque.

- The torque associated with a force is a vector. It has a magnitude and a direction.

- The direction of the torque can be found by using the right-hand rule to evaluate $\vec{r} \times \vec{F}$.

- For extended objects, the total torque is equal to the vector sum of the torque associated with each "component" of this object.





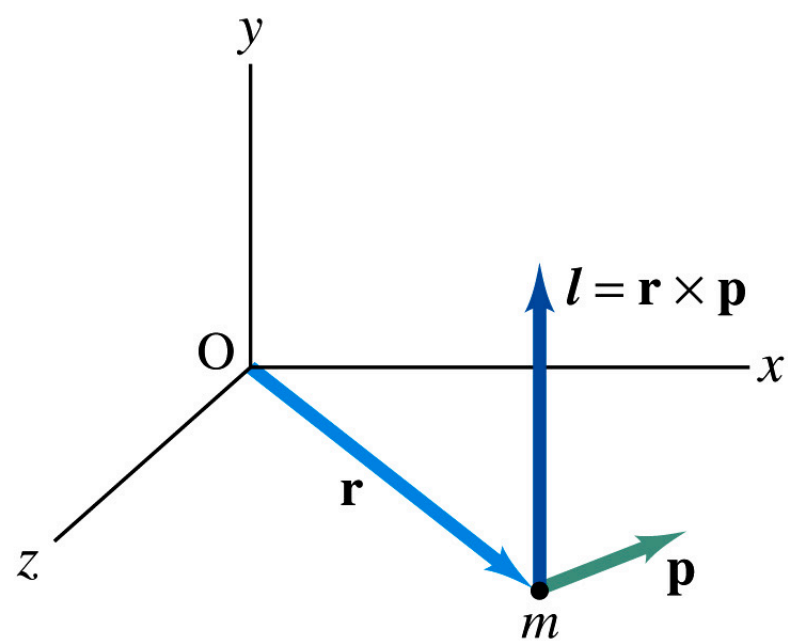
Angular momentum.

- We have seen many similarities between the way in which we describe linear and rotational motion.
- Our treatment of these types of motion are similar if we recognize the following equivalence:

linear	rotational
mass m	moment I
force $\vec{F}$	torque $\vec{\tau} = \vec{r} \times \vec{F}$

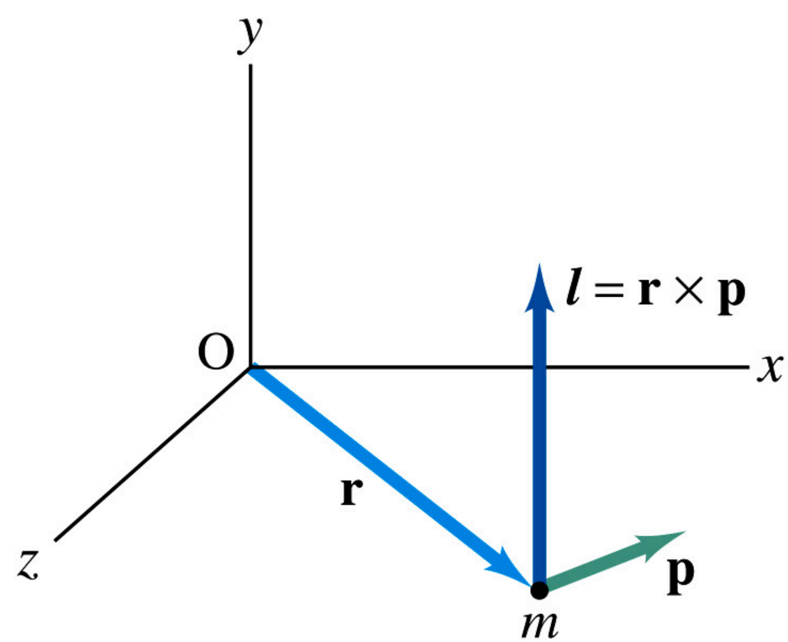
- What is the equivalent to linear momentum?

  Answer: **angular momentum**.



Angular momentum.

- The **angular momentum** is defined as the vector product between the position vector and the linear momentum.
- Note:
  - Compare this definition with the definition of the torque.
  - Angular momentum is a vector.
  - The unit of angular momentum is kg m^2/s.
  - The angular momentum depends on both the magnitude and the direction of the position and linear momentum vectors.
  - Under certain circumstances the angular momentum of a system is conserved!

