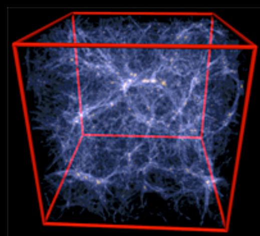
Today in Astronomy 102: Big Bang cosmology and the new flat Universe.

- Expansion rates, ages and fates of GR universes in Big Bang cosmology.
- Inflation: how vacuum fluctuations might have kick-started the Universe.
- Matter-dominated universes, and measurements of the mass density and age of the Universe in which we live: an open Universe?
- Acceleration in the Universe's expansion.
- Direct measurements of the Universe's curvature: it's flat from here to decoupling.



Simulation of structure in a universe dominated by cold dark matter and dark energy (Springel and Hernquist 2003).

I will be driving to Toronto in August to fly my favorite airplane.

PH-BFT at YYZ (Toronto)

And look at this one ....
My favorite plane and no smog in Beijing.

PH-BFT at PEK (Beijing)

Big bang cosmological models: finer points of the hieroglyphics

We won't be solving the Einstein field equation for the Universe, or use it on homework or exams, but to understand the differences between various universe models it is useful to look more carefully at the Hieroglyphics.

Universal expansion rate

Mass density (mass per unit volume)

$$\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8\pi G}{3}\rho - \frac{c^2}{3}\Lambda = -c^2\frac{k}{R^2}$$

Space (i.e. not spacetime) curvature

Typical distance between galaxies

Cosmological constant. Added by Einstein to allow static (constant in time) solutions.

Big bang cosmological models (continued)

$$\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8\pi G}{3}\rho - \frac{c^2}{3}\Lambda = -c^2\frac{k}{R^2}$$

This equation is a mathematical machine that can provide answers for all the terms in the equation (R, ρ , k, etc.) at all values of time (past, present, and future), if it is given "initial" conditions: the values of these quantities at any time during the Universe's history.

- Usually, of course, we provide it measured values for the terms, at the *present* time.

Big bang cosmological models (continued)

It is popular to define a **critical mass density**, $\rho_0 = 3H_0^2/8\pi G$
a **normalized** mass density, $\Omega_M = \rho/\rho_0$
and a normalized cosmological constant, $\Omega_\Lambda = c^2\Lambda/3H_0^2$
in terms of which the field equation is

$$\left(\frac{1}{H_0 R}\frac{dR}{dt}\right)^2 - \Omega_M - \Omega_\Lambda = -\frac{c^2}{(H_0 R)^2}k$$

The critical mass density comes out to

$$\rho_0 = \frac{3H_0^2}{8\pi G} = 7.9\times 10^{-30}\,\mathrm{gm\ cm^{-3}}$$

which is very small by Earthly standards (one ^{1}H/56 gallon).

Big bang cosmological models (continued)

Since Ω_M is a ratio of mass densities, it may be useful to think of Ω_Λ as a ratio of densities too. We often therefore define

$$\rho_\Lambda = \frac{c^2\Lambda}{8\pi G} \qquad \text{so that} \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_0}.$$

- And since ρ_Λ (and Ω_Λ) are expressed in the same units and terms as mass densities but are not densities of matter or radiation or anything related (like ρ, ρ_0, and Ω_M are), we need new words to name them.
  Currently the most popular name for the "substance" that corresponds to ρ_Λ and Ω_Λ is **dark energy**. $\rho_\Lambda c^2$ can be thought of as a dark energy density.

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Big bang cosmological models (continued)

$$\left(\frac{1}{H_0 R}\frac{dR}{dt}\right)^2 - \Omega_M - \Omega_\Lambda = -\frac{c^2}{(H_0 R)^2}k$$

It looks at first as if Ω_M and Ω_Λ should have the same effect on how R and k come out in the solutions, but they don't.

- Since mass is conserved, the normalized mass density Ω_M decreases as the universe expands.
- Ω_Λ, related as it is to the cosmological constant, stays the same as the Universe expands.
- As we will see, this property makes positive values of Ω_Λ lead inexorably to expansion, no matter what the value of Ω_M may be. (And to inexorable collapse, for negative values.)

