Physics 121.
Lecture 15.

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Lecture 15.

- Course Information
- Quiz.
- Topics to be discussed today:
  - Variables used to describe rotational motion
  - The equations of motion for rotational motion

Course Announcements.

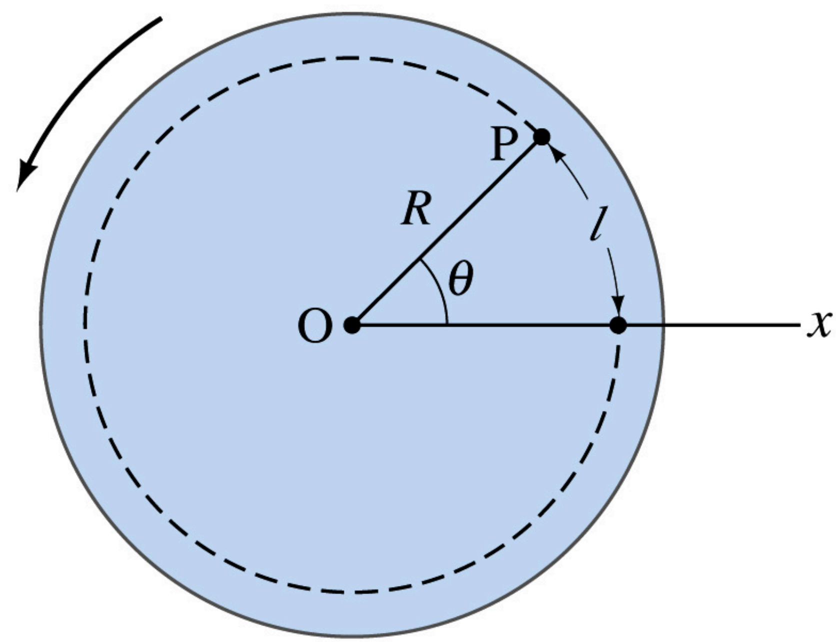
- Homework set # 6 is now available on the WEB and will be due on Saturday morning, April 4, at 8.30 am.
- There will be no additional office hours and workshops this week on Tuesday, Wednesday, Thursday, and Friday.
- Exam # 2 will be returned to you during workshops next week.

Quiz.

- Tell me about Exam # 2.

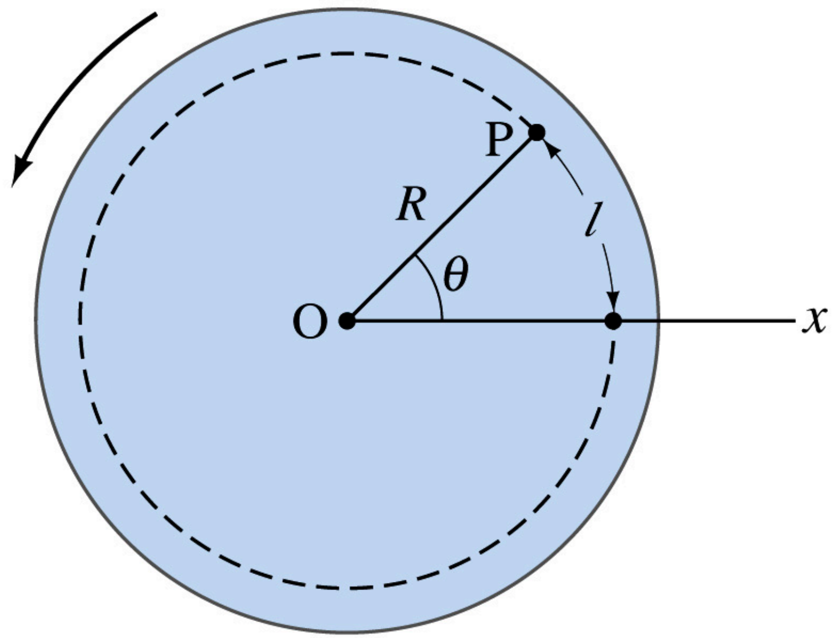
Rotational motion: variables.

- In our discussion of rotational motion we will first focus on the rotation of **rigid** objects around a **fixed** axis.
- The variables that are used to describe this type of motion are similar to those we use to describe linear motion:
  - Angular position
  - Angular velocity
  - Angular acceleration



