Physics 121.
Lecture 12.

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- Course Information
- Quiz
- Topics to be discussed today:
  - The center of mass
  - Conservation of linear momentum
  - Systems of variable mass

Physics 121.
Lecture 12.

- Homework set # 5 is now available on the WEB and will be due on Saturday morning, March 21, at 8.30 am. This is a change from the original due date since I will not discuss two-dimensional collisions until after spring break. As a result, the homework sets # 6 – 10 have been shifted also by one week. The information on the web, the calendar, and shown on webwork have been updated.

- Homework set # 5 includes a video analysis of two-dimensional collisions.

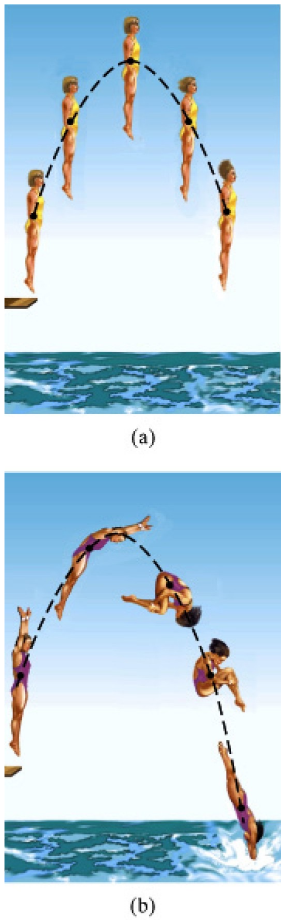
- The most effective way to work on the assignments is to tackle 1 or 2 problems a day.

Quiz.

- Let’s practice what we learned (and read).

The Center of Mass.

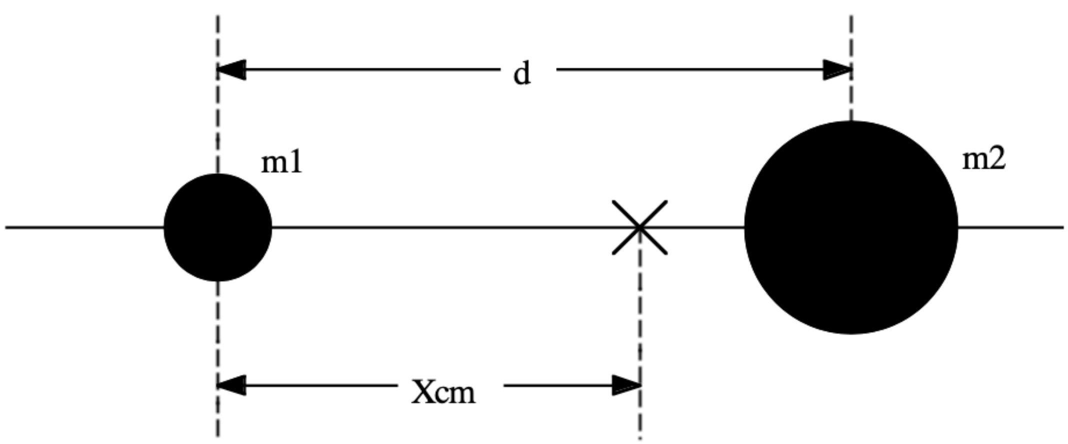
- Up to now we have ignored the shape of the objects were are studying.
- Objects that are not point-like appear to carry out more complicated motions than point-like objects (e.g. the object may be rotating during its motion).
- We will find that we can use whatever we have learned about motion of point-like objects if we consider the motion of the center-of-mass of the extended object.



But what is the center of mass and where is it located?