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Big bang cosmological models (continued)

In these terms we will now discuss how the GR Big Bang model,

$$\left(\frac{1}{H_0 R}\frac{dR}{dt}\right)^2 - \Omega_M - \Omega_\Lambda = -\frac{c^2}{(H_0 R)^2}k$$

applies, and how it is constrained by measurements of some of the quantities like R, k, and Ω_M :

- The very early universe and **inflation**.
- Matter-dominated universes, like the one we used to think we live in, and **dark matter**.
- The new flat Universe, and **dark energy**.

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Inflation: the cosmic microwave background is almost *too* isotropic.

No part of the cosmic microwave background differs in brightness from the average by more than 0.001%. It is hard to make gases, or the light they emit, that smooth or uniform. (Consider sunspots!)

- To do so would usually require that all parts of the gas be interacting with each other strongly, or that the gas be extremely well mixed.
- This would not seem possible for different parts of the decoupling surface. We were once part of that surface, and the parts of it that we see today have been out of contact with us (and each other) since the Big Bang, since we're only now receiving light from these parts and no signal or interaction can travel faster than light.

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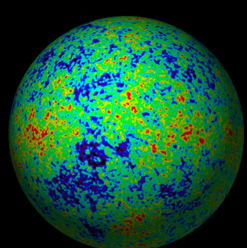
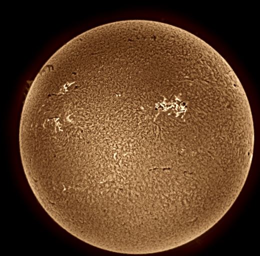
Inflation (continued)



Big Bang's visible surface: temperature varies by about 0.001% of the average. (WMAP/NASA)



Sun's visible surface: temperature varies by 10-20% of the average. (By Robert Gendler)

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Inflation (continued)

One theoretically-popular way out of this problem is to postulate a brief period of **inflation** early in the Universe's history. Briefly, this is thought to happen as follows.

- Shortly after the Big Bang, the **vacuum** could have had a much larger energy density, in the form of virtual pairs, than it does today. This possibility is allowed under certain theoretical models of numbers and interactions of elementary particles.
- At some time during the expansion, the vacuum underwent a **phase transition** (like freezing or condensing) to produce the lower-energy version we have today, presumably driven by the changes in spacetime curvature.

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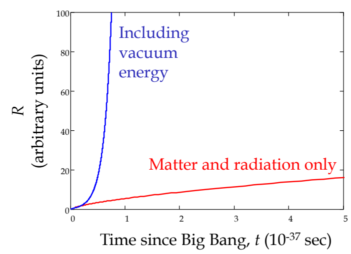
Inflation (continued)

- ❑ While the vacuum was in its high-energy-density state, it gave a large additional impulse to Universal expansion.
  - Vacuum fluctuations fill whatever volume the Universe has, independent of how much real matter it contains.
  - Thus the high-energy vacuum acts like dark energy, i.e. like a cosmological constant.
- ❑ Accounting for the vacuum's influence in general relativity leads to a **very much smoother and faster expansion**. During this period, spacetime's radius of curvature increases more like a bubble blowing up, than like a blast wave - hence the name **inflation** for the process.

Inflation (continued)

Solutions to the field equation for the early Universe:



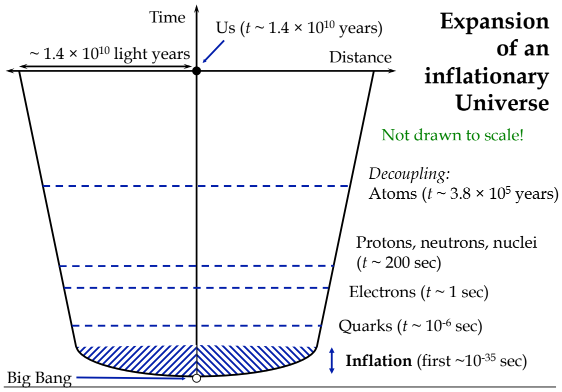
Inflation (continued)

The inflationary era would have been relatively brief, much shorter than the time between Big Bang and decoupling.