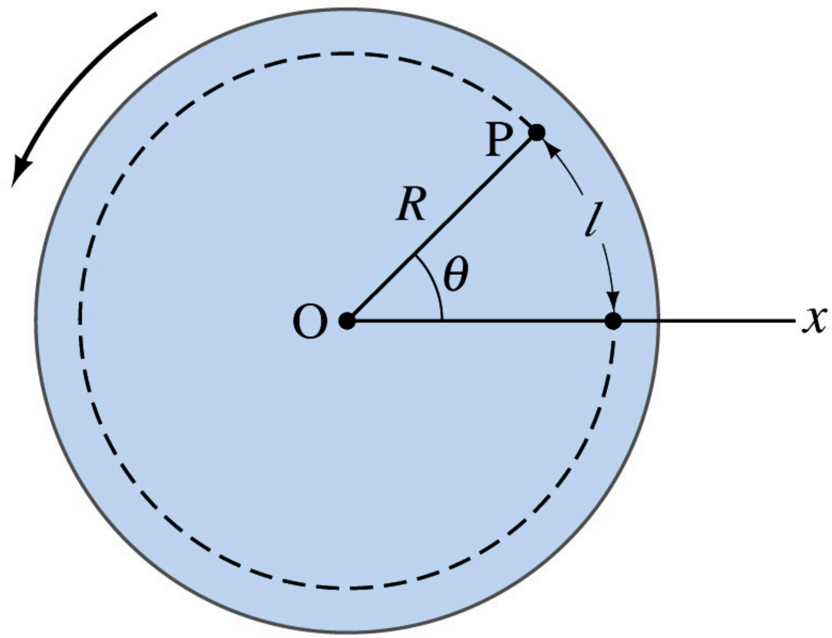
Rotational motion: variables.

- The angular position, measured in radians, is the angle of rotation of the object with respect to a reference position.
- The angular position of point *P* at this point in time is equal to θ. In order to uniquely define this position, we have assume that an the angular position is measured with respect to the *x* axis.

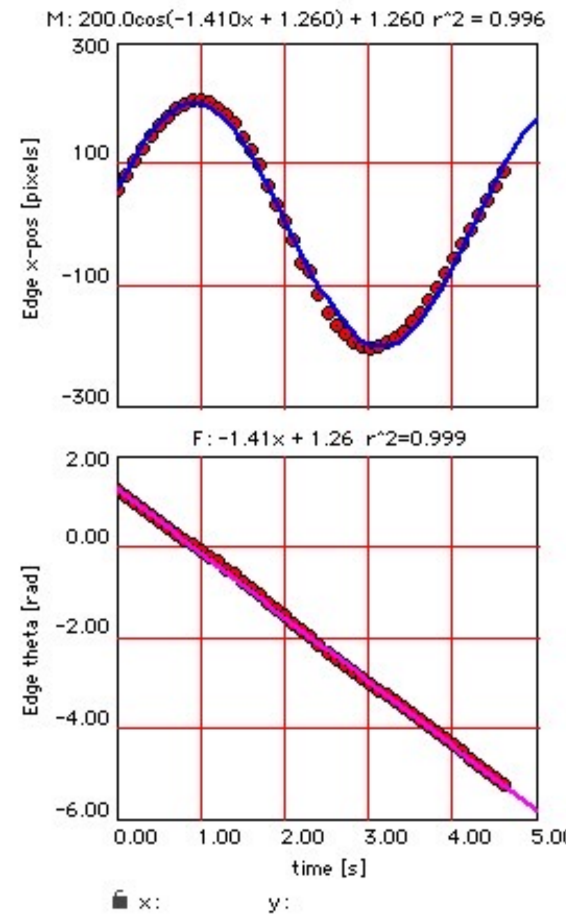


Rotational motion: variables.

- Note:
  - The angular position is always specified in radians!!!!
  - One radian is the angular displacement corresponding to a linear displacement $l = R$.
  - Make sure you keep track of the sign of the angular position!!!!!
  - **An increase in the angular position corresponds to a counter-clockwise rotation; a decrease corresponds to a clockwise rotation.**



Complex motion in Cartesian coordinates is simple motion in rotational coordinates.



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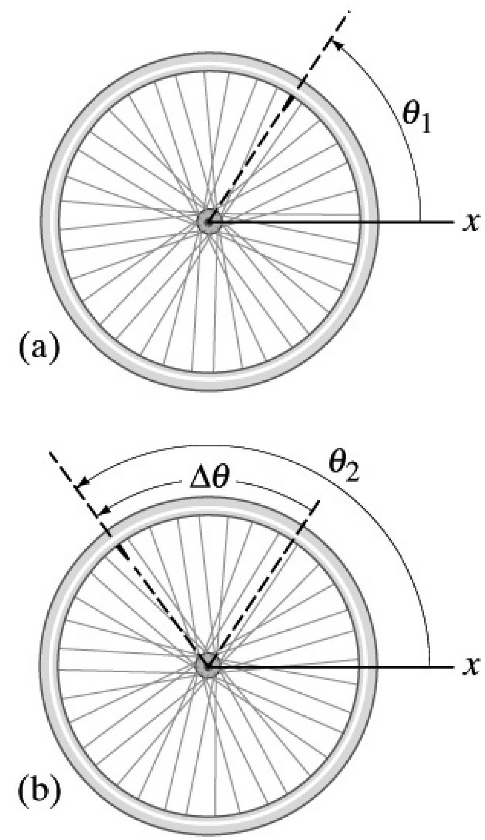
Rotational motion: variables.