We will start with considering one-dimensional objects. For an object consisting out of two point masses, the center of mass is defined as

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{1}{M}\sum_i m_i x_i$$

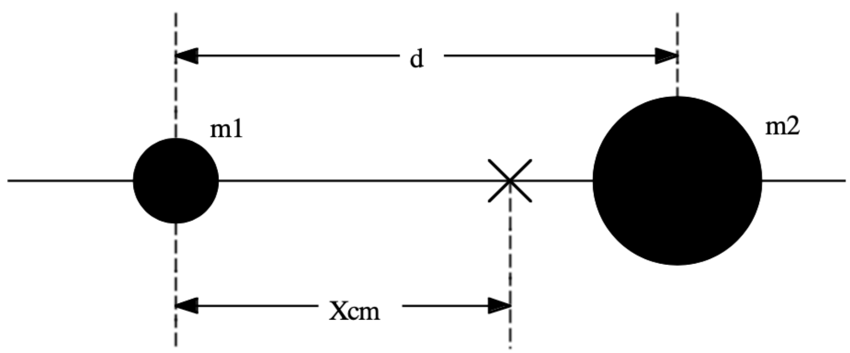


But what is the center of mass and where is it located?

- Let's look at this particular system.
- Since we are free to choose our coordinate system in a way convenient to us, we choose it such that the origin coincides with the location of mass m_1.
- The center of mass is located at

$$x_{cm} = \frac{m_2 d}{m_1 + m_2}$$

Note: the center of mass does not need to be located at a position where there is mass!

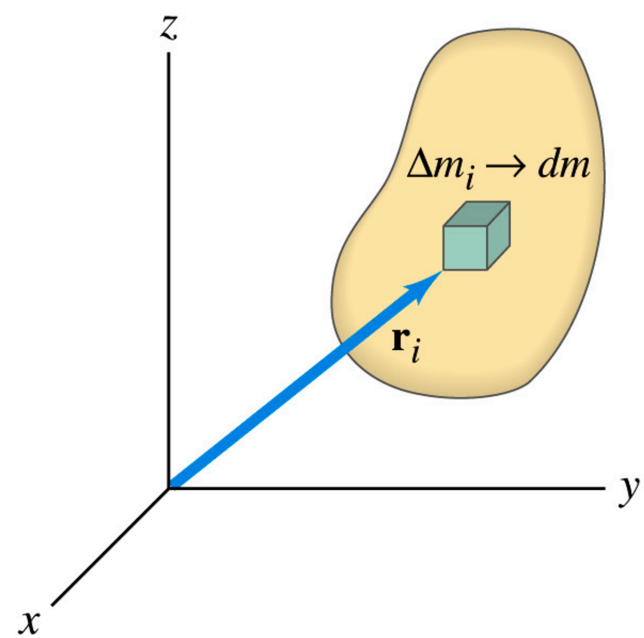


But what is the center of mass and where is it located?

- In two or three dimensions the calculation of the center of mass is very similar, except that we need to use vectors.

- If we are not dealing with discreet point masses, we need to replace the sum with an integral:

$$\vec{r}_{cm} = \frac{1}{M} \int_V \vec{r} dm$$



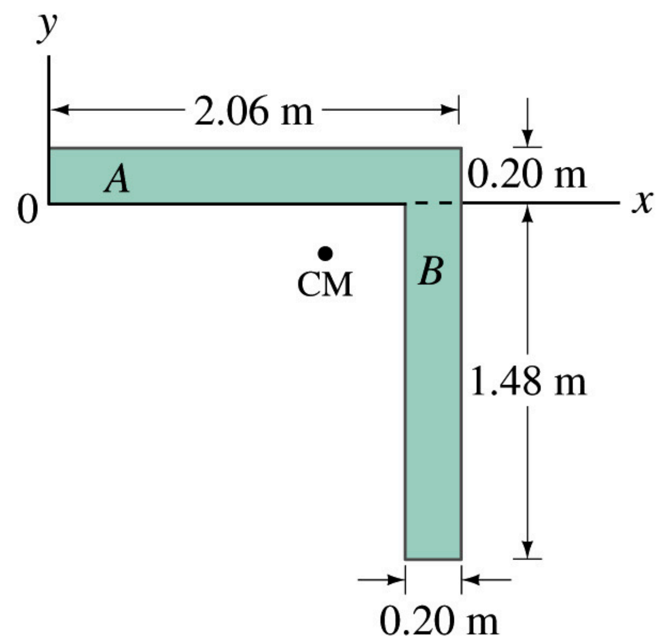
But what is the center of mass and where is it located?