- ❑ If it lasted through 100 doublings of the Universe's size, that would do it, and this takes only about 10^{-35} seconds.
- ❑ During the remaining "normal" expansion between the end of inflation (decay of the vacuum to its low energy density state) and decoupling, the bumps and wiggles normally present in blast waves still wouldn't have had enough time to develop.

We know of course that the Universe has become much less smooth since decoupling. The seeds for inhomogeneities like galaxies, stars and people were not sown before decoupling, however.

Expansion of an inflationary Universe



Matter-dominated universes

After inflation has come and gone, and decoupling has already happened, the energy density of everything we know about in the Universe (that is, taking $\Omega_\Lambda = 0$) is dominated by the rest mass of matter. Such a universe is called **matter-dominated.** Matter-dominated universes have three interesting properties:

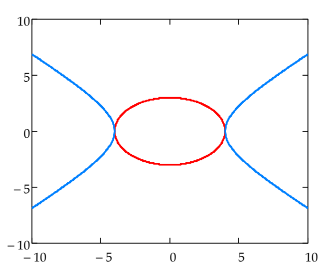
- ❑ The present matter density relative to the critical density, ρ_0, uniquely determines what the value of the curvature k is:

$$\text{If } \Omega_M = \frac{\rho}{\rho_0} > 1 \text{ then } k = 1 \text{ (positively-curved space)}$$

$$\Omega_M = 1 \text{ then } k = 0 \text{ (flat space)}$$

$$\Omega_M < 1 \text{ then } k = -1 \text{ (negatively curved space)}$$

2-D reminder of positive and negative curvature



The **ellipse** is positively curved, and is a closed path.
The **hyperbola** is negatively curved and reaches to infinity.

Matter-dominated universes (continued)

- The curvature, in turn, determines the boundedness of the universe: whether the universe is open or closed. If it is open, it is possible for paths to extend to infinity, like the hyperbola; if closed, all paths eventually return to the starting point, like the ellipse.
- And further, the matter density uniquely determines the age and fate of the universe.

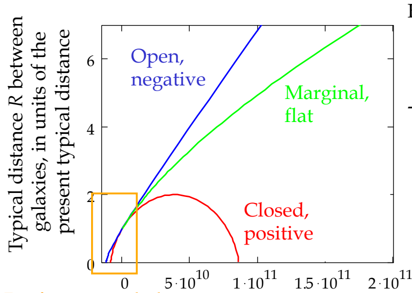
If $\Omega_M = \frac{\rho}{\rho_0} > 1$ then the universe is gravitationally bound.

$\Omega_M < 1$ then the universe is **not** gravitationally bound.

$\Omega_M = 1$ is called a critical, or marginal, universe.

If bound, the expansion will reverse and it will re-collapse; if not, the expansion will continue forever.

Matter-dominated universes (continued)



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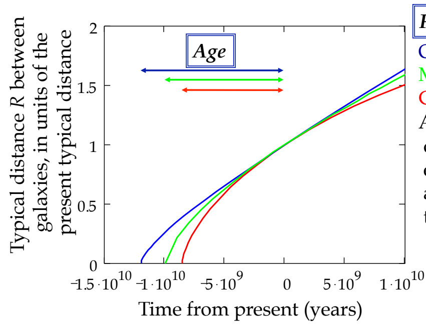
Here are some results of such calculations, for matter-dominated universes with three different present-day densities. Labels indicate boundedness and the sign of the spacetime curvature *k*.

Matter-dominated universes (continued)



Fate

Open
Marginal
Closed

All matched to observed expansion rate at present time.

How can we tell which "universe" is our Universe?

Assuming we live in a matter dominated Universe there are three "simple" measurements we can make to determine which model applies:

1. Measure the **density** directly, using observations of the motions of galaxies to determine how much gravity they experience. (Much like our way of measuring black-hole masses by seeing the orbital motion of companion stars.)
2. Measure the **ages of the oldest objects** in the Universe.
3. Measure the **acceleration or deceleration of galaxies**: the rate of change of the Hubble "constant."

