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# Today in Astronomy 102: the insides of black holes

- ❑ Physics and metaphysics, positivism and idealism: should we consider the interior of a black hole to be real?
- ❑ Assuming that we do: physics inside the event horizon, and the nature of the quantum-gravitational object at the center, the mass-density singularity.



**A Phoenix Aurora over Iceland**  
**Image Credit & Copyright: Hallgrímur P. Helgason**



## Brief review

Exotic matter is

- A. matter with negative mass.
- B. matter with negative energy density.
- C. matter made completely of antiparticles.
- D. matter made completely of vacuum.



## Brief review

Entropy is

- A. a measure of the number of rearrangements of a system that could be made without changing its overall appearance.
- B. a measure of the size and scale of vacuum fluctuations present in a system.
- C. a measure of the mass density of the horizon of a black hole.
- D. All of these.
- E. None of these.

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## Inside the horizon: physics (or metaphysics) of the mass-density singularity

We are about to discuss the physics of the interior of black holes, and first must deal with a rather obvious question:

□ Why should we, if information from the inside can never get out? Wouldn't such a study be uncomfortably close to metaphysics, rather than physics?

(Note: **Physics** = the study and description of the workings of the world accessible to our senses, measurements and reasoning.

**Metaphysics** = the study by logic of a world of ideal forms and eternally-existing, changeless objects; a world which is not accessible to our senses, only to our reasoning.

This distinction was first drawn, explicitly, by Aristotle, around 340 BC.)

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# Physics : metaphysics :: positivism : idealism

Humans who reflect upon the distinction between physics and metaphysics fall into two categories:

- **Positivists** (or **empiricists**) hold that the real world is the one accessible to the senses, and that this is the only real world, since all of our knowledge of reality has its origins in sense input. It is not helpful to speculate about any other world, since we can know nothing about it; **physics is the study of reality**.

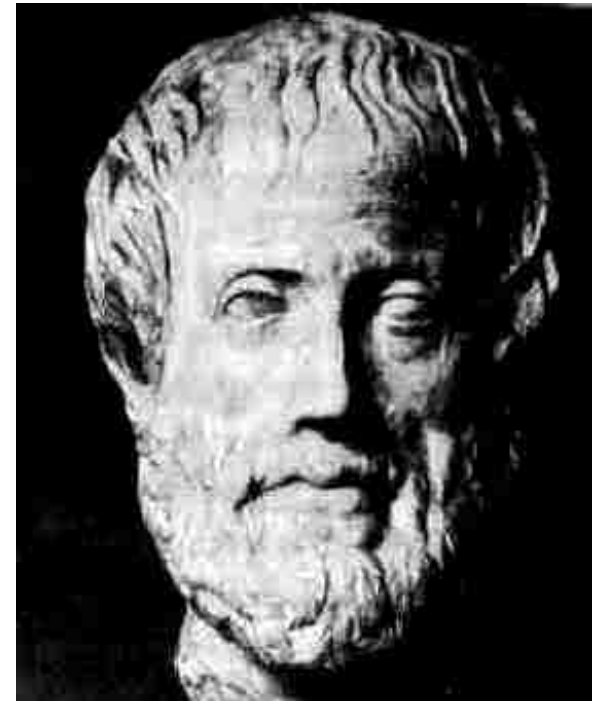


Friedrich  
Nietzsche, the  
definitive anti-  
-metaphysicist.

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# Physics : metaphysics :: positivism : idealism (continued)

- ❑ **Idealists** assert that the real world is the world of forms and ideal patterns, accessible to our logical acumen and our ability of abstraction, but inaccessible to our senses and measurements. The objects in the apparent world are merely ephemeral representations of the objects in the ideal world; **metaphysics is the study of reality.**
- ❑ **Science, by and large, is a positivistic activity**, since it requires experimental validation of theory.



Aristotle, grad  
student of the  
definitive  
metaphysicist.



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## Positivists and idealists

Some famous “positivists:”

Francis Bacon

David Hume

Johann W. v. Goethe

Auguste Comte

John Stuart Mill

Friedrich Nietzsche

Sigmund Freud

Albert Einstein

Bertrand Russell

Ludwig Wittgenstein

Logical  
positivism

Some famous idealists:

Parmenides

Plato

Aristotle

Classical  
philosophers

Avicenna

St. Thomas Aquinas

Rene Descartes

Immanuel Kant

Georg W.F. Hegel

Hegelian

Martin Heidegger

philosopher

Carl Jung

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## Inside the horizon: physics (or metaphysics) of the singularity (cont'd)

Why might even a positivist find it useful to study the interiors of black holes, even if it is initially a purely theoretical pursuit, without experimental constraint?