- If we look at an object carrying out a rotation around a fixed axis, we will see that the angular position becomes a function of time.
- To describe the rotational motion, we introduce the concepts of angular velocity and angular acceleration.
- **Remember**: for linear motion we found it useful to introduce the concepts of linear velocity and linear acceleration.



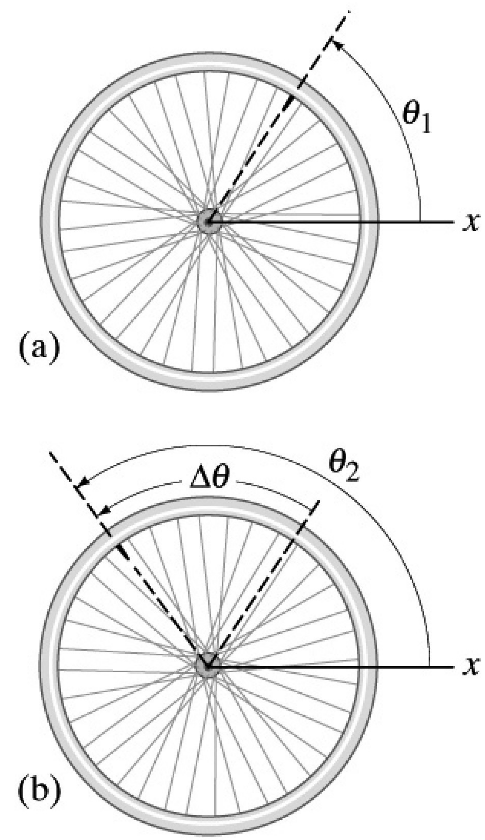
Angular acceleration (rad/s²)

Rotational motion: variables.

- For both velocity and acceleration, we can talk about instantaneous and average.

- Angular velocity:
  - Definition: $\omega = \frac{d\theta}{dt}$
  - Symbol: ω
  - Units: rad/s

- Angular acceleration:
  - Definition: $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
  - Symbol: α
  - Units: rad/s^2



Angular acceleration (rad/s^2)

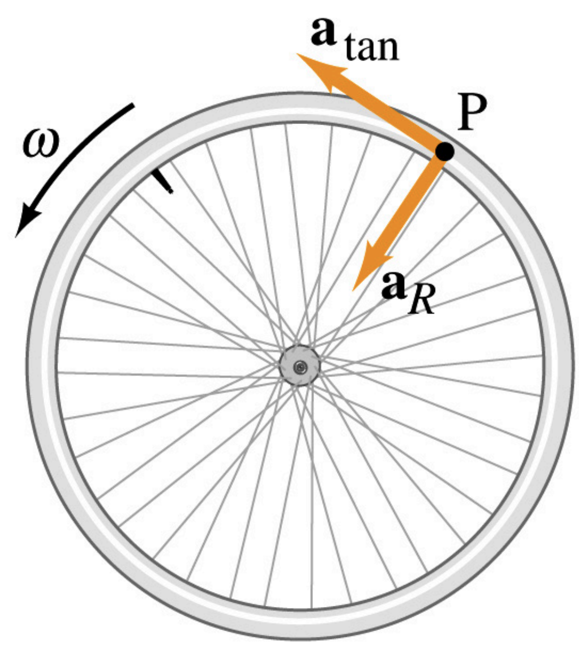
Rotational motion: constant acceleration.

- If the object experiences a constant angular acceleration, then we can describe its rotational motion with the following equations of motion:

$$\omega(t) = \omega_0 + \alpha t$$

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

- **Note how similar these equations are to the equation of motion for linear motion!!!!!!**



Rotational motion: constant acceleration. Example problem.

- A wheel starting from rest, rotates with a constant angular acceleration of 2.0 rad/s^2. During a certain 3.0 s interval it turns through 90 rad. (a) How long had the wheel been turning before the start of the 3.0 s interval ? (b). What was the angular velocity of the wheel at the start of the 3.0 s interval ?

- Define time $t = 0$ s as the time that the wheel is at rest. The angular velocity and the angle of rotation at a later time t are given by

$$\omega(t) = \alpha t \qquad \theta(t) = \frac{1}{2}\alpha t^2$$

Rotational motion: constant acceleration. Example problem.

- The change in the angular position $\Delta\theta$ during a time period Δt can now be calculated:

$$\Delta\theta(t) = \theta(t + \Delta t) - \theta(t) = \frac{1}{2}\alpha(\Delta t)^2 + \alpha t \Delta t$$

- Since the problem specifies $\Delta\theta$ and Δt we can now calculate the time *t* and the angular velocity at that time:

$$t = \frac{\Delta\theta - \frac{1}{2}\alpha(\Delta t)^2}{\alpha\Delta t} \quad \omega = \alpha t = \frac{\Delta\theta - \frac{1}{2}\alpha(\Delta t)^2}{\Delta t} = \frac{\Delta\theta}{\Delta t} - \frac{1}{2}\alpha\Delta t$$

5 Minute 16 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 5 minute 16 second intermission.

- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.