The first two ways are least difficult and provide most of our data. In order…

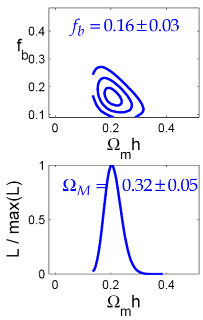
The mass density of the Universe

Astronomers have been perfecting measurements of this quantity for decades.

- Observational bounds on Ω_M, made from "nearby" galaxy redshift surveys over the past twenty years, consistently indicate

$$\Omega_M = 0.3 \pm 0.1$$

and that **only about 16% of this is normal matter**. We will take $\Omega_M = 1/3$.



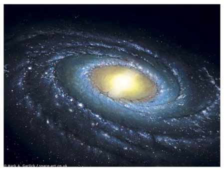
Likelihood L of various values of $\Omega_M h$, where h = 0.65 (the Hubble constant in units of 100 km/sec/Mpc), and the resulting probabilities for the fraction of the mass that's in the form of normal ("baryonic") matter. From the Sloan Digital Sky Survey (Pope et al. 2004); see www.sdss.org.

Dark matter

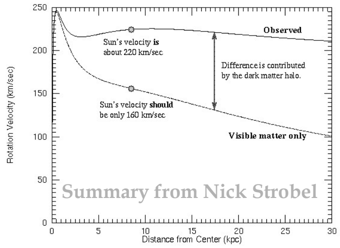
You heard that right: we detect lots of matter, through that matter's gravity and its influence on the motions of galaxies – a third of the amount it would take to close the Universe – but only a small fraction of this mass, 0.16 of it (16%) exists in the form of normal matter (i.e. atoms).

- The rest (84%!) is called **dark matter** because it signals its existence only by its gravity (so far), not by emitting light.
- We don't know what it is made of; all we know is that it can't contain protons and neutrons. (It can't be photons or neutrinos or electrons either.)
- Thus we search for its nature among the zoo of elementary particles that can be produced and detected in high-energy physics experiments.

Evidence for dark matter: Galactic rotation curves.



Milky Way Illustrated.
Illustration Credit & Copyright: Mark Garlick (Space Art)



The gravity of the visible matter in the Galaxy is not enough to explain the high orbital speeds of stars in the Galaxy. For example, the Sun is moving about 60 km/sec too fast. The part of the rotation curve contributed by the visible matter only is the bottom curve. The discrepancy between the two curves is evidence for a **dark matter halo.**

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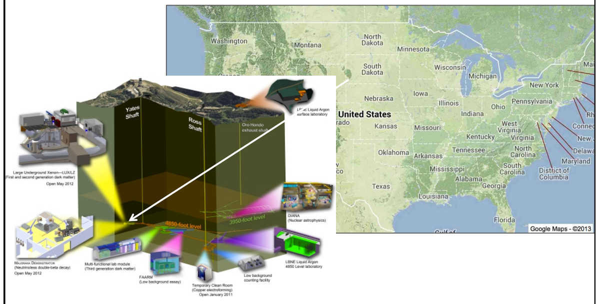
Sanford Underground Research Facility (SURF) – Lead, SD.



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The LUX Dark Matter Detector. LUX installed in the Davis Laboratory.

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LUX Dark Matter Results

The mass density of the Universe (continued)

- So if the Universe is matter-dominated, its curvature is negative, it is open, and it will continue to expand.
- It is, however, a strong theoretical prediction of many models of elementary particles and of the early Universe, especially those involving **inflation**, that Ω_M **should be exactly 1**, and that for unknown reasons the present measurements of Ω_M are faulty. Observers and theoreticians used to argue incessantly about this.
- There are **no good experimental results or theoretical arguments to suggest that the universe is matter-dominated and closed**. We don't think our Universe is a black hole.

Age of matter-dominated universes

A general result of the solutions for matter-dominated universes is that the age is always given, in terms of the present value of the Hubble "constant", as

$$t = A\frac{1}{H_0}$$

where the value of the factor A depends on Ω_M, but is less than or equal to 1.

- The factor A is equal to 1 if Ω_M is very small compared to 1. The larger the value of Ω_M, the smaller the value of A. Open universes have values of A between 2/3 and 1, and closed universes have values of A smaller than 2/3.
- Jargon: $t = 1/H_0$ is often called "one **Hubble time**."