- ❑ **Naked mass-density singularities may exist.** Computer solutions to Einstein's field equation sometimes appear to produce the central mass-density singularity, but without event horizons – at least temporarily.
- ❑ **It may be possible to enter and exit certain combinations of black holes.** We will investigate one type of these, called wormholes.
- ❑ **The Big Bang may be similar to a black hole interior.** The Universe started out as a singularity; this may have observable consequences.



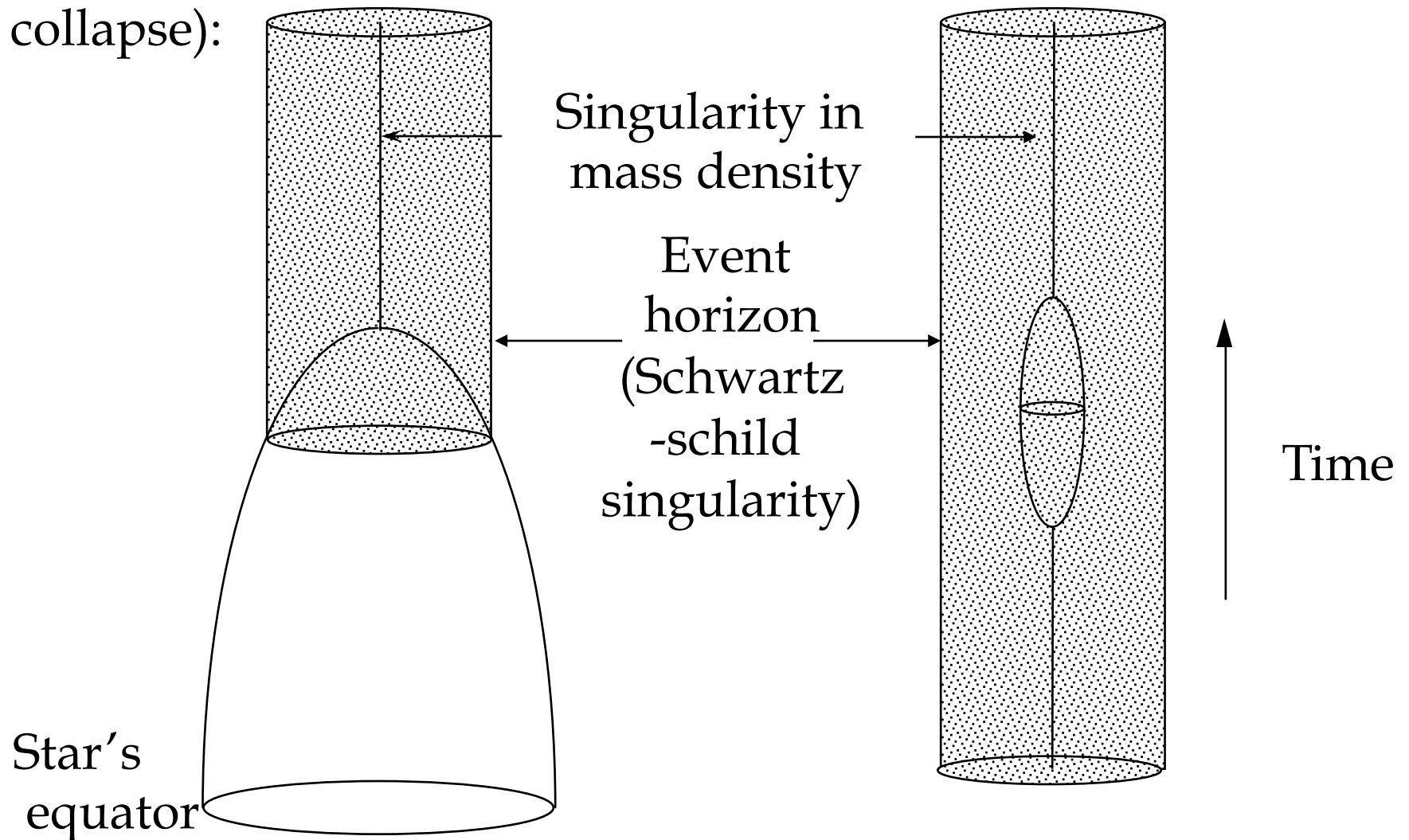
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## Inside a black hole

- ❑ Solutions of the Einstein field equations for the outsides of black holes occur that are stable in time (**static**), like the solutions originally obtained by Schwarzschild.
- ❑ However, for a mass (or collection of masses) distributed within a space smaller than the corresponding event horizon, there turn out to be **no static solutions** to the field equations. The solutions are of two kinds:
  - **collapsing** solutions: all the matter quickly converges on the center as time goes on, and a **singularity** in the mass density appears there in the solutions.
  - **expanding** solutions: the matter can expand briefly, within the horizon volume before collapsing to form a singularity (again).