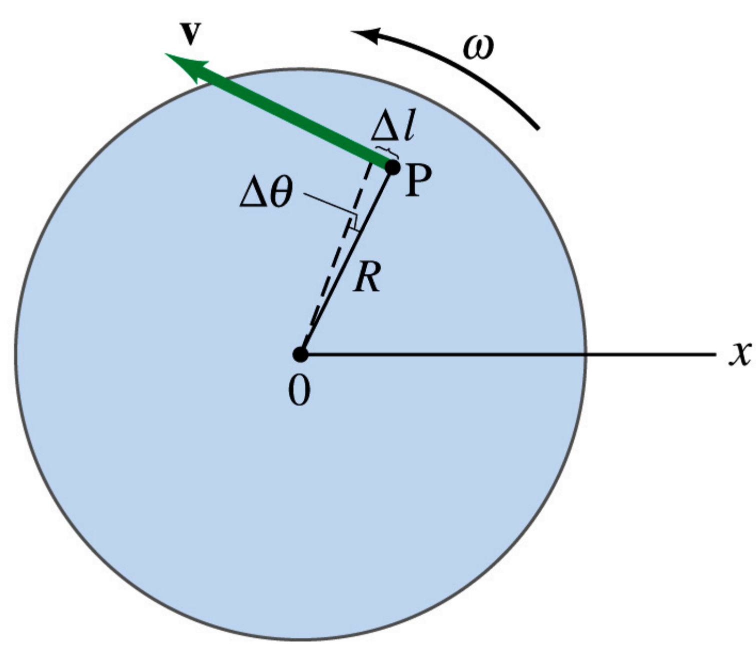
Rotational motion: variables.

- The linear velocity of a part of the rigid body is related to the angular velocity of the object.
- Consider point P:
  - It this point makes one complete revolution, it travels a distance $2\pi R$.
  - When the angular position changes by $\Delta\theta$, point P moves a distance

$$\Delta l = 2\pi R \frac{\Delta\theta}{2\pi} = R\Delta\theta$$

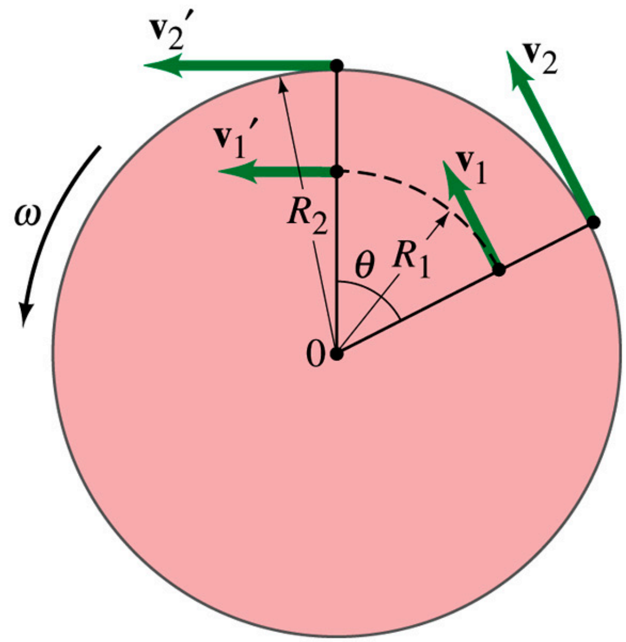
  - The linear velocity of point P is equal to

$$v = \frac{dl}{dt} = R\frac{d\theta}{dt} = R\omega$$



Rotational motion: variables.

- Things to consider when looking at the rotation of rigid objects around a fixed axis:
  - Each part of the rigid object has the same angular velocity.
  - Only those parts that are located at the same distance from the rotation axis have the same linear velocity.
  - The linear velocity of parts of the rigid object increases with increasing distance from the rotation axis.