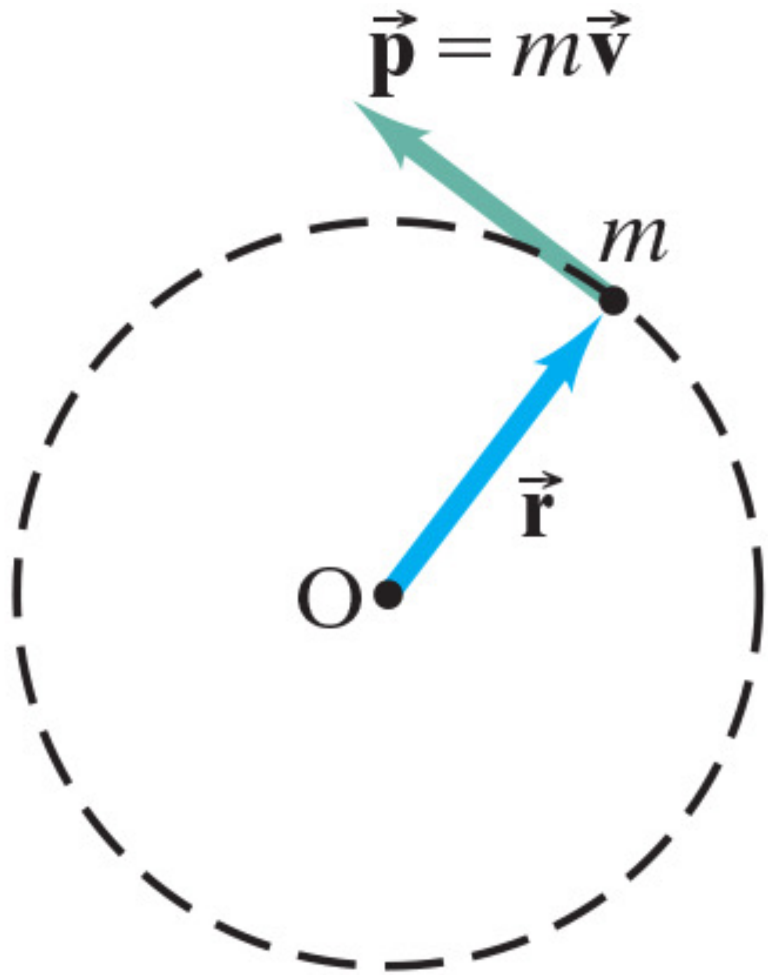


Angular momentum.

- Consider an object carrying out circular motion.
- For this type of motion, the position vector will be perpendicular to the momentum vector.
- The magnitude of the angular momentum is equal to the product of the magnitude of the radius $\vec{r}$ and the linear momentum $\vec{p}$:

$$L = mvr = mr^2 \frac{v}{r} = I\omega$$

- Note: compare this with the linear momentum $p = mv$!

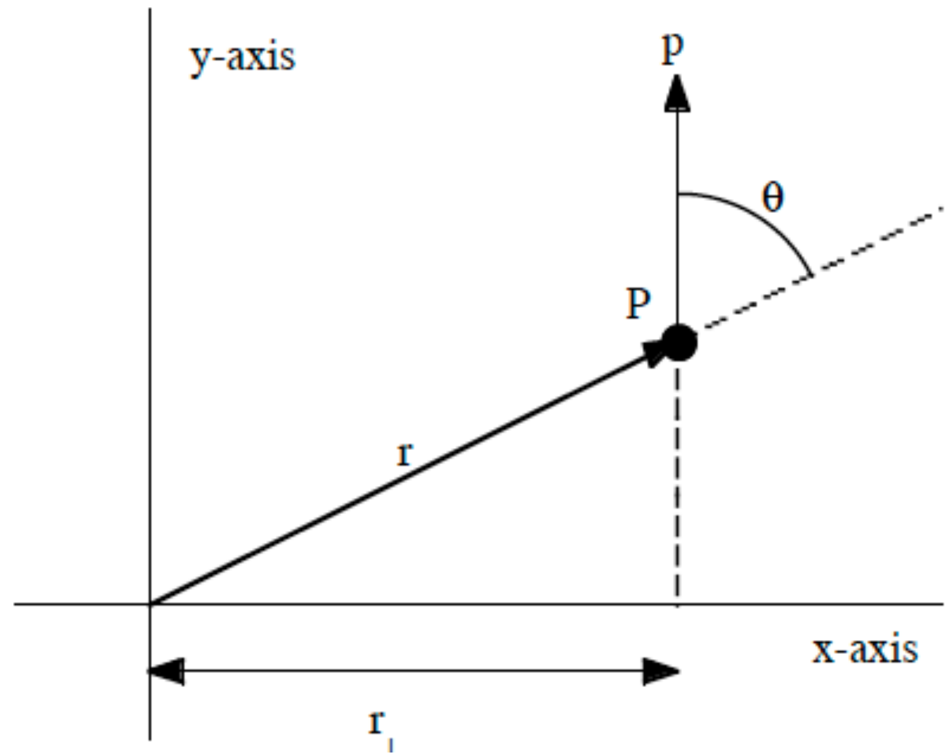


Angular momentum.