Collapsing solution (example of black hole formation in stellar collapse):

Initially expanding solution:



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## Inside a black hole (continued)

Recall the following comments about singularities in the equations of physics and astronomy:

A formula is called singular if, when you put the numbers into it in a calculation, the result is infinity, or is not well defined. The particular combination of numbers is called the singularity.

Singularities often arise in the formulas of physics and astronomy. They usually indicate either:

- ❑ that not all of the necessary physical laws have been accounted for in the formula (no big deal), or
- ❑ that the singularity is not realizable (also no big deal), or
- ❑ that a mathematical error was made in obtaining the formula (just plain wrong).

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## Inside a black hole (continued)

This inevitable-collapse and singularity-formation behavior was first demonstrated theoretically for collapsing, spherical stars in 1939 by J.R. Oppenheimer and his group:

- ❑ Oppenheimer and Volkoff obtained field-equation solutions for static (neutron) stars larger than the horizon. (see lecture notes for Lecture 12)
- ❑ Oppenheimer and Snyder dealt with the realm past the limit of neutron degeneracy pressure, and showed that all solutions collapsed as time went on, and ended with a singularity.

This, again, is the **mass-density singularity** (not to be confused with the Schwarzschild singularity = event horizon).

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# The physics of mass-density singularities

Implication of the Oppenheimer-Snyder solution:

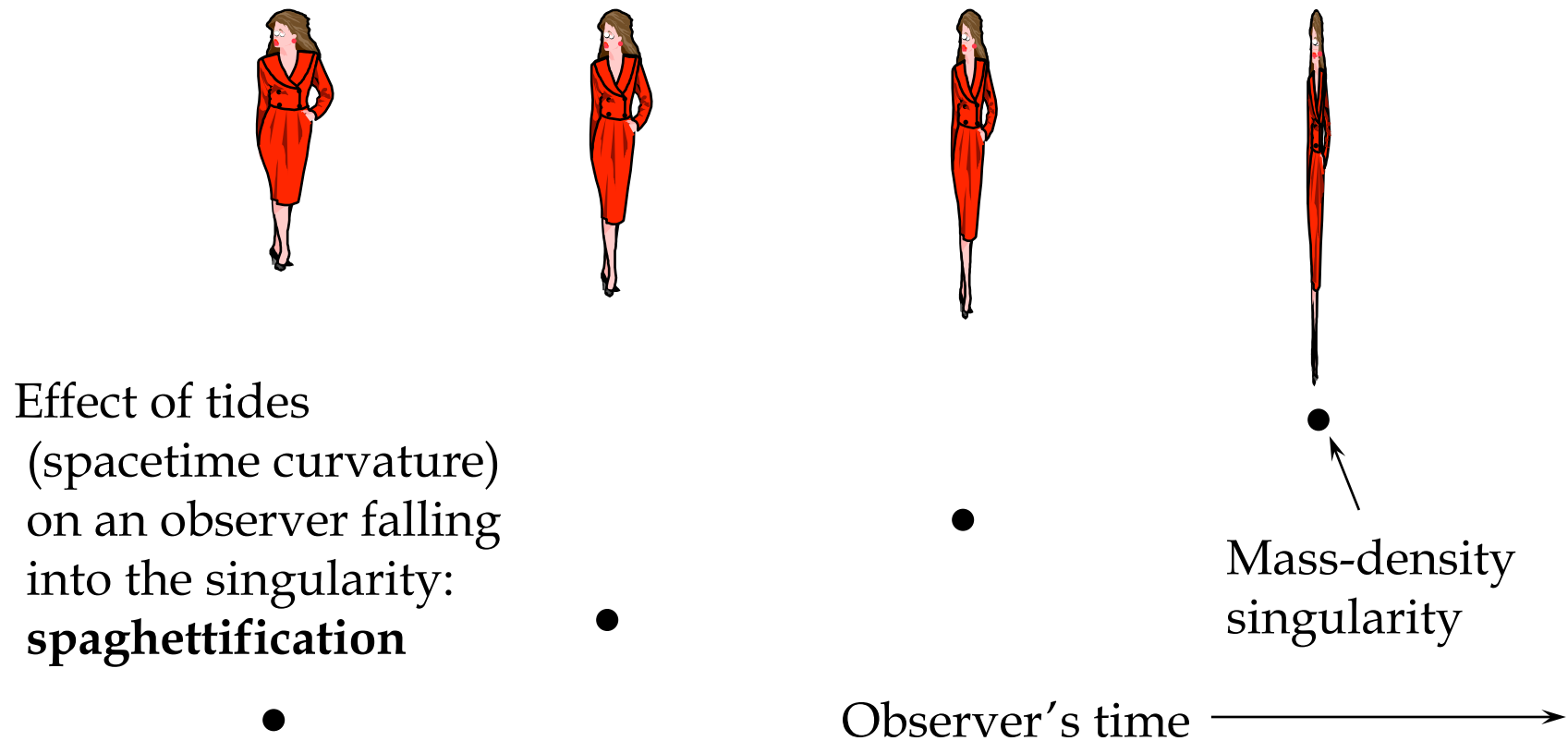
- ❑ Any matter inside the horizon falls into the center, collapsing to a single point in spacetime (i.e. a mass-density singularity). Formally, **spacetime ends at this point.** ►
- ❑ All paths (**geodesics**; the paths followed by photons) that matter can follow terminate in the mass-density singularity. Thus the region inside the horizon is indeed disconnected from the rest of the universe. ►

