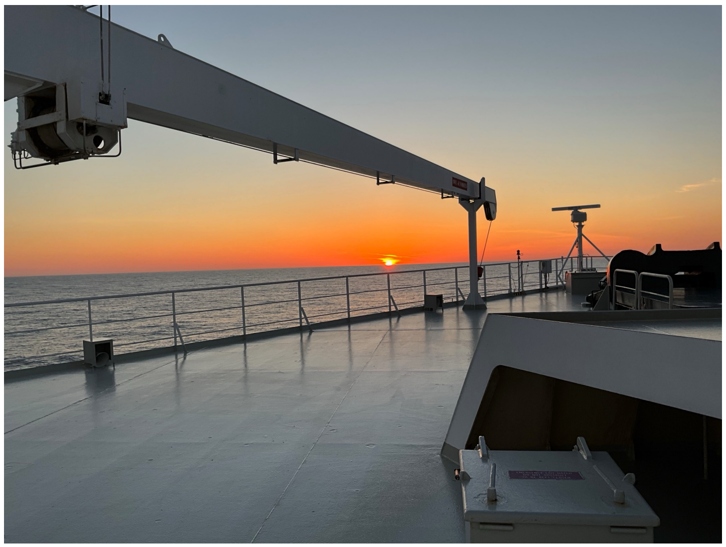
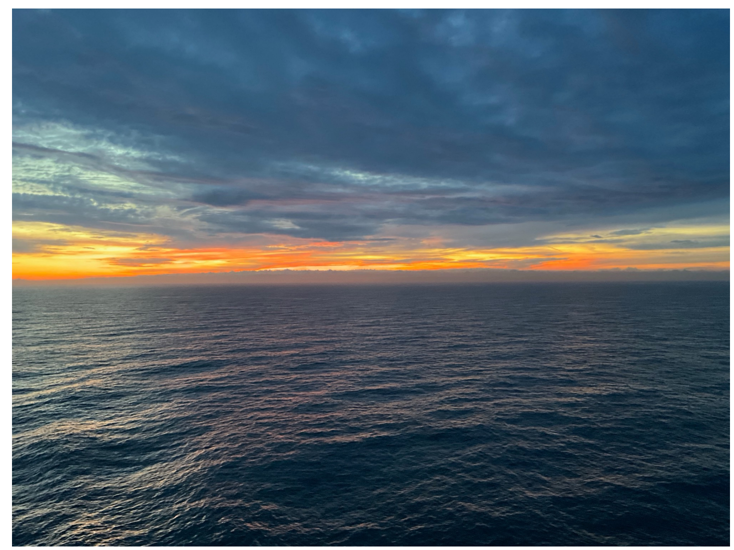
Physics 121.
Lecture 16.





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Physics 121.
Lecture 16.

- Course Information
- Quiz
- Topics to be discussed today:
  - Rotational Variables (Review)
  - Torque
  - Rolling Motion

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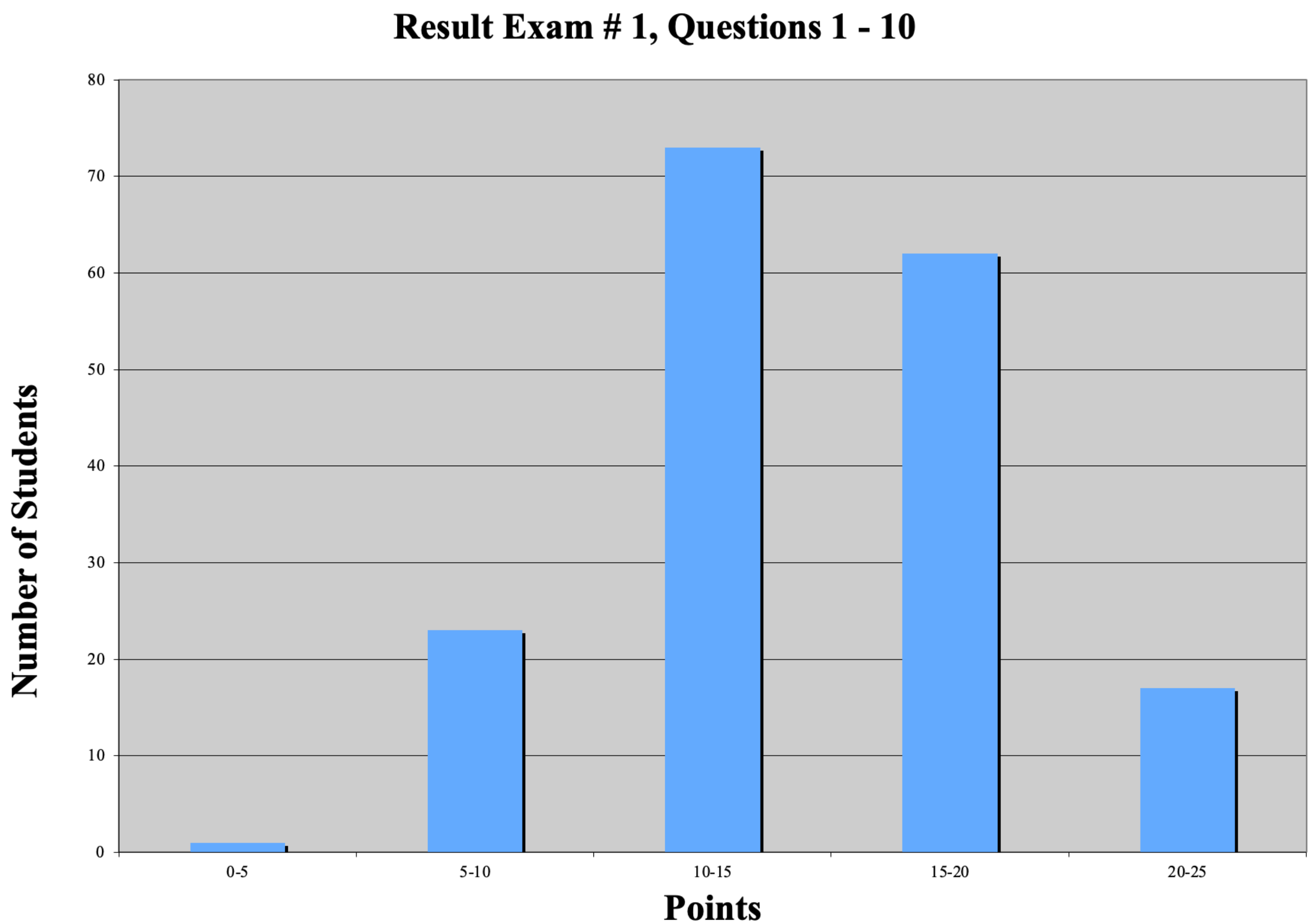
Physics 121.
Lecture 16.

- Homework set # 6 is now available on the WEB and will be due on Saturday morning, April 4, at 8.30 am.
- There will be no workshops or office hours today and tomorrow.
- Exam 2 will be returned during workshops next week.
- Three students did not enter a student ID on the scantron form. These students will receive a 0 for Questions 1 – 10.

First result of Exam # 2: Questions 1 – 10.