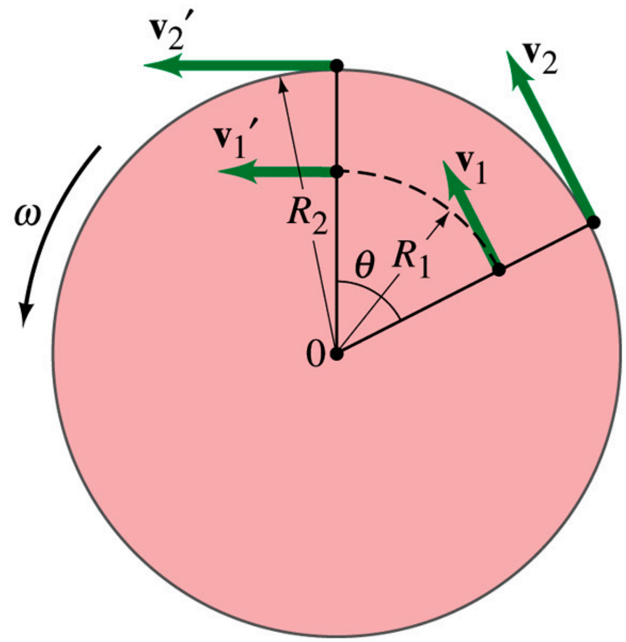
Relation between rotational and linear variables.

- Although in rotational motion we prefer to use rotational variables, we can also express the motion in terms of linear variables:

$$s = r\theta$$

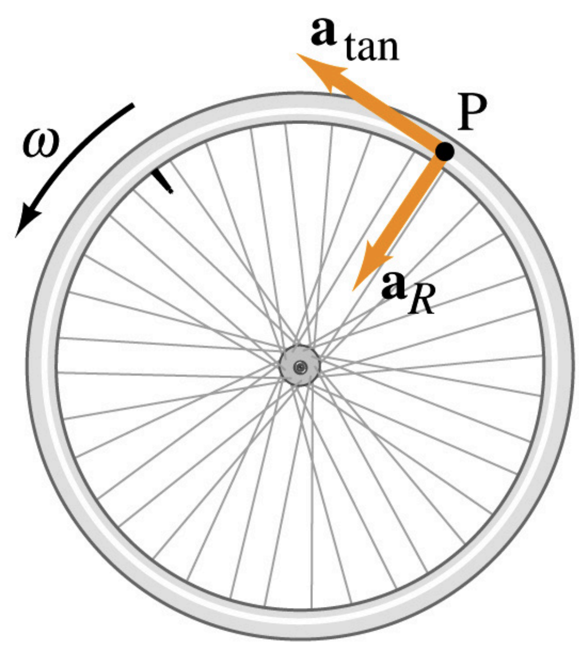
$$v = \frac{ds}{dt} = \frac{d}{dt}(r\theta) = r\frac{d\theta}{dt} = r\omega$$

$$a_t = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r\frac{d\omega}{dt} = r\alpha$$



Rotational motion: acceleration.

- Note: the acceleration $a_t = r\alpha$ is only one of the two component of the acceleration of point P. The two components of the acceleration of point P are:
  - The radial component: this component is always present since point P carried out circular motion around the axis of rotation.
  - The tangential component: this component is present only when the angular acceleration is not equal to 0 rad/s^2.

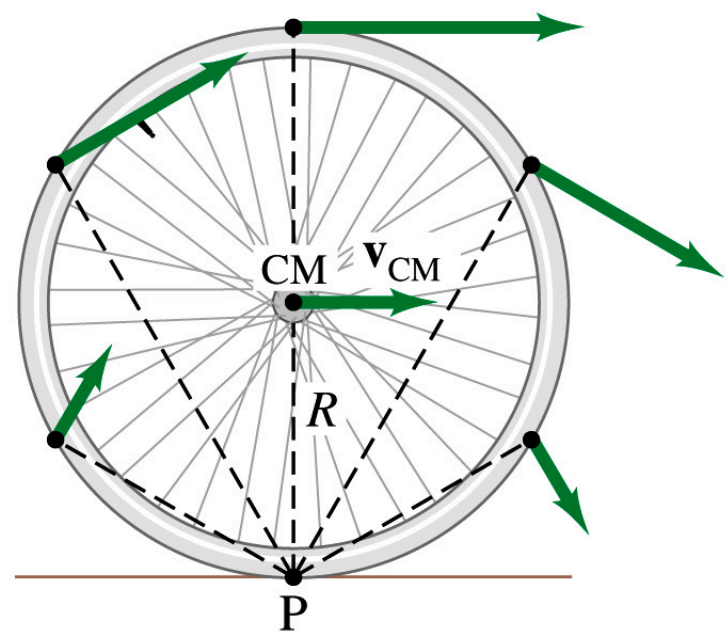


Rolling motion.

- To describe rolling motion we need to use both translational and rotational motion.