We know that no mathematical error was made by Oppenheimer and Snyder. Is the mass-density singularity realizable in nature, like the Schwarzschild singularity (#1), or have crucial physical effects been left out of the calculation that would prevent this singularity from forming (#2)? ►

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# Approaching the mass-density singularity

At the singularity, the **curvature of spacetime is infinite**, as is anything that gets stronger with more spacetime curvature; tidal forces, for example, also become infinite.




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## No time at the mass-density singularity

- Recall the Minkowski absolute interval from special relativity (which applies to flat spacetime):

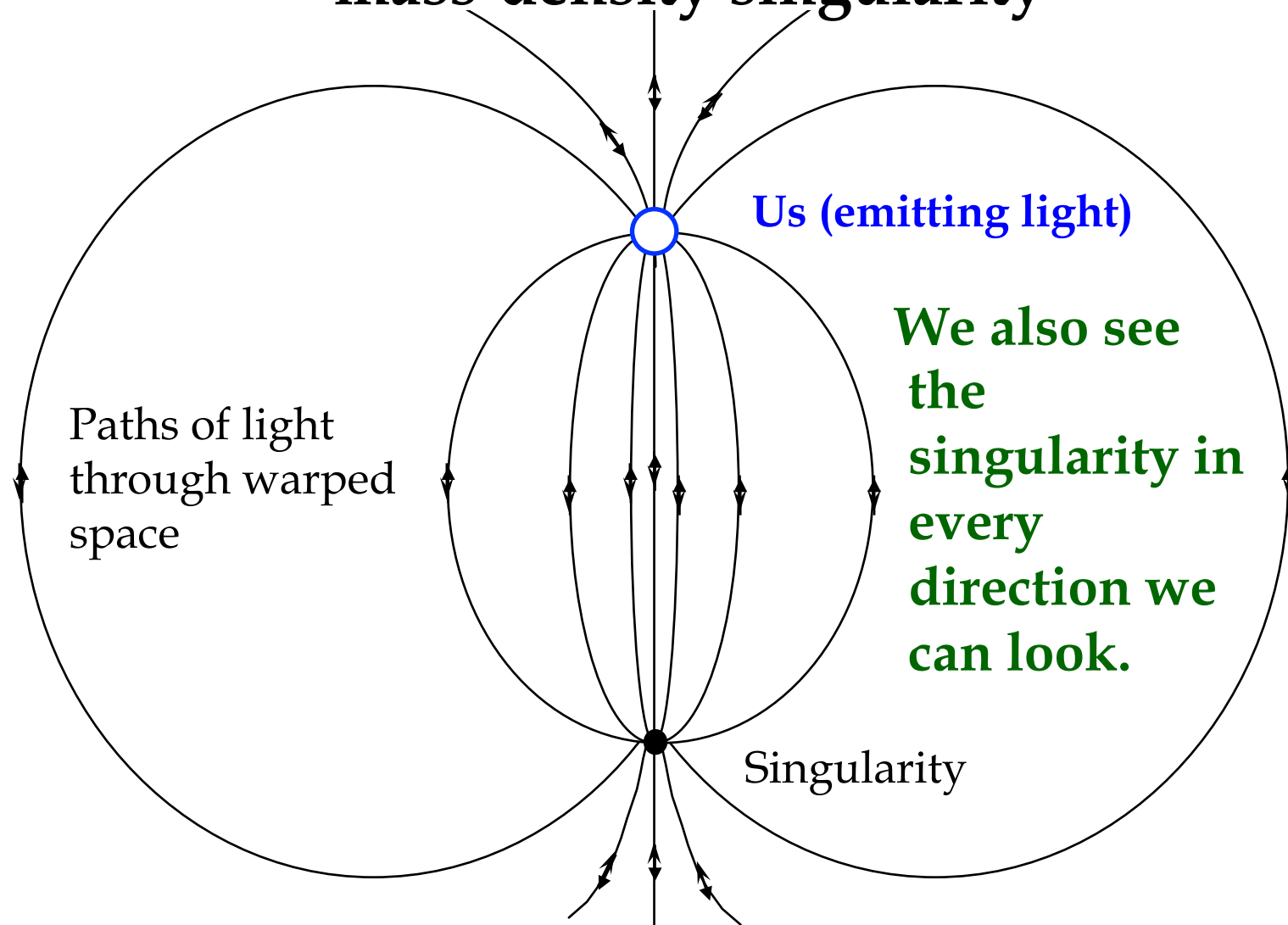
$$\Delta s = \sqrt{\Delta x^2 - c^2 \Delta t^2}$$