- We can also calculate the position of the center of mass of a two or three dimensional object by calculating its components separately:

$$x_{cm} = \frac{1}{M}\sum_i m_i x_i$$

$$y_{cm} = \frac{1}{M}\sum_i m_i y_i$$

$$z_{cm} = \frac{1}{M}\sum_i m_i z_i$$

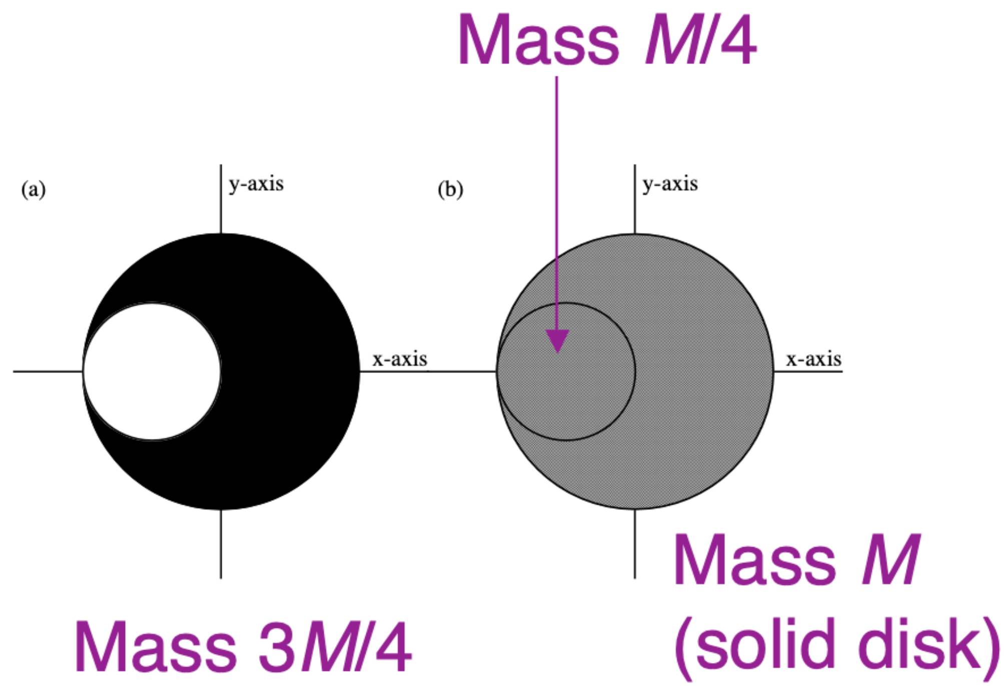


- Note: the center of mass may be located outside the object.

Calculating the position of the center of mass. Example problem.

- Consider a circular metal plate of radius $2R$ from which a disk of radius R has been removed. Let us call it object X. Locate the center of mass of object X.

- Since this object is complicated, we can simplify our life by using the principle of superposition:
  - If we add a disk of radius R, located at $(-R, 0)$ we obtain a solid disk of radius $2R$.
  - The center of mass of the solid disk is located at $(0, 0)$.



Calculating the position of the center of mass. Example problem.