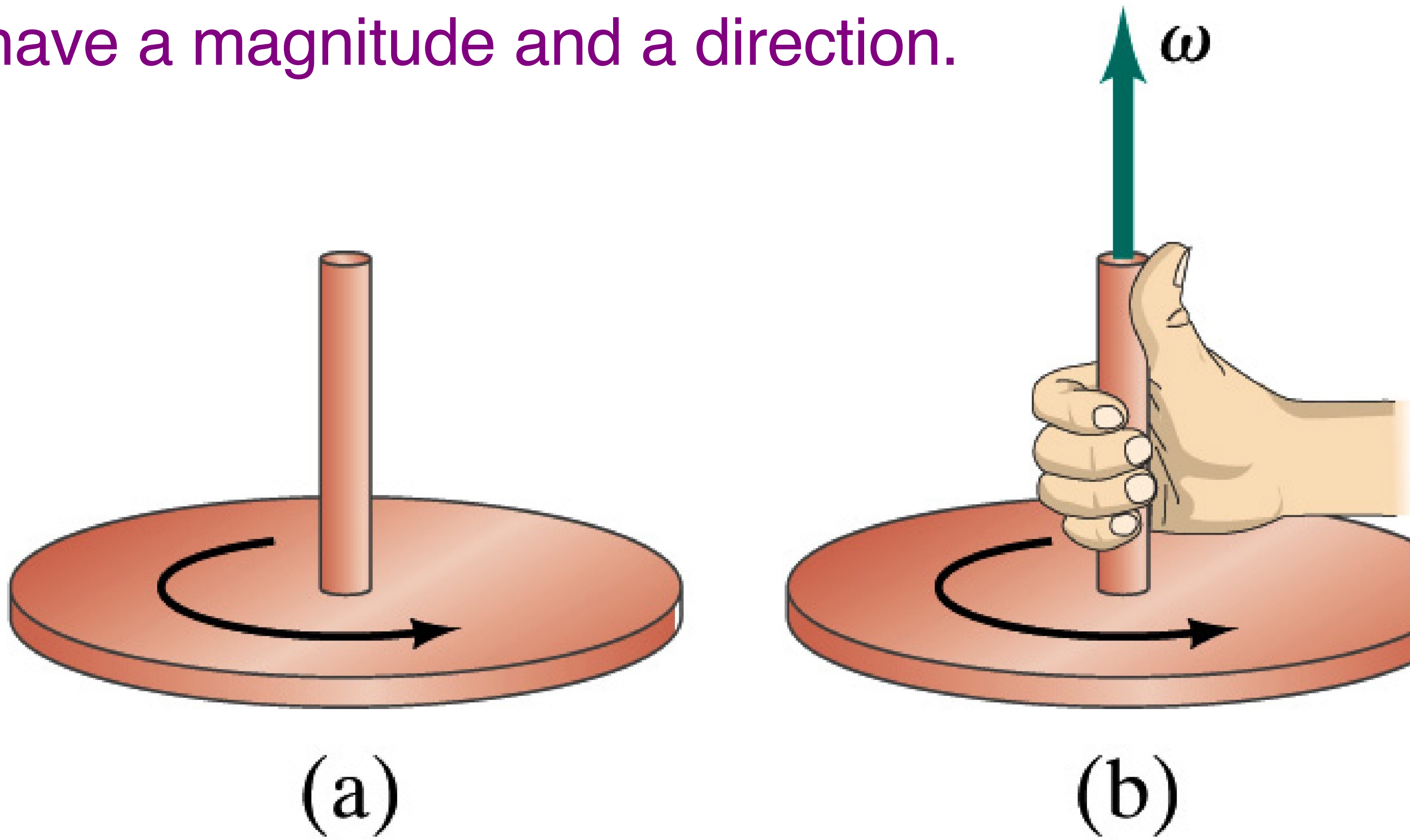
- The rolling motion can be described in terms of pure rotational motion with respect to the contact point P which is always at rest.

- The rotation axis around P is called the instantaneous axis and will move when the wheel rolls.



Direction of the angular velocity. Use your right hand!

Angular velocity and acceleration are vectors!
They have a magnitude and a direction.