The spacetime coordinate that we experience as **time** enters the formula with a **minus** sign in flat spacetime. Coordinates that we experience as **space** or distance enter with **plus** signs.

- In the more complicated form for the absolute interval in field-equation solutions just outside the singularity, all coordinates enter the equation with plus signs. The four dimensions of spacetime all act like space; **there is no such thing as time at the mass-density singularity.**
- (We will see this again: it is why the answer to “what was there before the Big Bang?” is “there’s no such thing.”) 



# All paths within the event horizon lead to the mass-density singularity





## The vacuum near the singularity

Curvature, and related things like tidal force (= temperature, at the horizon) get very large close to the mass-density singularity. This will lead to effects on the vacuum:

- A. Vacuum fluctuations are very strongly exotic (antigravity).
- B. Virtual particles of large mass have larger influence on the surroundings than they would at the horizon.
- C. A continual bubbling of matter from the singularity, kind of like Hawking radiation but settling back in short order.
- D. All of these.
- E. None of these.



## Mid-lecture Break (2 min. 46 sec.)

- ☐ Homework #5 is now available on WeBWork.
- ☐ Due date for homework # 5 has been changed to April 16 at 8.30 am.



**The Great Nebula in Carina**

**Image Credit & Copyright: Damian Peach/SEN**

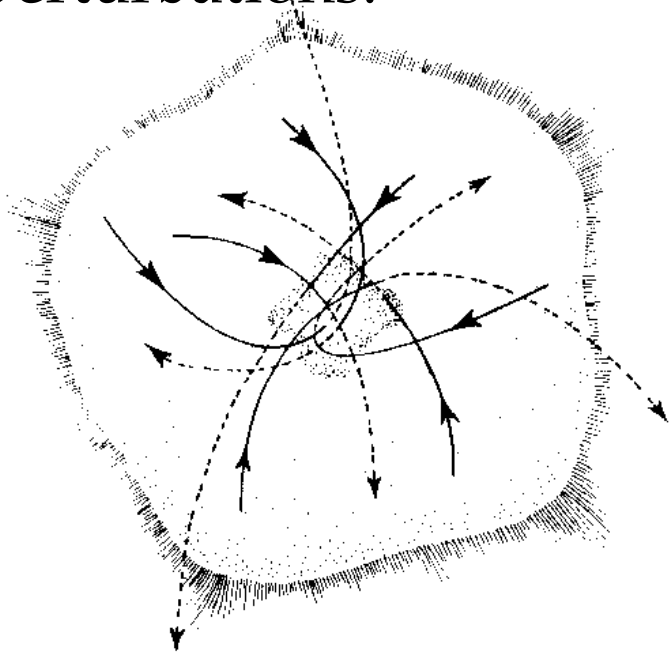
© Damian Peach



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## Possibility #1: is the mass-density singularity realizable?