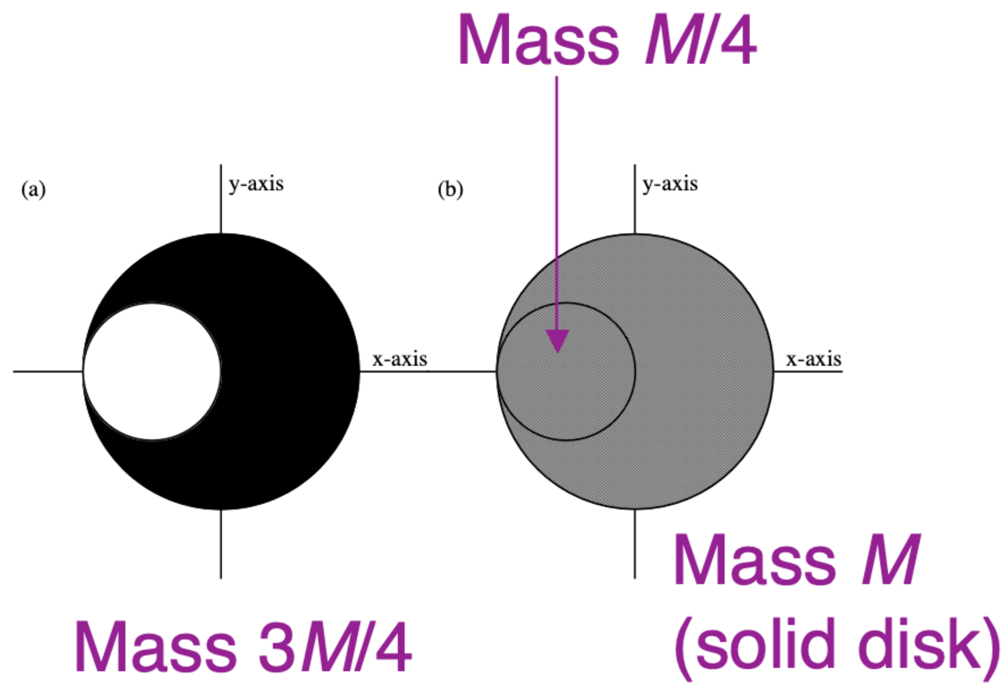
- The center of mass of the solid disk can be expressed in terms of the disk *X* and the disk *D* we used to fill the hole in disk *X*:

$$x_{cm,C} = \frac{x_{cm,X} m_X + x_{cm,D} m_D}{m_X + m_D} = 0$$

- This equation can be rewritten as

$$x_{cm,X} = -\frac{x_{cm,D} m_D}{m_X} = \frac{R m_D}{m_X}$$

- Here we have used the fact that the center of mass of disk *D* is located at (-*R*, 0).
- Thus, $x_{cm,X} = R/3$.



Motion of the center of mass.

- To examine the motion of the center of mass we start with its position and then determine its velocity and acceleration:

$$M\vec{r}_{cm} = \sum_i m_i \vec{r}_i$$

$$M\vec{v}_{cm} = \sum_i m_i \vec{v}_i$$

$$M\vec{a}_{cm} = \sum_i m_i \vec{a}_i$$

Motion of the center of mass.

- The expression for $M\vec{a}_{cm}$ can be rewritten in terms of the forces on the individual components:

$$M\vec{a}_{cm} = \frac{d}{dt}(M\vec{v}_{cm}) = \frac{d\vec{P}_{cm}}{dt} = \sum_i m_i \vec{a}_i = \sum_i \vec{F}_i = \vec{F}_{net,ext}$$

- We conclude that the motion of the center of mass is only determined by the external forces. Forces exerted by one part of the system on other parts of the system are called internal forces. According to Newton's third law, the sum of all internal forces cancel out (for each interaction there are two forces acting on two parts: they are equal in magnitude but pointing in an opposite direction and cancel if we take the vector sum of all internal forces).

Motion of the center of mass and linear momentum.

- Now consider the special case where there are no external forces acting on the system. In this case, $Ma_{cm,x} = Ma_{cm,y} = Ma_{cm,z} = 0$ N.

- In this case, $Mv_{cm,x}$, $Mv_{cm,y}$, and $Mv_{cm,z}$ are constant.

- The product of the mass and velocity of an object is called the linear momentum $\vec{P}$ of that object.

- In the case of an extended object, we find the total linear momentum by adding the linear momenta of all of its components:

$$\vec{P}_{tot} = \sum_i \vec{p}_i = \sum_i m_i \vec{v}_i = M\vec{v}_{cm}$$

Conservation of linear momentum.

- The change in the linear momentum of the system can now be calculated:

$$\frac{d\vec{P}_{cm}}{dt} = M\frac{d\vec{v}_{cm}}{dt} = M\vec{a}_{cm} = \sum_i m_i\vec{a}_i = \sum_i \vec{F}_i = \vec{F}_{net,ext}$$

- This relations shows us that **if there are no external forces, the total linear momentum of the system will be constant** (independent of time).