Age of matter-dominated universes (continued)

- If Ω_M is assumed to be much smaller than 1, the age would be

$$t = \frac{1}{H_0} = \frac{\text{sec Mly}}{20\text{ km}} = \frac{\text{sec}\times\left(10^6\times3\times10^5\text{ km sec}^{-1}\times\text{year}\right)}{20\text{ km}}$$
$$= 1.5\times10^{10}\text{ years.}$$ (as we saw a few lectures back)

- If Ω_M is assumed to be 1, the factor A turns out to be exactly 2/3, and the age is

$$t = \frac{2}{3}\frac{1}{H_0} = 1.0\times10^{10}\text{ years}$$

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Age of matter-dominated universes (continued)

For the best experimental value, $\Omega_M = 1/3$, we get

$$t = 1.2\times10^{10}\text{ years}$$

Other constraints on the Universe's age, independent of density determinations:

- We know that the Universe must be older than the solar system, which is 4.6×10^9 years old, so an age of 1.2×10^{10} years would be OK on this score.
- The ages of white dwarf stars and globular star clusters turn out to be accurately measurable; the oldest of these are 1.2×10^{10} years old ($\pm$ about 0.1×10^{10} years).

This agrees with $\Omega_M = 1/3$ (though smaller would be more comfortable)**, and is in conflict with $\Omega_M = 1$.**

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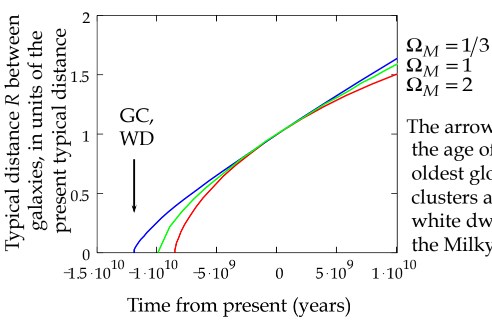
Age of matter-dominated universes (concluded)



The arrow marks the age of the oldest globular clusters and white dwarfs in the Milky Way.

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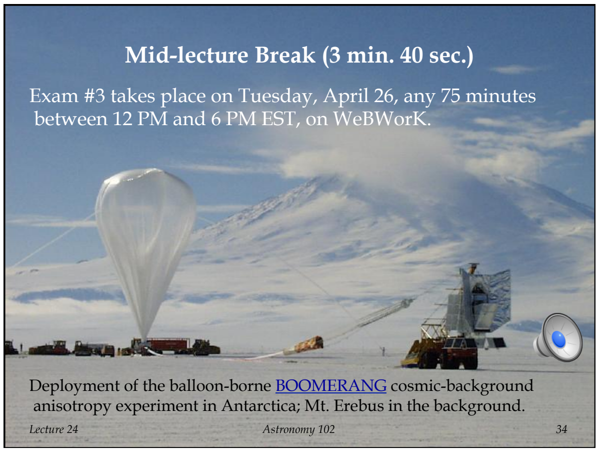
Mid-lecture Break (3 min. 40 sec.)

Exam #3 takes place on Tuesday, April 26, any 75 minutes between 12 PM and 6 PM EST, on WeBWorK.



Deployment of the balloon-borne BOOMERANG cosmic-background anisotropy experiment in Antarctica; Mt. Erebus in the background.

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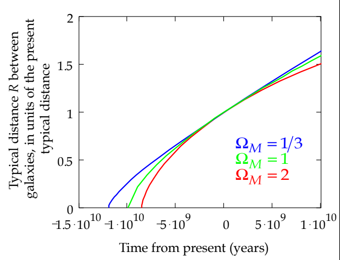
The third method: measurement of the acceleration of distant galaxies

The third way of finding which Big Bang model fits our Universe best is the measurement of acceleration of distant galaxies.

- Looking back through time at distant galaxies, one should be able to measure the shape of R vs. t if one can measure distances accurately enough.