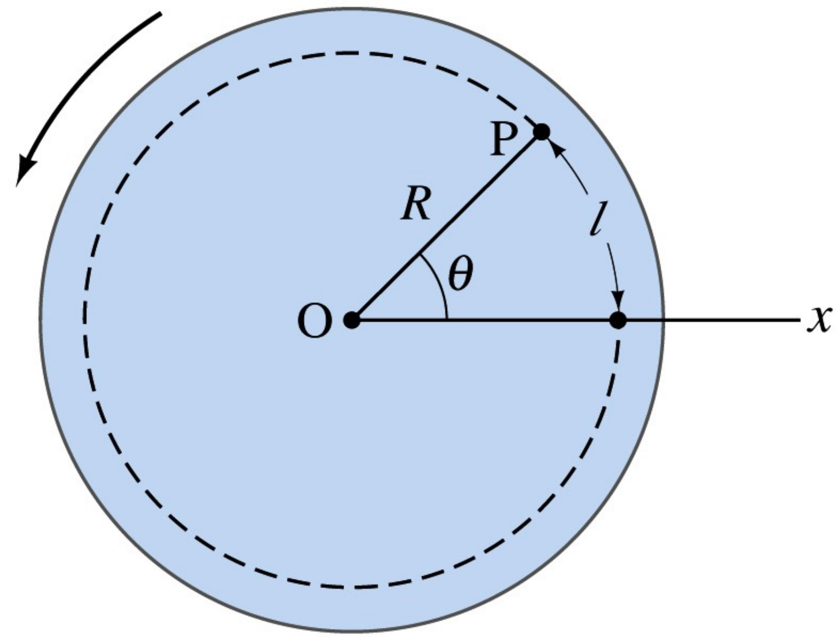


Quiz.

- Practice what we have learned.

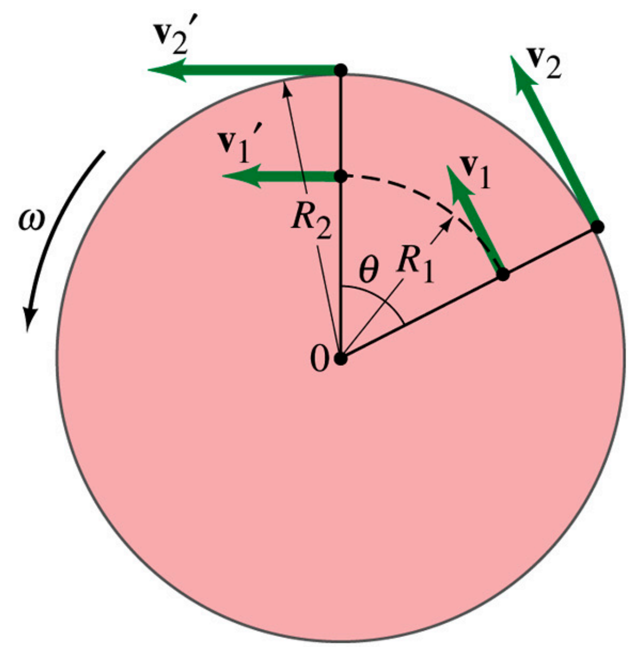
Rotational variables. A quick review.

- The variables that are used to describe rotational motion are:
  - Angular position θ
  - Angular velocity $\omega = d\theta/dt$
  - Angular acceleration $\alpha = d\omega/dt$
- The rotational variables are related to the linear variables:
  - Linear position $l = R\theta$
  - Linear velocity $v = R\omega$
  - Linear acceleration $a = R\alpha$



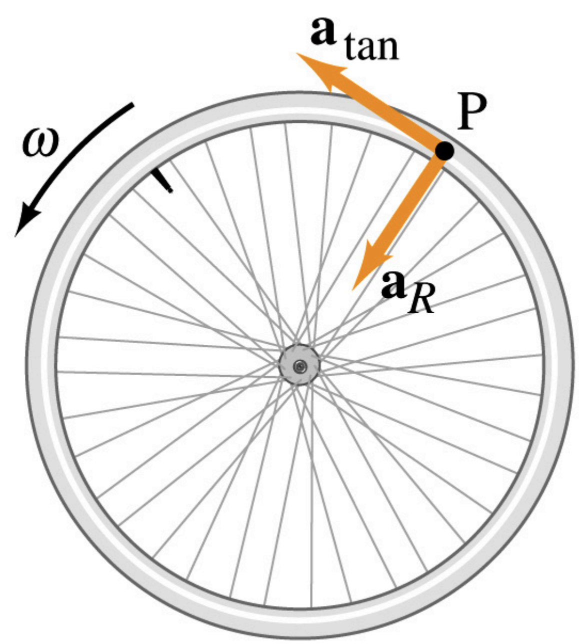
Rotational variables.
A quick review.

- Things to consider when looking at the rotation of rigid objects around a fixed axis:
  - Each part of the rigid object has the same angular velocity.
  - Only those parts that are located at the same distance from the rotation axis have the same linear velocity.
  - The linear velocity of parts of the rigid object increases with increasing distance from the rotation axis.



Rotational variables.
A quick review.

- Note: the acceleration $a_t = r\alpha$ is only one of the two component of the acceleration of point P. The two components of the acceleration of point P are:
  - The radial component: this component is always present since point P carried out circular motion around the axis of rotation.
  - The tangential component: this component is present only when the angular acceleration is not equal to 0 rad/s^2.