- Note: the system can change as a result of internal forces!

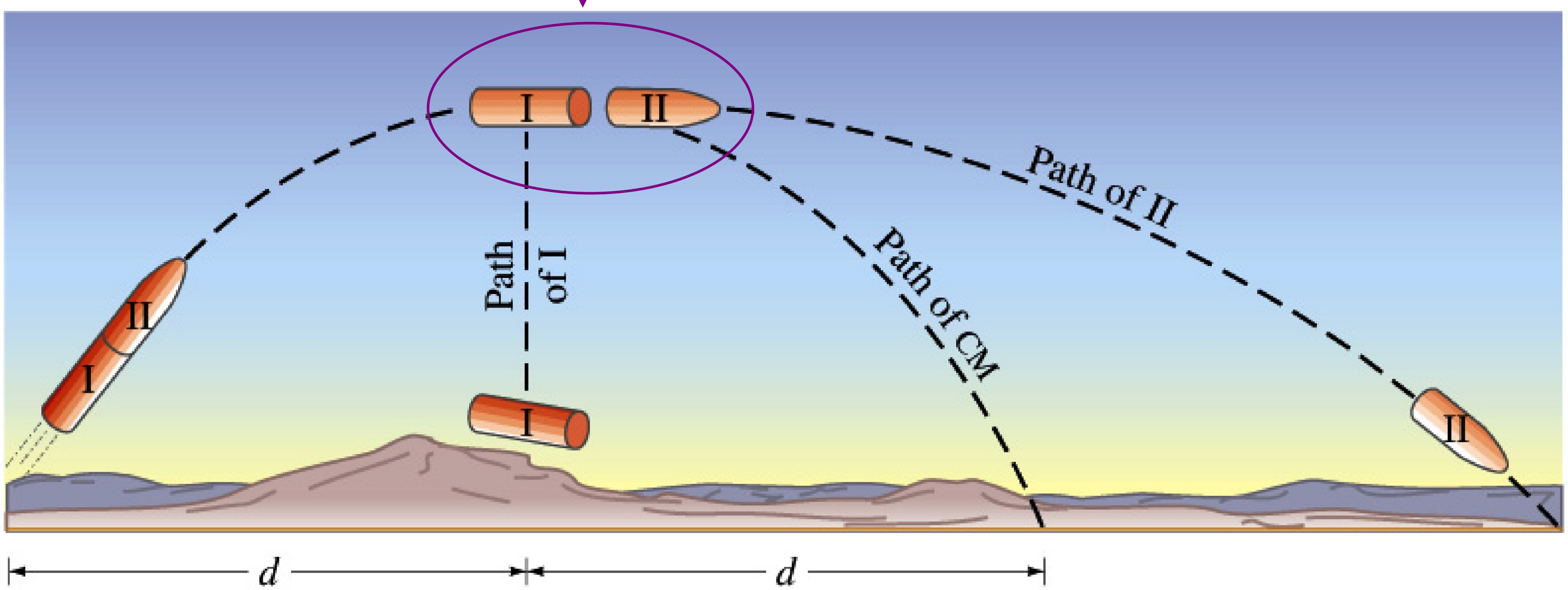
3 Minute 56 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 3 minute 56 second intermission.

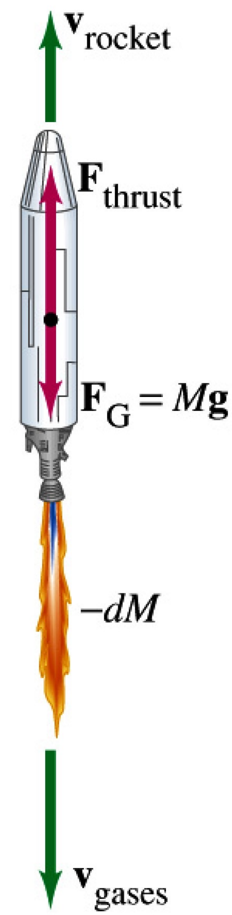
- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.

Conservation of linear momentum. Applications.

Internal forces are responsible for the breakup.