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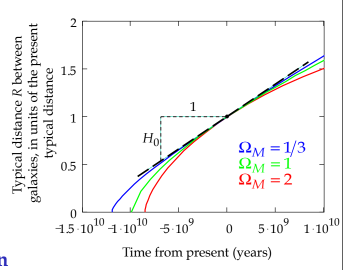
Measurement of the acceleration of distant galaxies (continued)

The **slope** of the curves we've been plotting (R/R_0 vs. t), at the present time, turns out simply to be the Hubble constant.

The matter-dominated models curve away from the straight-line Hubble law at large distances, in the direction of larger values of H_0: **deceleration** of the Universal expansion.



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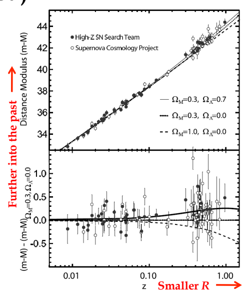
Measurement of the acceleration of distant galaxies (continued)

In the late 1990s it became possible to measure distances and redshifts for galaxies containing supernovae of type Ia, at distances large enough to reveal departure from the straight-line Hubble law.

To the great surprise of most astronomers, very distant SN Ia were fainter than expected – from which it is inferred that they are significantly more distant than expected. The curve bent in the direction of **smaller** H_0.



Riess, Fillipenko et al. 1998, Perlmutter et al. 1999

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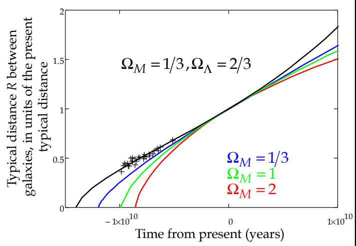
Measurement of the acceleration of distant galaxies (continued)

Those who observed these supernovae were quick to point out that the bend in the curve was in the direction of **acceleration** of the Universal expansion, rather than the anticipated deceleration, which implies substantial **dark energy** in the Universe as well as matter.



SN Ia -galaxy observations by Riess et al. (2007), transposed onto matter-dominated (blue +) and accelerating (black +) models.

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Measurement of the acceleration of distant galaxies (continued)

Because the effect being measured is small, and because the supernovae and galaxies being observed are so distant, these results were a bit controversial.

- ❑ Most of the controversy had to do with the assumption that SN Ia have the same “yield” – give off the same amount of light – whether they happened recently or ten billion years ago.
- ❑ The abundance of elements heavier than helium decreases substantially as one looks back further in the past.
- ❑ This in principle can alter the amount of light given off by a SN Ia, and even the direction the light is beamed, and the physics of these blasts is sufficiently complicated that theoretical models of them have not been conclusive.

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Measurement of the acceleration of distant galaxies (continued)

Reaction typical, though physicist famous:

…I encountered a hard-bitten veteran gravitation physics colleague in the elevator of the Princeton physics building and asked him if he believed the purported evidence of accelerating expansion. "No," he replied. Neither do I. Why not? Two reasons: (1) Because the speed-up argument relies too trustingly on the supernovae being standard candles. (2) Because such an expansion would, it seems to me, contradict a view of cosmology too simple to be wrong.

John Archibald Wheeler (who preferred the closed Universe with $\Omega_M > 0,\ \Omega_\Lambda = 0$)

A fourth method: measurement of the curvature of space.

Nobody really fussed about the acceleration controversy too much, though, because measurements of the **curvature of space** between here/now and the epoch of Decoupling were on the horizon.

- Acceleration enthusiasts and detractors alike looked forward to these new measurements as conclusive, as they would determine *k* and Ω_{total} independent of observations of supernovae and galaxies.
- The curvature of space in the nearby Universe is too small to measure in the foreseeable future, but observations of the small-scale structure ("**anisotropies**") of the cosmic microwave background (CMB) offer a way to measure the curvature on a grand scale.

Measurement of the curvature of space (continued)

- Recall that the anisotropies are very small; none differ by more than 0.001% in brightness from the average brightness of the CMB.
- The COBE satellite could not detect small enough angular scales to solve this problem.
- Astronomers had been trying for two decades to detect anisotropies on angular scales to measure curvature, using ground-based telescopes, but without much success. Fluctuation in atmospheric transmission, and civilization-created radio interference, kept ruining the observations.
- Finally in the late 1990s and early 2000s the problems were overcome by leaving the absorbing part of the atmosphere.