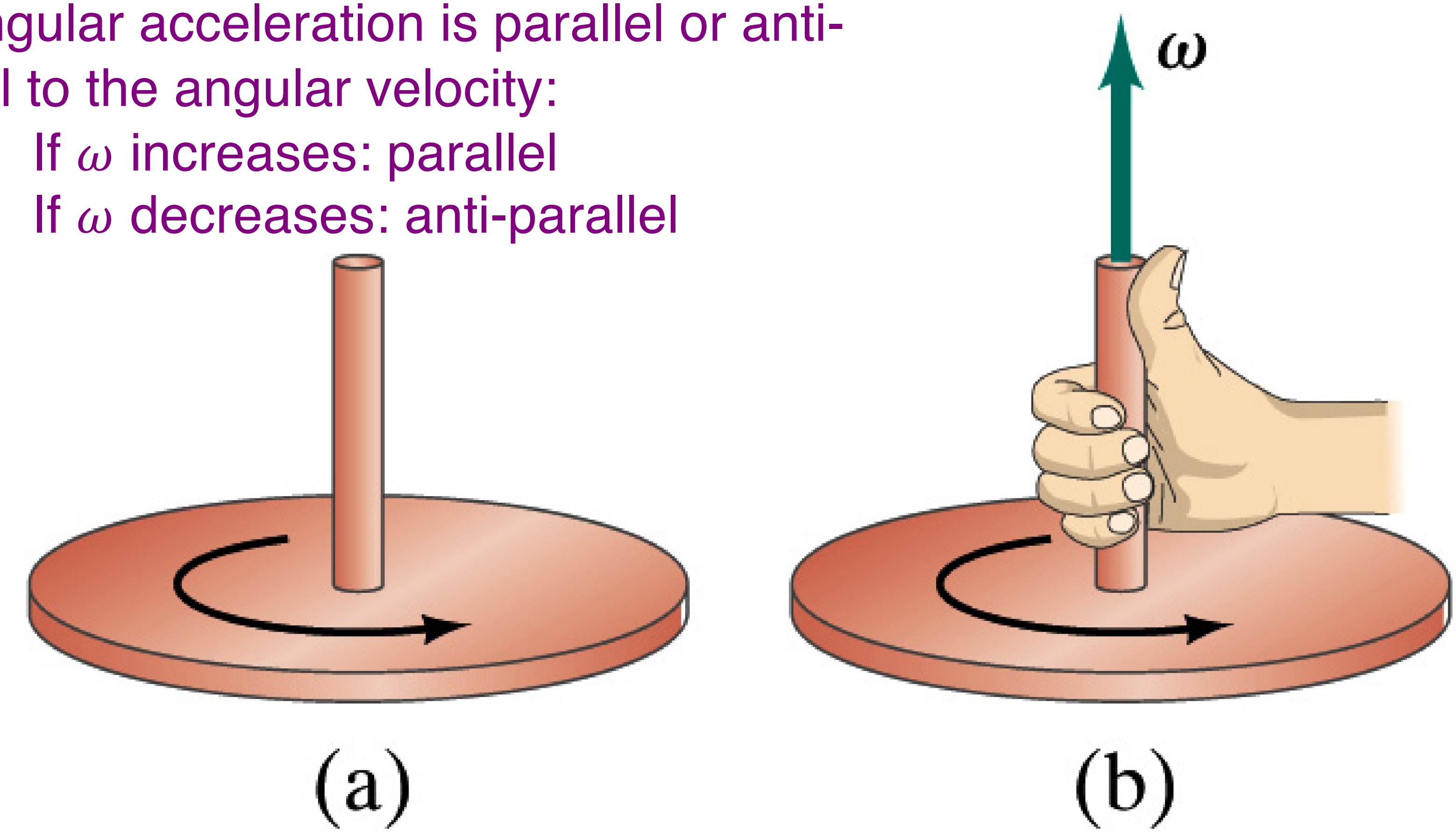
Rotational variables.
A quick review.

Angular velocity and acceleration are vectors! They have a magnitude and a direction. The direction of ω is found using the right-hand rule.

The angular acceleration is parallel or anti-parallel to the angular velocity:

If ω increases: parallel

If ω decreases: anti-parallel



Rotational kinetic energy.

- Since the components of a rotating object have a non-zero (linear) velocity we can associate a kinetic energy with the rotational motion:

$$K = \sum_i \frac{1}{2} m_i v_i^2 = \sum_i \frac{1}{2} m_i (\omega r_i)^2 = \frac{1}{2} \left(\sum_i m_i r_i^2 \right) \omega^2$$

- The kinetic energy is proportional to the square of the rotational velocity ω. Note: the equation is similar to the translational kinetic energy ($1/2\ mv^2$) except that instead of being proportional to the the mass m of the object , the rotational kinetic energy is proportional to the **moment of inertia *I*** of the object:

$$I = \sum_i m_i r_i^2 \quad \Longrightarrow \quad K = \frac{1}{2} I \omega^2$$

Note: units of I: kg m^2

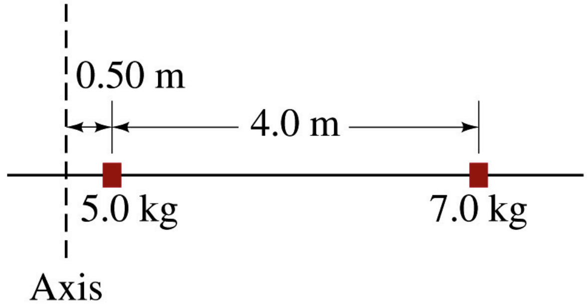
The moment of inertia. Calculating *I*.

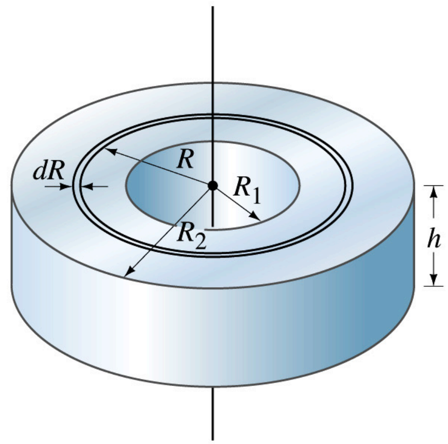
- The moment of inertia of an objects depends on the mass distribution of object and on the location of the rotation axis.
- For discrete mass distribution it can be calculated as follows:

$$I = \sum_i m_i r_i^2$$

- For continuous mass distributions we need to integrate over the mass distribution:

$$I = \int r^2 dm$$



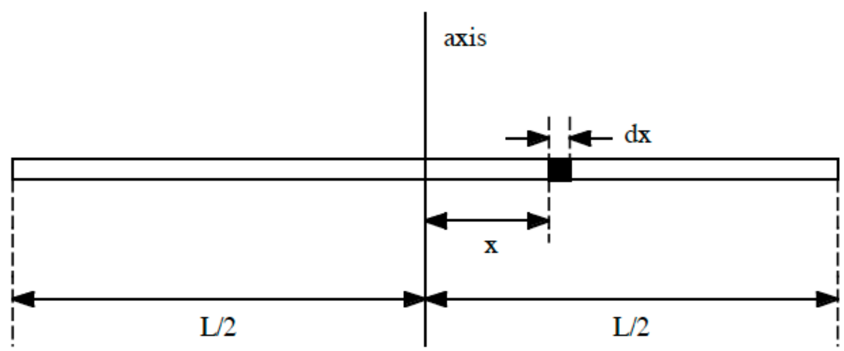


Moment of inertia. Sample problem.

- Consider a rod of length L and mass m. What is the moment of inertia with respect to an axis through its center of mass?

- Consider a slice of the rod, with width dx, located a distance x from the rotation axis. The mass dm of this slice is equal to

$$dm = \frac{m}{L} dx$$

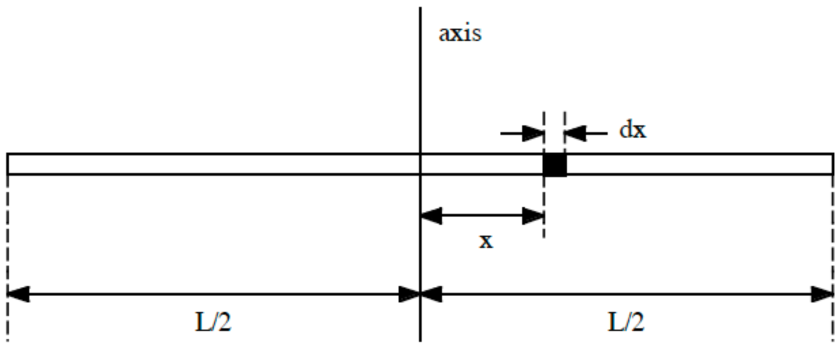


Moment of inertia. Sample problem.