- An object does not need to carry out rotational motion to have an angular moment.
- Consider a particle P carrying out linear motion in the xy plane.
- The angular momentum of P (with respect to the origin) is equal to

$$\vec{L} = \vec{r} \times \vec{p} = mrv\sin\theta\hat{k} = pr_{\perp}\hat{k}$$

and will be constant (if the linear momentum is constant).



Conservation of angular momentum.

- Consider the change in the angular momentum of a particle:

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r}\times\vec{p}) = m\left(\vec{r}\times\frac{d\vec{v}}{dt} + \frac{d\vec{r}}{dt}\times\vec{v}\right) = m(\vec{r}\times\vec{a} + \vec{v}\times\vec{v})$$

$$\frac{d\vec{L}}{dt} = \vec{r}\times m\vec{a} = \vec{r}\times\sum\vec{F} = \sum\vec{\tau}$$

- Consider what happens when the net torque is equal to 0:

$$\frac{d\vec{L}}{dt} = 0 \implies L = \text{constant}$$

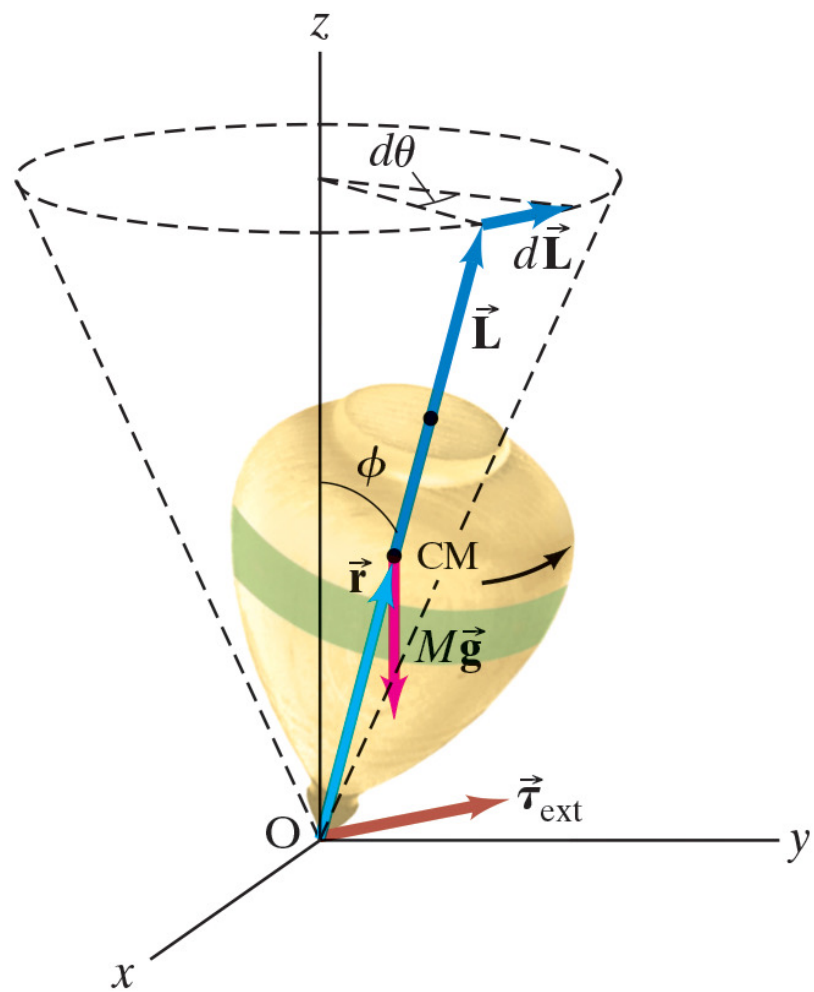
- When we take the sum of all torques, the torques due to the internal forces cancel and the sum is equal to torque due to all external forces.
- Note: notice again the similarities between linear and rotational motion.

Conservation of angular momentum.

- The connection between the angular momentum $\vec{L}$ and the torque $\vec{\tau}$

$$\sum\vec{\tau} = \frac{d\vec{L}}{dt}$$

  is only true if $\vec{L}$ and $\vec{\tau}$ are calculated with respect to the same reference point (which is at rest in an inertial reference frame).
- The relation is also true if $\vec{L}$ and $\vec{\tau}$ are calculated with respect to the center of mass of the object (note: center of mass can accelerate).



Conservation of angular momentum.

- Ignoring the mass of the bicycle wheel, the external torque will be close to zero if we use the center of the disk as our reference point.
- Since the external torque is zero, angular momentum thus should be conserved.
- I can change the orientation of the wheel by applying internal forces. In which direction will I need to spin to conserve angular momentum?

Quiz and concept test.

- Let us practice what we have learned.

Done for today!
More about rotational motion on Thursday.

The Orion Nebula from CFHT
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