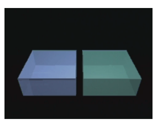
Measurement of the curvature of space (continued)

- Observations from extremely dry sites, like the South Pole (e.g. ACBAR) or the high Atacama desert in Chile (e.g. CBI).
- Long-duration observations from high-altitude balloons.
  - Several-day flights give useful results (e.g. MAXIMA), but better observations are enabled, and made uniquely difficult, by steady circumpolar winds in the arctic and antartic: with luck, the balloon blows around to its starting point in about a month. Best example is BOOMERANG.
- Satellite observations, à la COBE: the Wilkinson Microwave Anisotropy Probe (WMAP), launched in 2001. These measurements turned out to be definitive.

Measurements of the Universe's space curvature

The small-angular-scale anisotropies in the cosmic microwave background provide the means to measure the curvature of the Universe rather directly. Reasons:

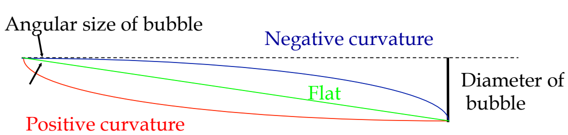
- Before decoupling, the Universe consisted of ionized gas in equilibrium with photons. This gas-photon mixture took the form of **bubbles** with very slightly different densities and temperatures.
- If a bubble were compressed by its neighbors, it heated up and pushed back on its neighbors all the harder. Thus the bubbles could **oscillate** in size and temperature.
- The speed with which these bubbles oscillate is limited by the **speed of sound** in the gas.
- The cosmic microwave background is a snapshot of the final state of these bubbles, and the anisotropies outline the bubbles.



Animation courtesy of NASA and the WMAP Science Team. See map.gsfc.nasa.gov.

Measurements of the Universe's space curvature (continued)

- It turns out that the bubbles that are the most numerous are the ones that have only gone through half an oscillation between the Big Bang and decoupling. Their diameters can be calculated precisely. (We know the speed of sound *and* the speed of light.)
- By observing their angular size and knowing their diameters we can determine the curvature of space between decoupling and here-and -now.

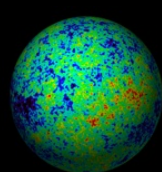


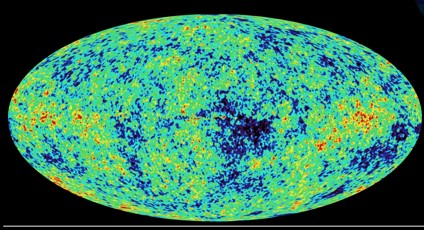
Measurements of the Universe's space curvature (continued)

Animation courtesy of the WMAP Science Team (NASA/GSFC). See map.gsfc.nasa.gov .

Measurements of the Universe's space curvature (continued)

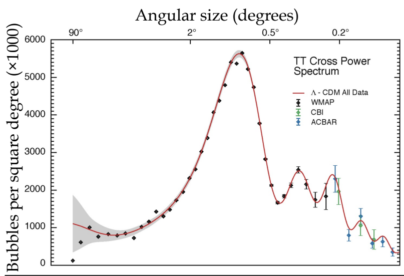
Map of the sky (plane of Milky Way along the equator) on scales small enough to measure curvature for all current models of the Universe, by WMAP.





Measurements of the Universe's space curvature (continued)

Result: $k = 0$ – **the Universe is flat** – and $\Omega = \Omega_M + \Omega_\Lambda = 1$.

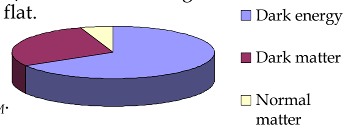


Black points: results from WMAP ($\Omega_M = 0.3$, Bennett et al. 2003). In **red**: expectations for $\Omega_\Lambda = 0.7, H_0 =$ 22 km sec^{-1}Mly^{-1}.

Measurements of the Universe's space curvature (continued)

How did the Universe come to be flat?