- The moment of inertia dI of this slice is equal to

$$dI = x^2 dm = \frac{m}{L} x^2 dx$$

- The moment of inertia of the rod can be found by adding the contributions of all of the slices that make up the rod:



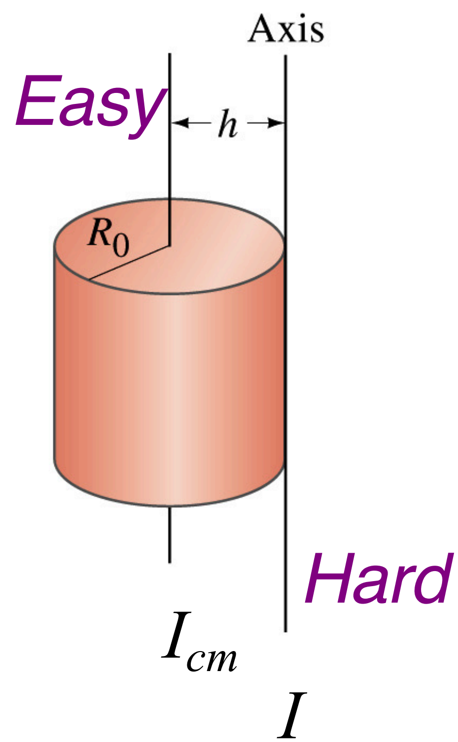
$$I = \int_{-L/2}^{L/2} dI = \int_{-L/2}^{L/2} \frac{m}{L} x^2 dx =$$

$$= \frac{m}{L}\left\{\frac{1}{3}\left(\frac{L}{2}\right)^2 - \frac{1}{3}\left(-\frac{L}{2}\right)^2\right\} = \frac{1}{12} mL^2$$

Moment of inertia. Parallel-axis theorem.

- Calculating the moment of inertial with respect to a symmetry axis of the object is in general easy.
- It is much harder to calculate the moment of inertia with respect to an axis that is not a symmetry axis.
- However, we can make a hard problem easier by using the parallel-axis theorem:

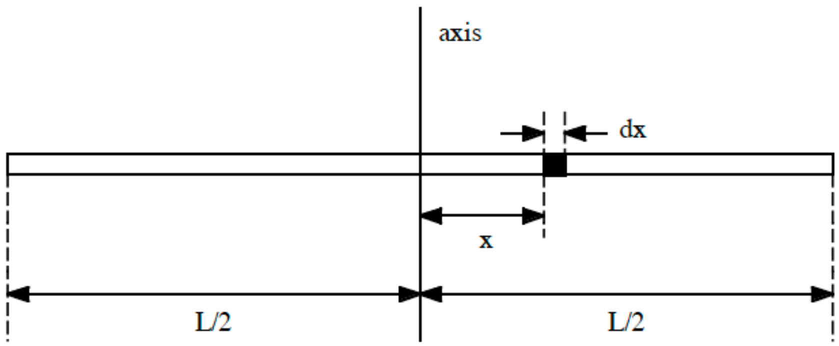
$$I = I_{cm} + Mh^2$$



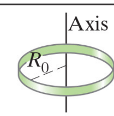
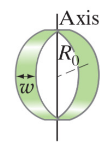
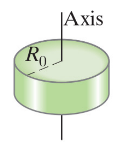
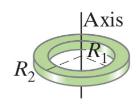
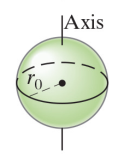
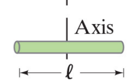
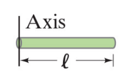
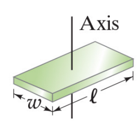
Moment of inertia. Sample problem.

- Consider a rod of length L and mass m. What is the moment of inertia with respect to an axis through its left corner?
- We have determined the moment of inertia of this rod with respect to an axis through its center of mass. We use the parallel-axis theorem to determine the moment of inertia with respect to the current axis:

$$I = I_{cm} + m\left(\frac{L}{2}\right)^2 = \frac{1}{3}mL^2$$



Moment of inertia. Do not memorize them! You will get Figure 10.21 on our exams!

	Object	Location of axis		Moment of inertia
(a)	**Thin hoop**, radius R_0	Through center		MR_0^2
(b)	**Thin hoop**, radius R_0 width w	Through central diameter		$\frac{1}{2}MR_0^2 + \frac{1}{12}Mw^2$
(c)	**Solid cylinder**, radius R_0	Through center		$\frac{1}{2}MR_0^2$
(d)	**Hollow cylinder**, inner radius R_1 outer radius R_2	Through center		$\frac{1}{2}M(R_1^2 + R_2^2)$
(e)	**Uniform sphere**, radius r_0	Through center		$\frac{2}{5}Mr_0^2$
(f)	**Long uniform rod**, length ℓ	Through center		$\frac{1}{12}M\ell^2$
(g)	**Long uniform rod**, length ℓ	Through end		$\frac{1}{3}M\ell^2$
(h)	**Rectangular thin plate**, length ℓ, width w	Through center		$\frac{1}{12}M(\ell^2 + w^2)$

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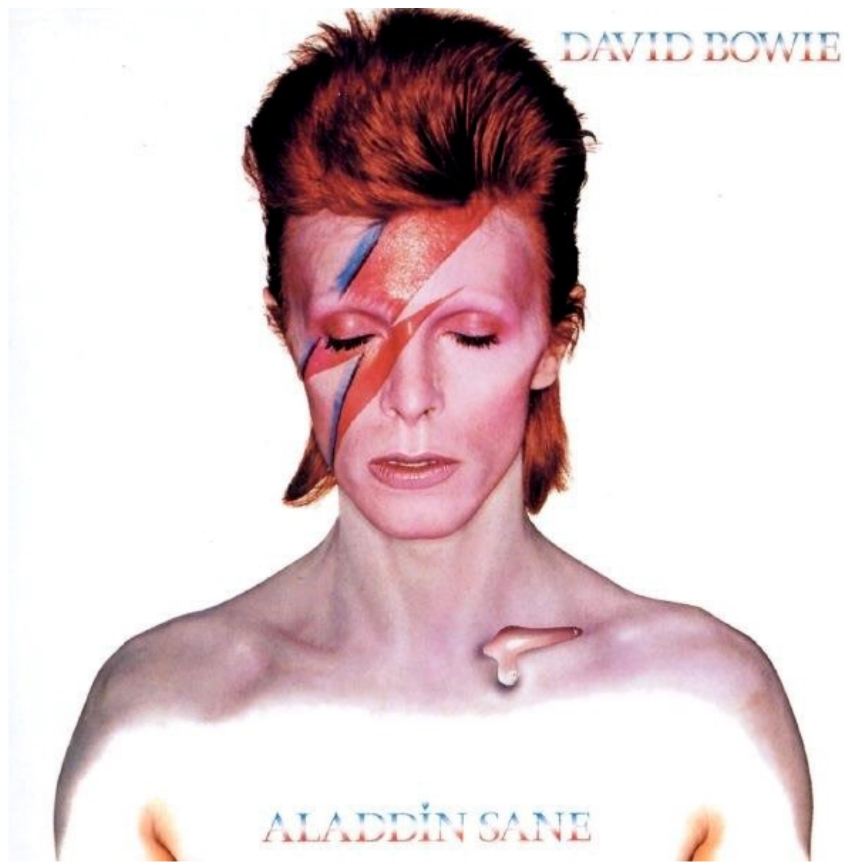
4 Minute 9 Second Intermission.