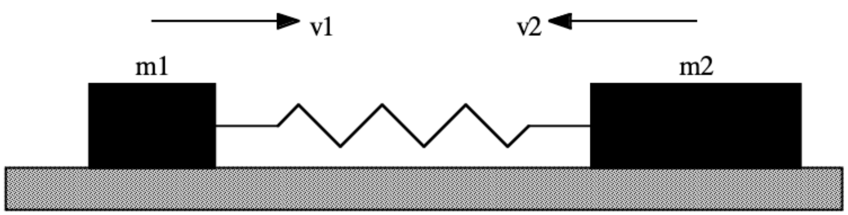


Conservation of linear momentum. Applications.



Conservation of linear momentum. An example.

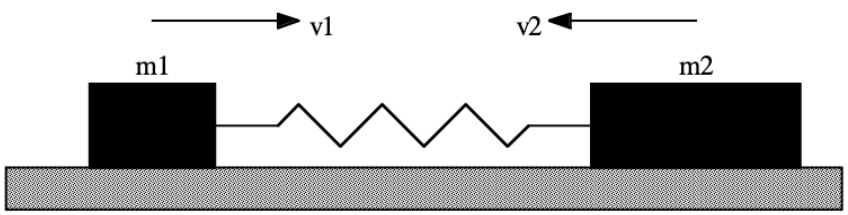
- Two blocks with mass m_1 and mass m_2 are connected by a spring and are free to slide on a frictionless horizontal surface. The blocks are pulled apart and then released from rest. What fraction of the total kinetic energy will each block have at any later time?



Conservation of linear momentum. An example.

- The system of the blocks and the spring is a closed system, and the horizontal component of the external force is 0 N. The horizontal component of the linear momentum is thus conserved.
- Initially the masses are at rest, and the total linear momentum is thus 0 kg m/s.
- At any point in time, the velocities of block 1 and block 2 are related:

$$v_2 = -\frac{m_1}{m_2} v_1$$



Conservation of linear momentum. An example.

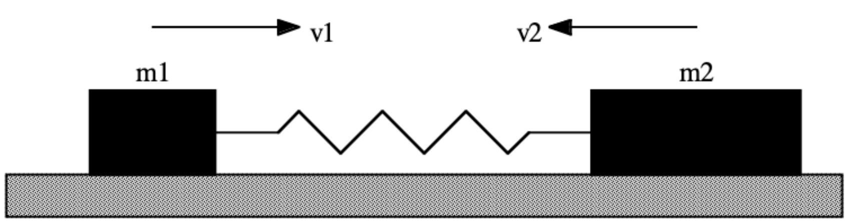
- The kinetic energies of mass m_1 and m_2 are thus equal to

$$K_1 = \frac{1}{2} m_1 v_1^2$$

$$K_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2}\frac{1}{m_2}(m_2 v_2)^2 = \frac{1}{2}\frac{m_1}{m_2}\frac{(m_1 v_1)^2}{m_1} = \frac{m_1}{m_2} K_1$$

- The fraction of the total kinetic energy carried away by block 1 is equal to

$$f_1 = \frac{K_1}{K_t} = \frac{K_1}{K_1 + K_2} = \frac{K_1}{K_1 + \dfrac{m_1}{m_2} K_1} = \frac{m_2}{m_1 + m_2}$$



Systems with variable mass.

- Rocket motion is an example of a system with a variable mass:

$$M(t+dt) = M(t) + dM$$

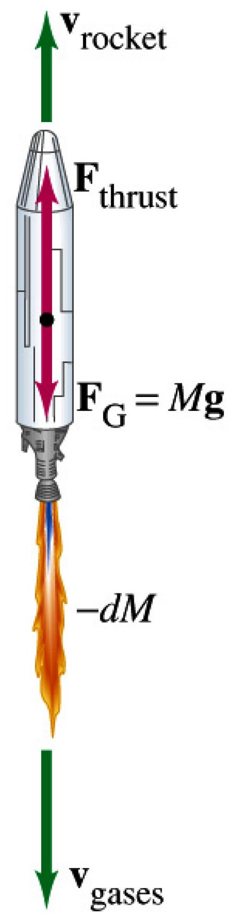
Mass of exhaust.
dM < 0 kg

- As a result of dumping the exhaust, the rocket will increase its velocity:

$$v(t+dt) = v(t) + dv$$

- Since this is an isolated system, linear momentum must be conserved. The initial momentum is equal to

$$p_i = M(t)v(t)$$



Systems with variable mass.