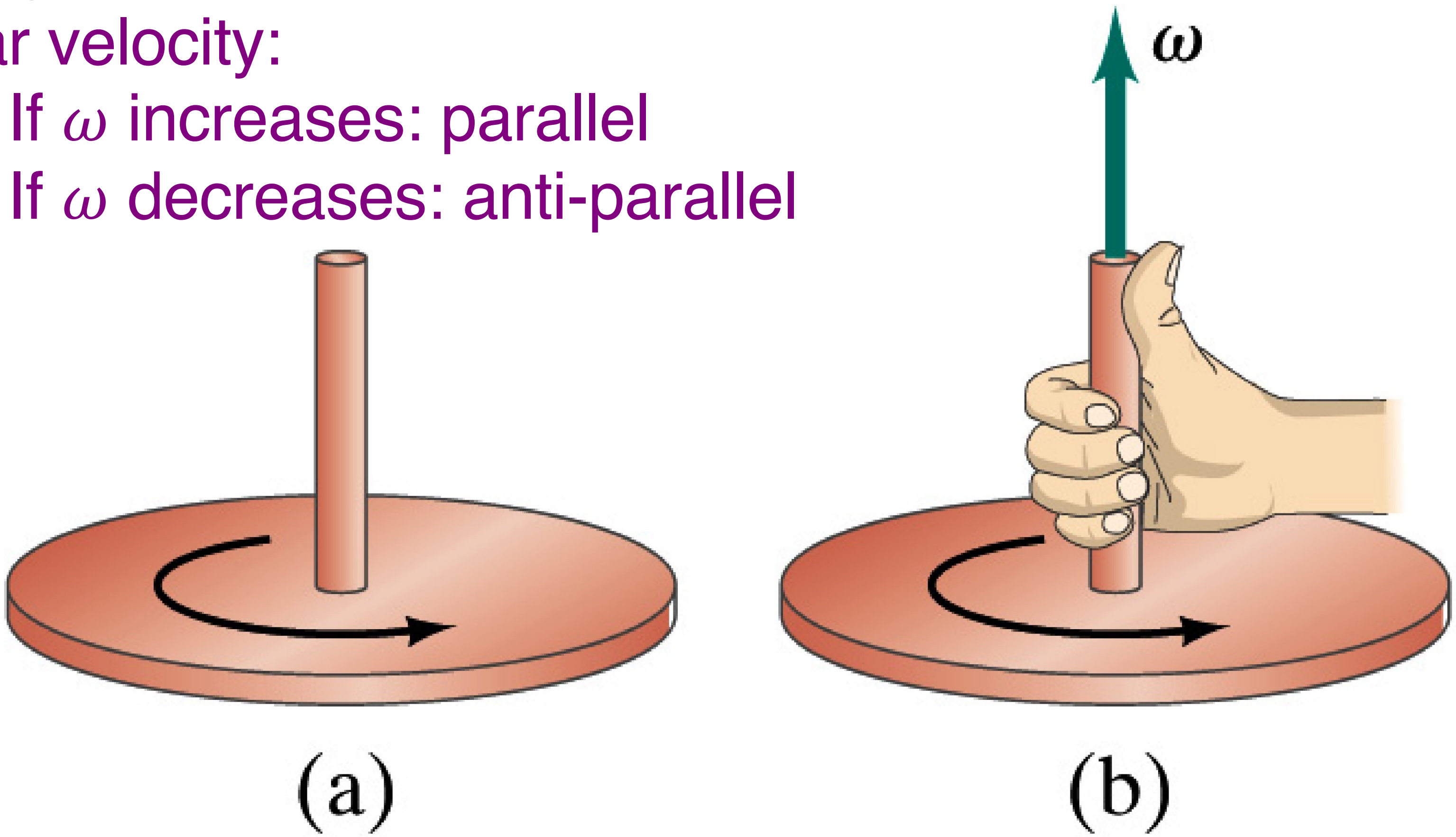


Direction of the angular acceleration.

The angular acceleration α is parallel or anti-parallel to the angular velocity:

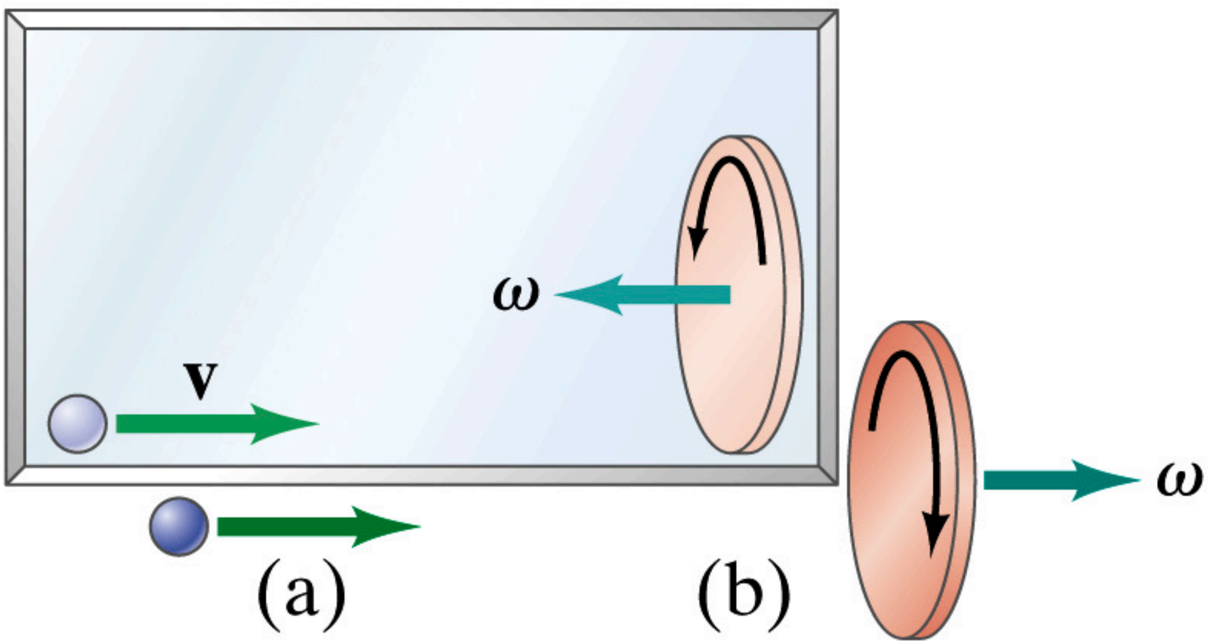
If ω increases: parallel

If ω decreases: anti-parallel

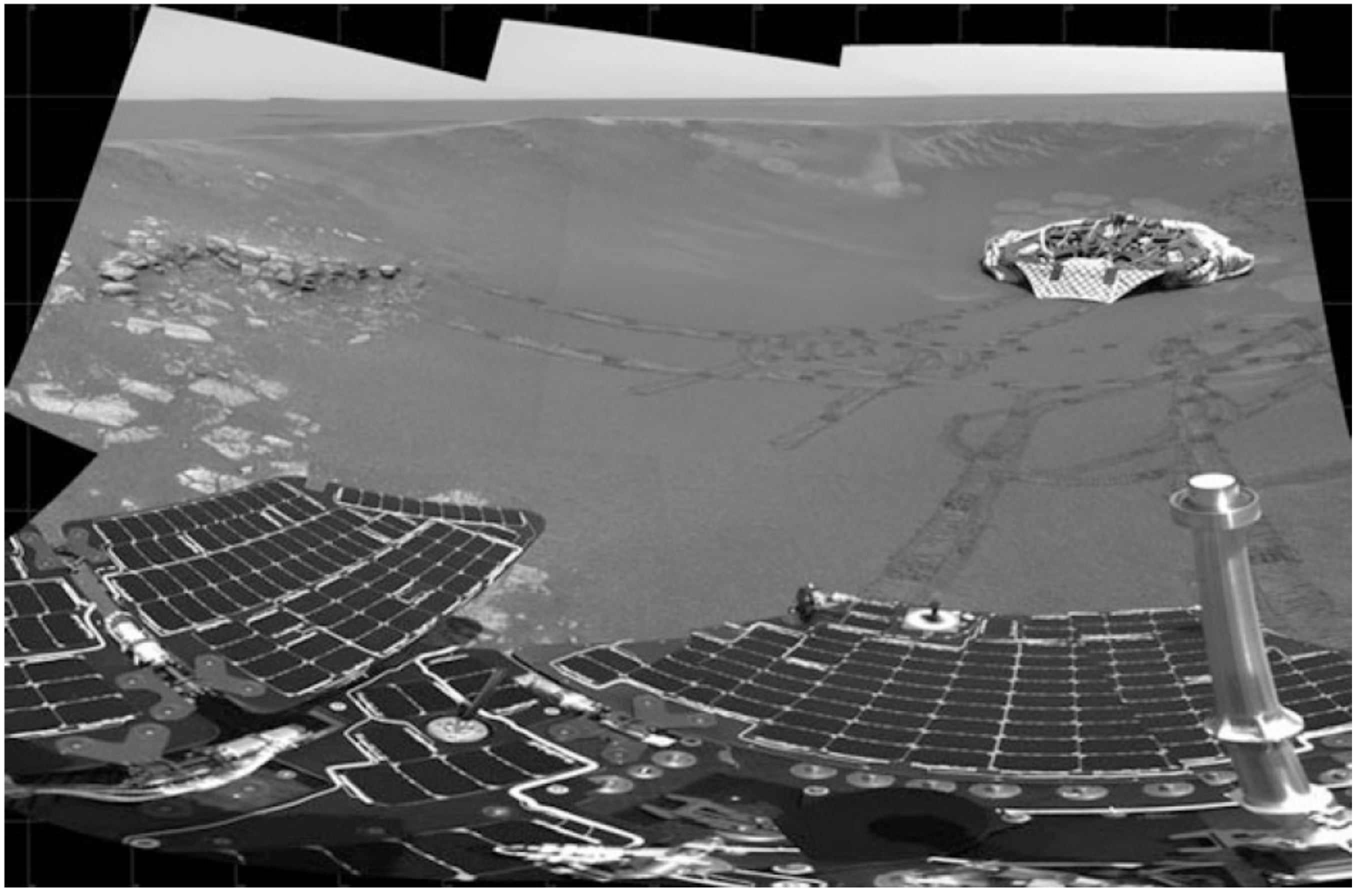


Pseudo vectors.

Angular velocity and accelerations are not real vectors! They do no behave as real vectors under reflection: they are what we call pseudo vectors.



Done for today!
Much more about rotations on Thursday!



Opportunity rover indicates ancient Mars was wet.
Credit: Mars Exploration Rover Mission, JPL, NASA