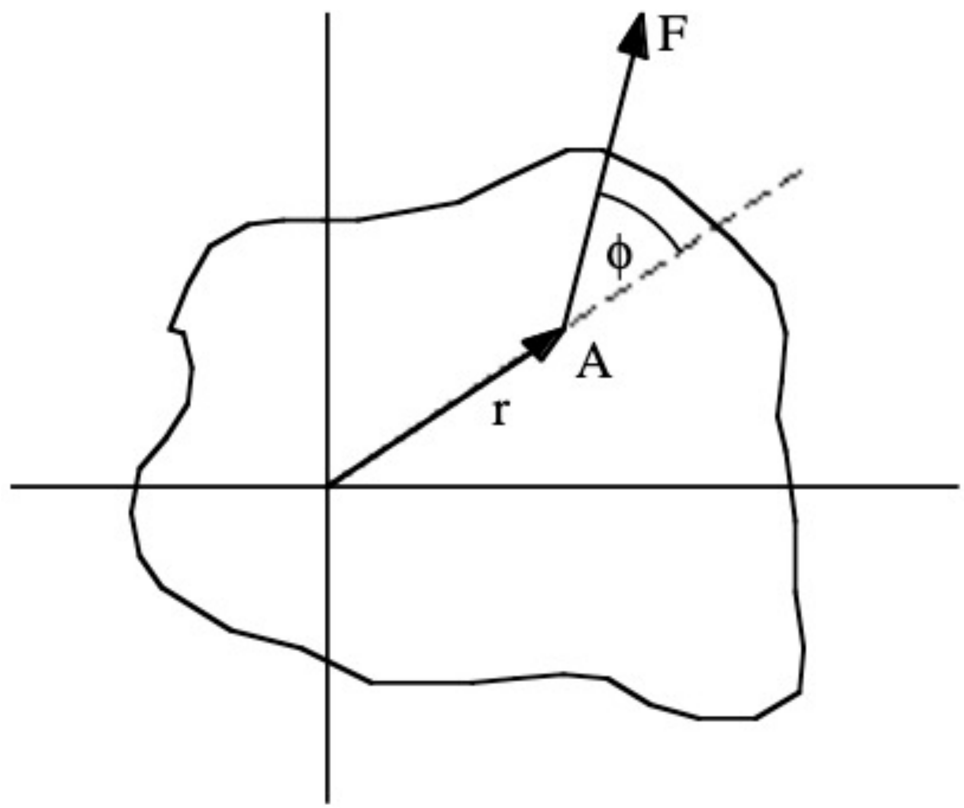
- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 4 minute 9 second intermission.

- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.



Torque.

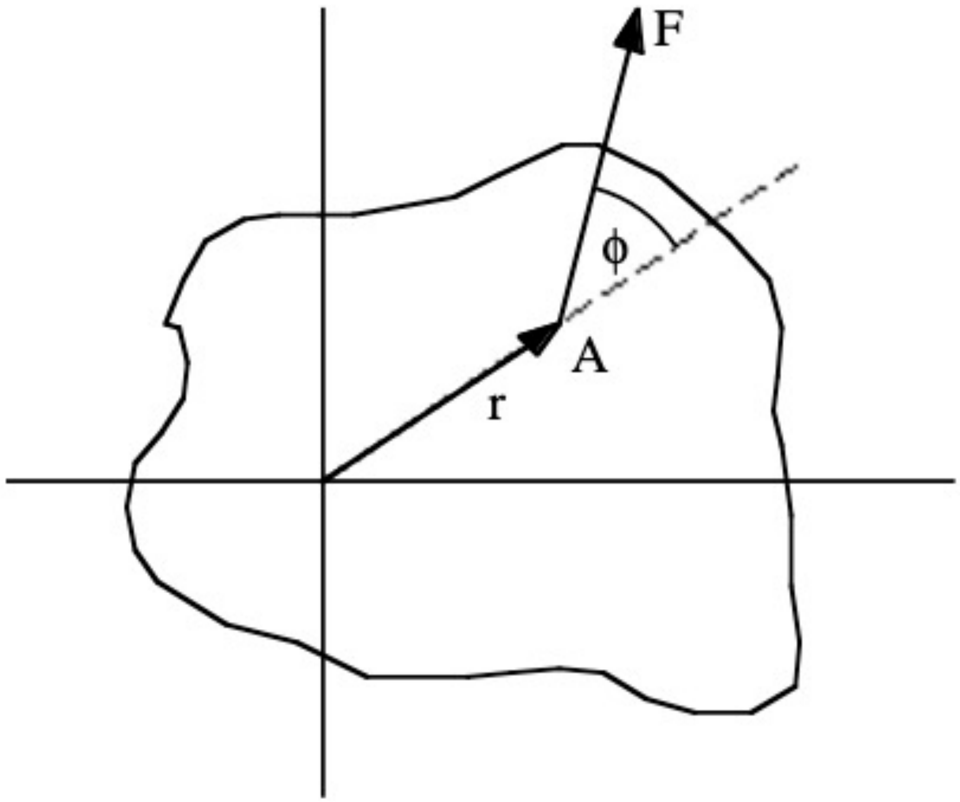
- Consider a force $\vec{F}$ applied to an object that can only rotate.
- The force $\vec{F}$ can be decomposed into two two components:
  - A radial component directed along the direction of the position vector r. The magnitude of this component is $F\cos\phi$. This component will not produce any motion.
  - A tangential component, perpendicular to the direction of the position vector r. The magnitude of this component is $F\sin\phi$. This component will result in rotational motion.



Torque.

- If a mass m is located at the position on which the force is acting (and we assume any other masses can be neglected), it will experience a linear acceleration equal to $F\sin\phi/m$.
- The corresponding angular acceleration is equal to$F\sin\phi/mr$.
- Since in rotational motion the moment of inertia plays an important role, we will rewrite the angular acceleration in terms of the moment of inertia:

$$\alpha = \frac{rF\sin\phi}{mr^2} = \frac{rF\sin\phi}{I}$$



Torque.

- Consider rewriting the previous equation in the following way:

$$rF\sin\phi = I\alpha$$

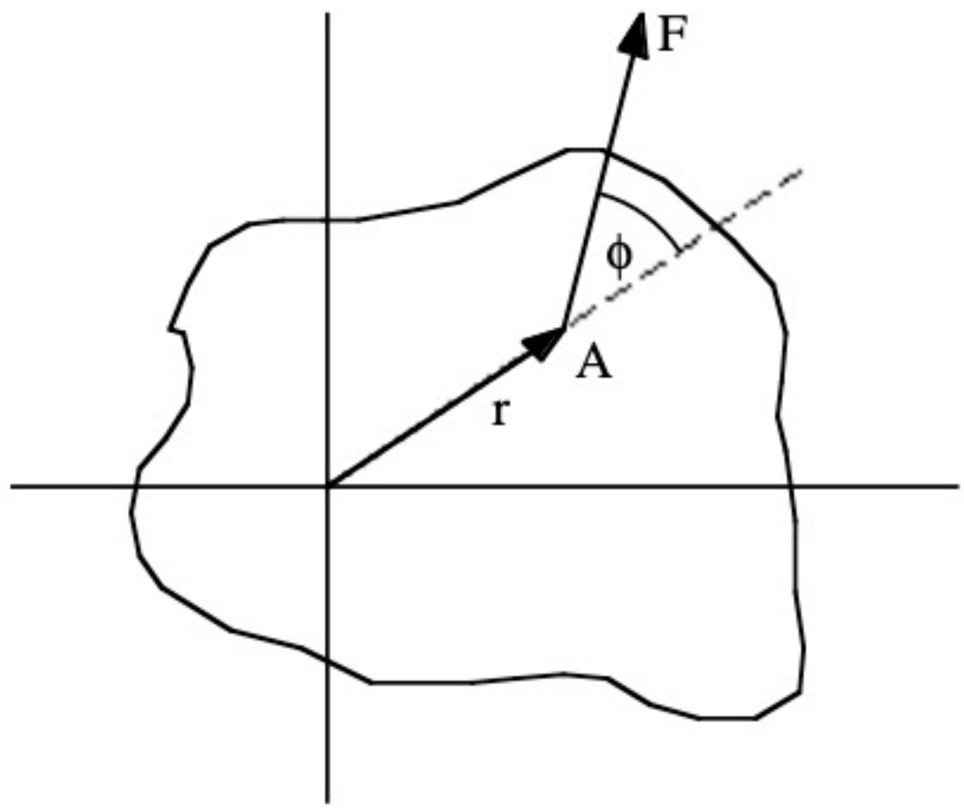
- The left-hand-side of this equation is called the torque τ of the force F:

$$\tau = I\alpha$$

- This equation looks similar to Newton's second law for linear motion:

$$F = ma$$

Linear	**Rotational**
Mass m	Moment I
Force F	Torque τ

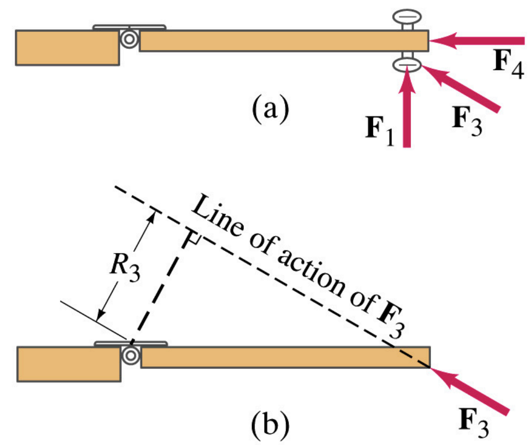


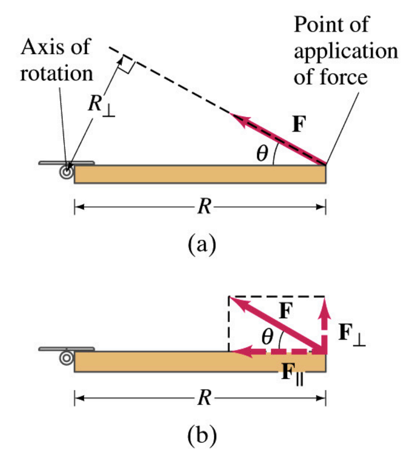
Torque.

- In general, the torque associated with a force F is equal to

$$|\vec{\tau}| = rF\sin\phi = |\vec{r}\times\vec{F}|$$

- The arm of the force (also called the moment arm) is defined as $r\sin\phi$. The arm of the force is the perpendicular distance of the axis of rotation from the line of action of the force.
- If the arm of the force is 0, the torque is 0, and there will be no rotation.
- The maximum torque is achieved when the angle ϕ is 90°.

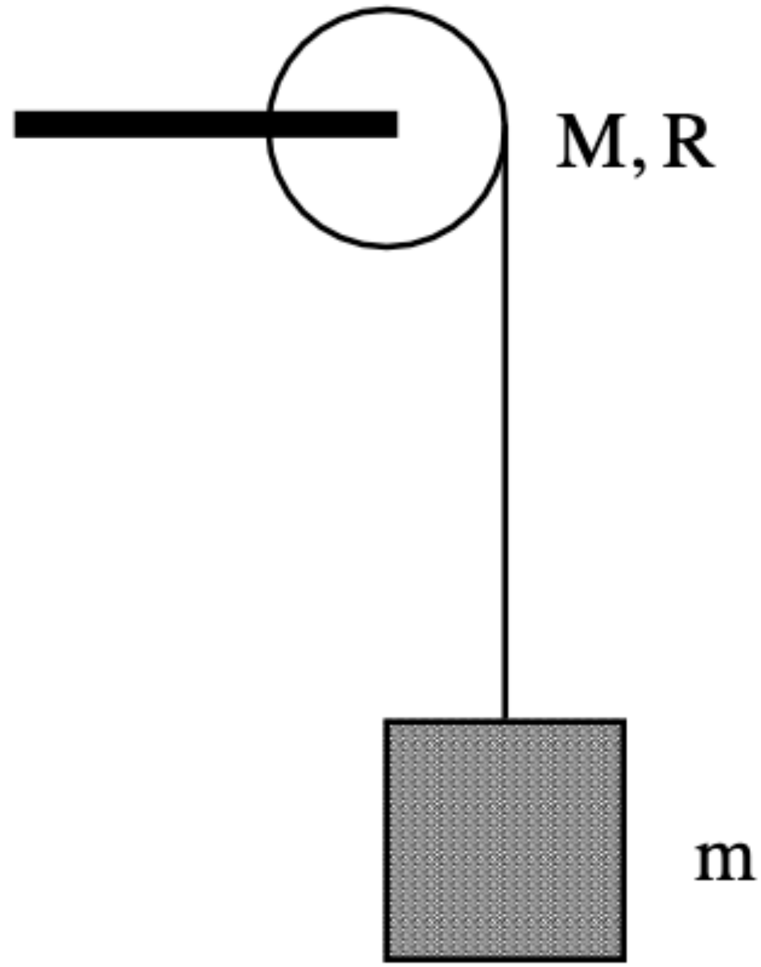




Rotational motion. Sample problem.

- Consider a uniform disk with mass M and radius R. The disk is mounted on a fixed axle. A block with mass m hangs from a light cord that is wrapped around the rim of the disk. Find the acceleration of the falling block, the angular acceleration of the disk, and the tension of the cord.