- The final linear momentum of the system is given by

$$p_f = (M(t) + dM)(v(t) + dv) + (-dM)U$$

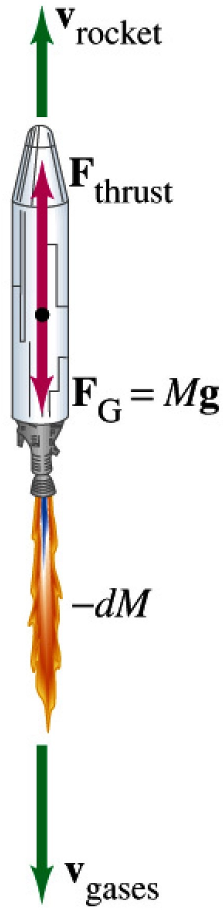
where U is the velocity of the exhaust.

- Conservation of linear momentum therefore requires that

$$M(t)v(t) = (M(t) + dM)(v(t) + dv) + (-dM)U$$

- The exhaust has a fixed velocity U_0 with respect to the engine. U_0 and U are related in the following way:

$$U - U_0 = v(t) + dv$$



Systems with Variable Mass.

- Conservation of linear momentum can now be rewritten as

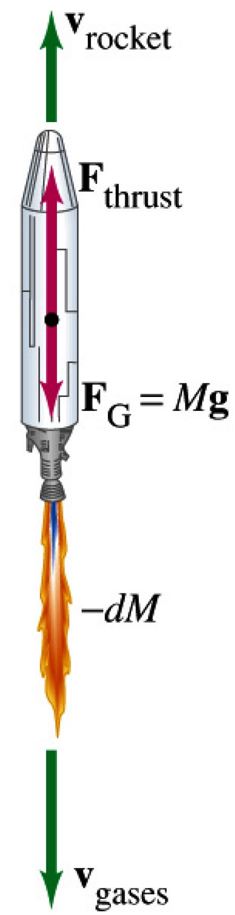
$$M(t)v(t) =$$
$$= (M(t) + dM)(v(t) + dv) + (-dM)(v(t) + dv + U_0)$$

or

$$M(t)v(t) = M(t)(v(t) + dv) - (dM)U_0$$

- We conclude

$$(dM)U_0 = M(t)dv$$



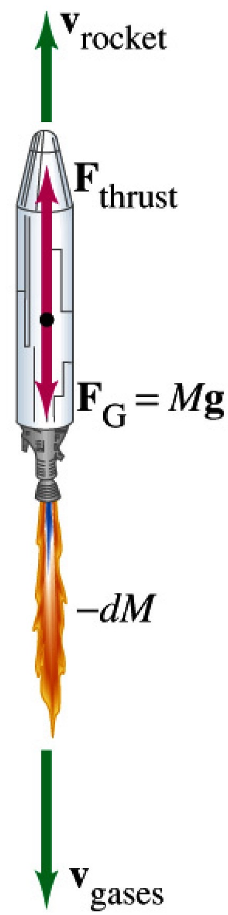
Systems with variable mass.