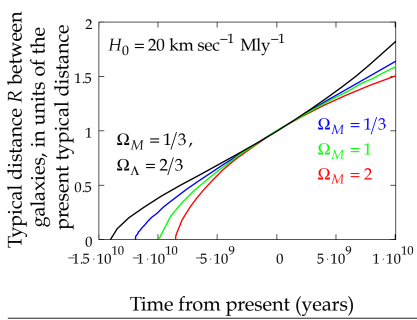
- We know that $\Omega_M = 1/3$: there isn't enough matter in the Universe to make it flat.
- The **simplest** way out seems to be dark energy, in the amount $\Omega_\Lambda = 1 - \Omega_M$.



- This, as discussed above, would account for the apparent acceleration of the Universal expansion, seen in the high-redshift SN Ia results.

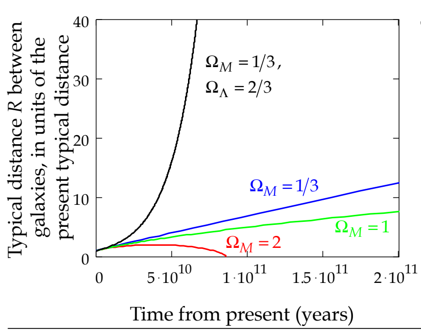
Einstein is having a really good laugh about this.

Age of the new flat Universe



Here the "new" Universe is compared to the matter-dominated models. Its present age turns out to be **1.4×10^{10} years**.

Fate of the new flat Universe



The new Universe is open and expanding exponentially; in just a few Hubble times most of the Universe we can see today will be redshifted into invisibility.

Summary: best (experimental) determination of the state of the Universe

- The Universe has a present-day relative mass density of about $\Omega_M = 1/3$, not nearly enough to close the Universe.
- If matter were to dominate its energy, the Universe would be negatively-curved and **open**, and about 1.2×10^{10} years (12 billion years) would have elapsed since the Big Bang.
- But the cosmic background small-scale anisotropies indicate that the Universe is **flat** between here and the decoupling surface. Easiest to explain if $\Omega_\Lambda = 2/3$; the Universe's dynamics are dominated by dark energy.
- Thus, the **Universe is open**, the present expansion will continue and will increase dramatically over time, and **the Universe is about 1.4×10^{10} years (14 billion years) old**.

Caveats

There are still doubters, though; they might even be in the majority. Two substantial reasons to doubt that this – a dark-energy dominated Universe – is the whole story:

- *How much do you trust Occam's razor?* This is the simplest model that explains the observations, but begs the question of what dark energy actually is, and stands unique among complex systems in the simplicity of its description. (A Universe simpler than a star or planet?)
- *If the model is true, we're in a privileged position.* We now find ourselves poised on the boundary between the matter-driven and dark-energy-driven eras of Universal expansion. Ask Copernicus what we risk by thinking we live at the center of the Universe…

Let's assume it's true, though. What's next?

Within a few tens of billions of years:

- The rapidly-increasing Universal expansion will not soon result in the expansion of compact, tightly-bound things like you, the Earth, or the Milky Way.
- But the exponential expansion will render invisible parts of the Universe that are currently visible.
  - As space expands more rapidly, widely separated parts that light could currently travel between within the age of the Universe, can no longer make the trip. We will lose sight of our surroundings, beginning with the most distant galaxies. Eventually…

What's next? (continued)

- The Milky Way and its closest companions will be all that can be seen of the Universe of galaxies. It will die alone, as eventually its matter is converted to black holes and radiation.

❑ Eventually it will be impossible even to verify the origins of the Universe.

- The Cosmic Microwave Background will become redshifted so extremely – its temperature becoming so close to absolute zero – that it will become impossible to detect.
- No galaxies in view: no Universal expansion to characterize.

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What's next? (continued)

This makes our current position seem even more privileged: we can still **demonstrate** that the Universe began in the explosion of a mass-density singularity, that the ensuing expansion has been in progress for 14 billion years, and that the Universe is spatially flat and open. In another 100 billion years, those experimental facts could become undemonstrable, and come to be regarded as fables.

NASA/WMAP Science Team

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Done!
Aurora and Unusual Clouds Over Iceland.

Image Credit & Copyright: Stéphane Vetter (Nuits sacrées)

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