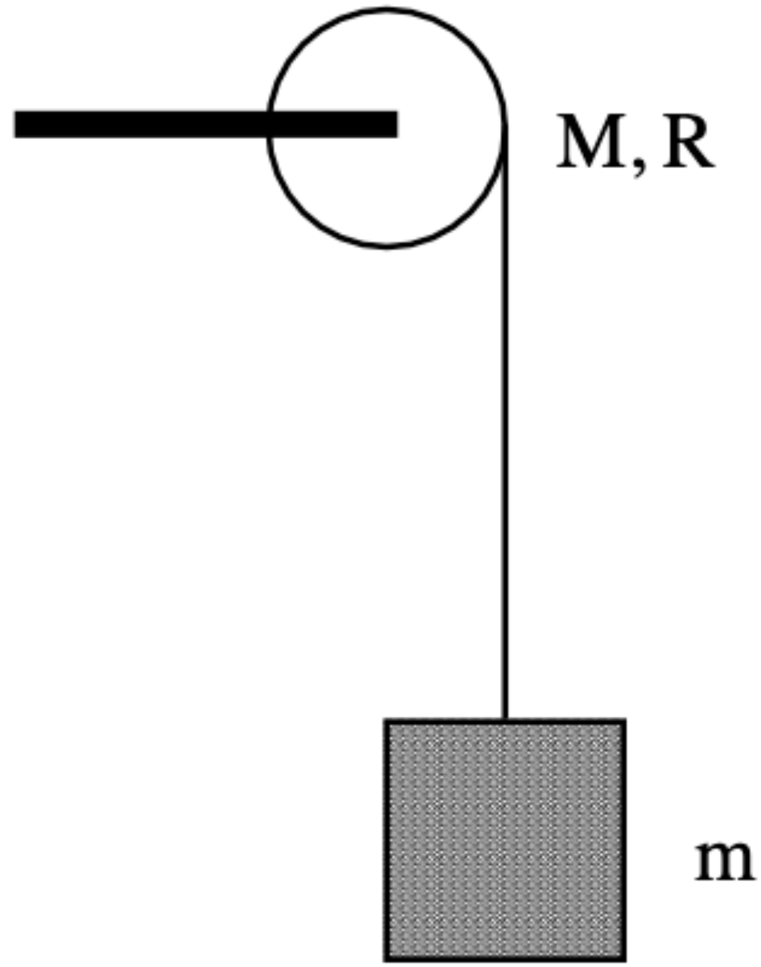
- Expectations:
  - Linear acceleration should approach g when M approaches 0 kg.



Rotational motion. Sample problem.

- Start with considering the forces and torques involved.
- Define the sign convention to be used.
- The block will move down and we choose the positive y axis in the direction of the linear acceleration.
- The net force on mass m is equal to

$$ma = mg - T$$



Rotational motion. Sample problem.

- The net torque on the pulley is equal to

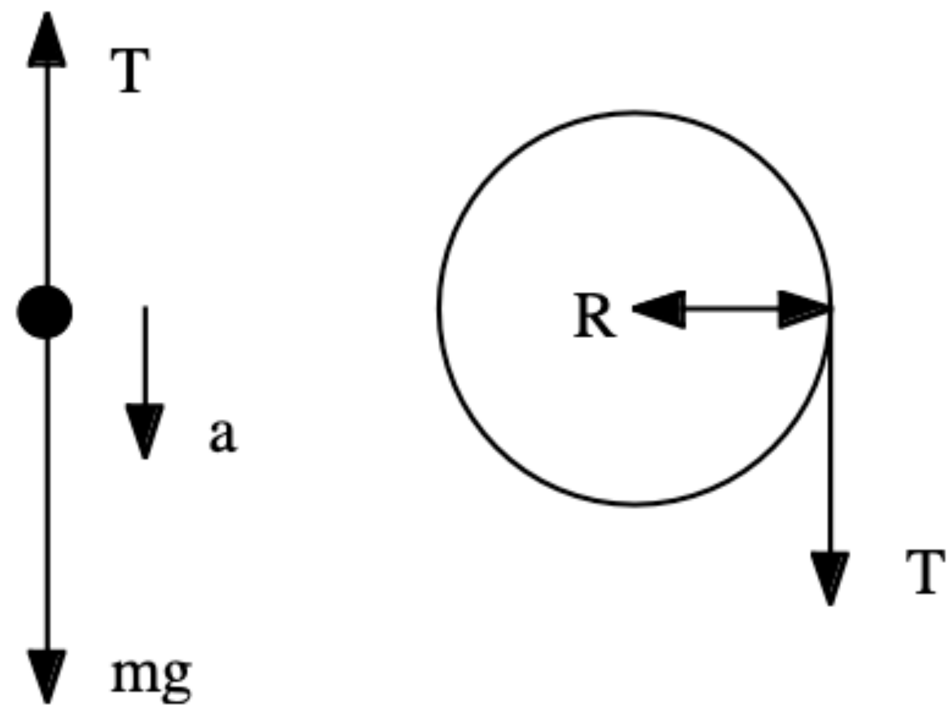
$$\tau = RT$$

- The resulting angular acceleration is equal to

$$\alpha = \frac{\tau}{I} = \frac{RT}{\frac{1}{2}MR^2} = \frac{2T}{MR}$$

- Assuming the cord is not slipping we can determine the linear acceleration:

$$a = \frac{2T}{M}$$



Rotational motion.
Sample problem.

- We now have two expressions for a:

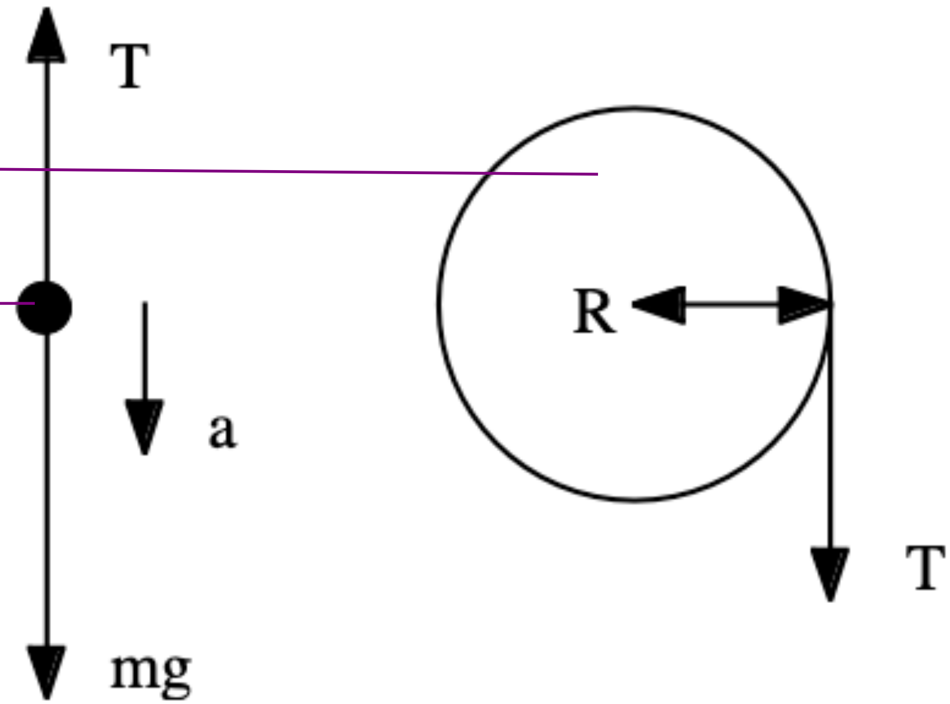
$$a = \frac{2T}{M}$$

$$a = g - \frac{T}{M}$$

- Solving these equations we find:

$$T = \frac{M}{M + 2m} mg$$

$$a = \frac{2m}{M + 2m} g$$



Note: *a* = *g* when *M* = 0 kg!!!

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Rolling motion.