- The previous equation can be rewritten as

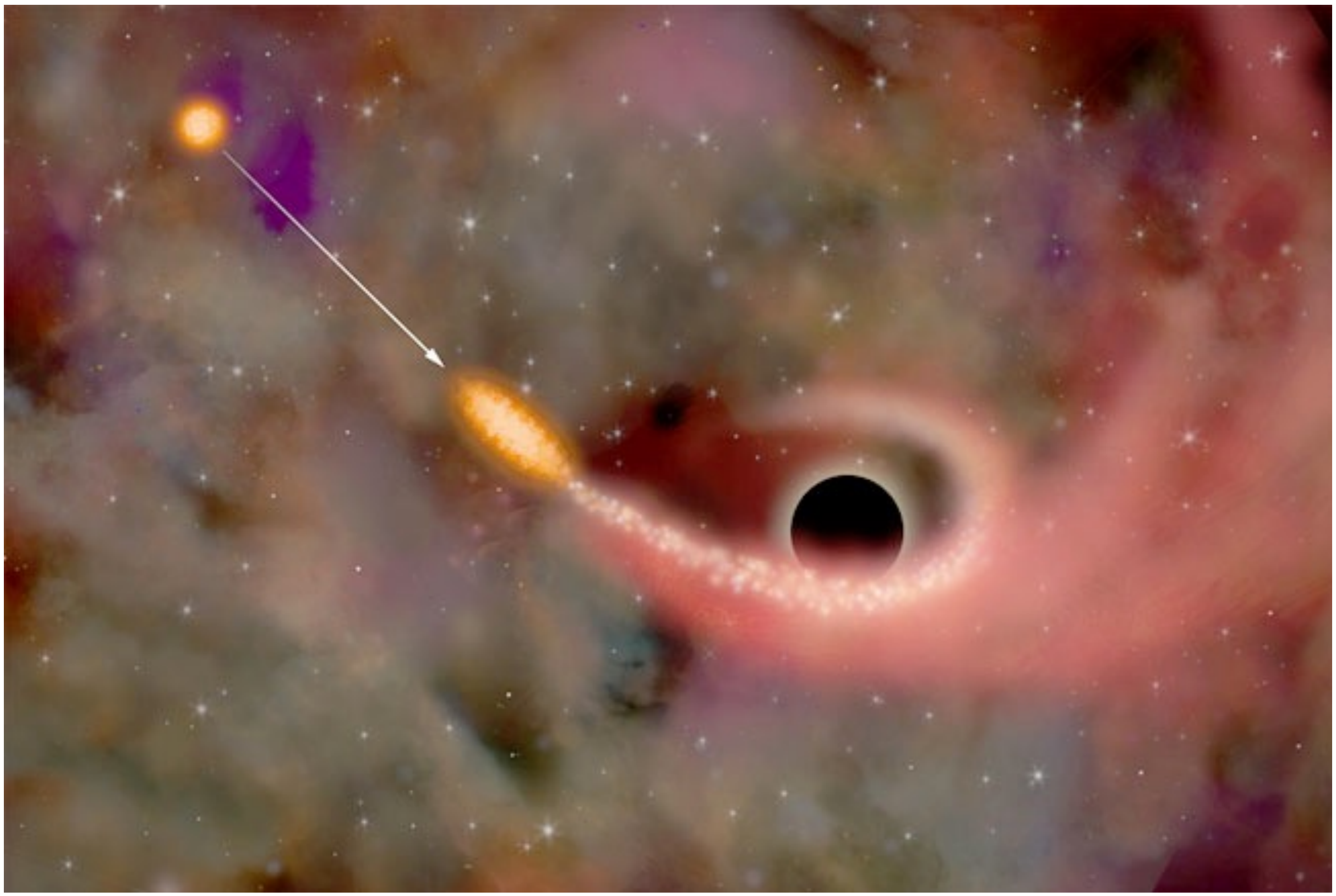
$$\frac{dM}{dt} U_0 = M(t) \frac{dv(t)}{dt}$$

  In this equation:
  - $dM/dt = -R$ where R is the rate of fuel consumption.
  - $U_0 = -u$ where u is the (positive) velocity of the exhaust gasses relative to the rocket.
  - dv/dt is the acceleration of the rocket.
- This equation can be rewritten as $RU_0 = Ma_{\text{rocket}}$ which is called the **first rocket equation**. This equation can be used to find the velocity of the rocket (**second rocket equation**):

$$v_f = v_i + u \ln \frac{M_i}{M_f}$$



Done for today!
Thursday: collisions.



X-Rays Indicate Star Ripped Up by Black Hole. Illustration Credit: M. Weiss, [CXC](), [NASA]()