- Rolling motion is a combination of translational and rotational motion.
- The kinetic energy of rolling motion has thus two contributions:
  - Translational kinetic energy:

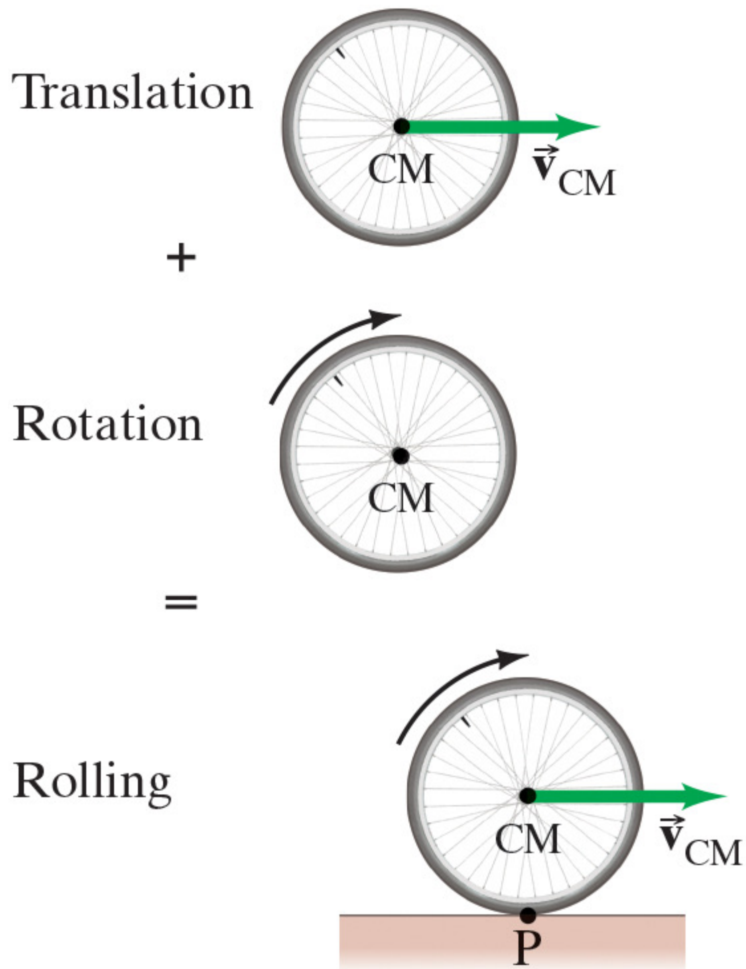
$$\frac{1}{2}Mv_{cm}^2$$

  - Rotational kinetic energy:

$$\frac{1}{2}I_{cm}\omega^2$$

- Assuming the wheel does not slip:

$$\omega = \frac{v}{R}$$



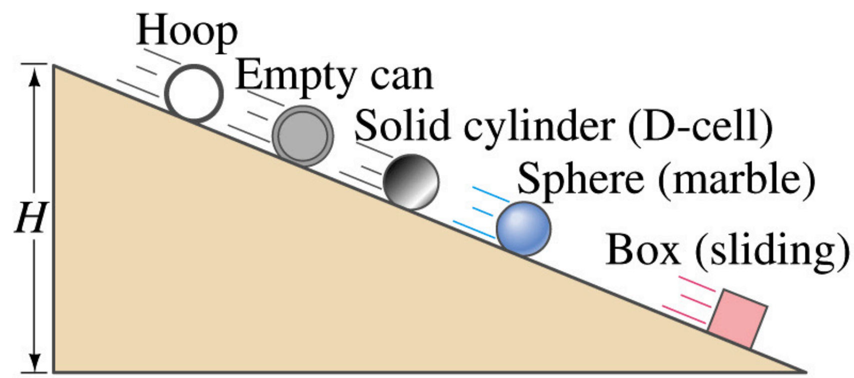
Rolling motion.

- Consider two objects of the same mass but different moments of inertia, released from rest from the top of an inclined plane:

  - Both objects have the same initial mechanical energy (assuming their CM is located at the same height).

  - At the bottom of the inclined plane, they will have both rotational and translational kinetic energy.

  - Which object will reach the bottom first?



Rolling motion.

- Initial mechanical energy:

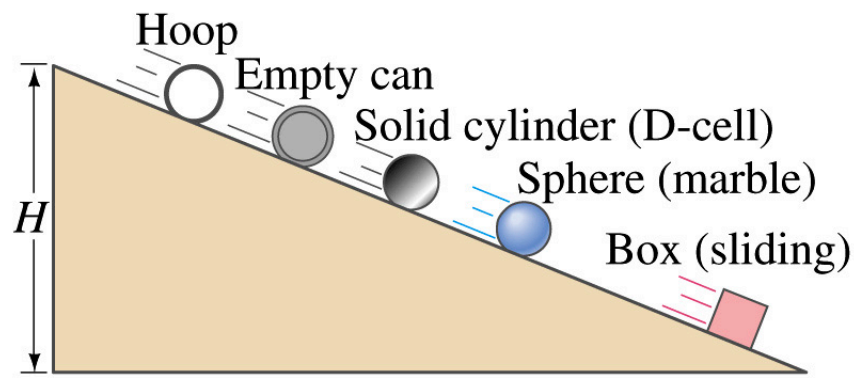
$$E_i = mgH$$

- Final mechanical energy:

$$E_f = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

- Assuming no slipping, we can rewrite the final mechanical energy as

$$E_f = \frac{1}{2}\left(m + \frac{I_{cm}}{R^2}\right) v_{cm}^2$$



Rolling motion.

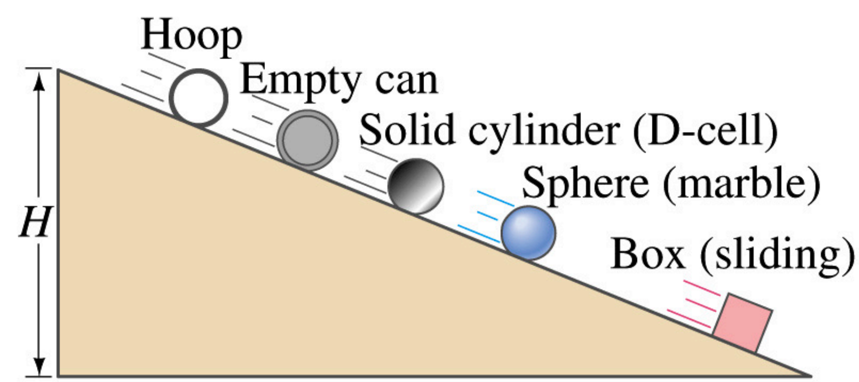
- Conservation of energy implies:

$$\frac{1}{2}\left(m+\frac{I_{cm}}{R^2}\right)v_{cm}^2=mgH$$

or

$$\frac{1}{2}\left(1+\frac{I_{cm}}{mR^2}\right)v_{cm}^2=gH$$

- The smaller I_{cm}, the larger v_{cm} at the bottom of the incline.
- Example:
  - Ring: $I_{cm}=MR^2$, $v_{cm}=\sqrt{gH}$
  - Disk: $I_{cm}=\frac{1}{2}MR^2$, $v_{cm}=\sqrt{\frac{4}{3}gH}$



Done for today!
More about rotational motion next week.

N49's Cosmic Blast
Credit: Hubble Heritage Team (STScI / AURA), Y. Chu (UIUC) et al., NASA