**Khalatnikov and Lifshitz (1961):** No. You only get that for a perfectly spherical non-spinning star implosion; any deviation from this, however minor, leads to explosion. In other words, the Oppenheimer solution is unstable to small perturbations.



Infalling particles in an asymmetric collapse each fall toward a different point; since they don't meet in the center, they just “sling” against each other's gravity, and explode.

Figure: Thorne, *Black holes and time warps*

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## Possibility #1: is the mass-density singularity realizable? (continued)

**Penrose (1964):** Yes it is. It is possible to prove mathematically, and quite generally, the **horizon-singularity theorem**:

**Any** solution to the Einstein field equation that involves the formation of a horizon also involves the formation of a central mass-density singularity.

**Belinsky, Khalatnikov and Lifshitz (BKL, 1964):** Oops. There is a stable, singular solution after all, that works no matter how asymmetric the star was. Penrose is right.

- ❑ Stable solution: BKL, or mixmaster, singularity (BKL, Misner). Curvature inside horizon oscillates in time and space; the oscillation increases in strength as one approaches the singularity.

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## Possibility #2: have all the necessary physical laws been included?

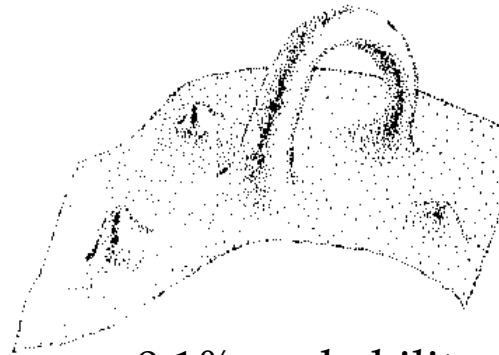
**Wheeler:** No, obviously, because quantum mechanics has been left out.

- ❑ No matter how massive the black hole is, its quantum-mechanical wavelength must still be nonzero.
- ❑ If the mass collapses to a size comparable to, or smaller than, its wavelength, then its wave properties become prominent. This seems to be the case for the mass-density singularity: it is a **quantum-gravitational object**.
- ❑ The wave properties, whatever their details turn out to be, will serve to spread the singularity out.
- ❑ The details are not yet known, unfortunately. There is no successful, consistent quantum theory of gravity, yet.

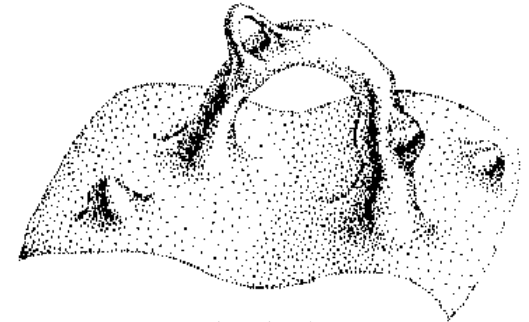
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## Possibility #2: have all the necessary physical laws been included? (continued)

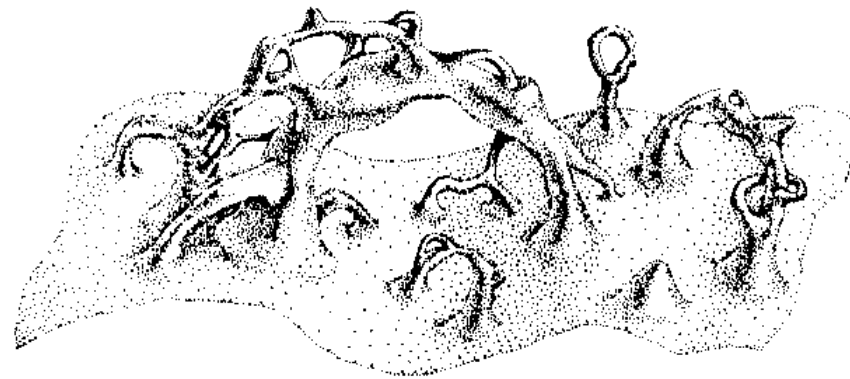
- Best guess: the mass-density singularity consists of a randomly -connected four-dimensional space (no time): **quantum foam**. Here are embedding diagrams for configurations of two of the four dimensions.



0.1% probability



0.4% probability



0.02% probability

Figure: Thorne, *Black holes and time warps*

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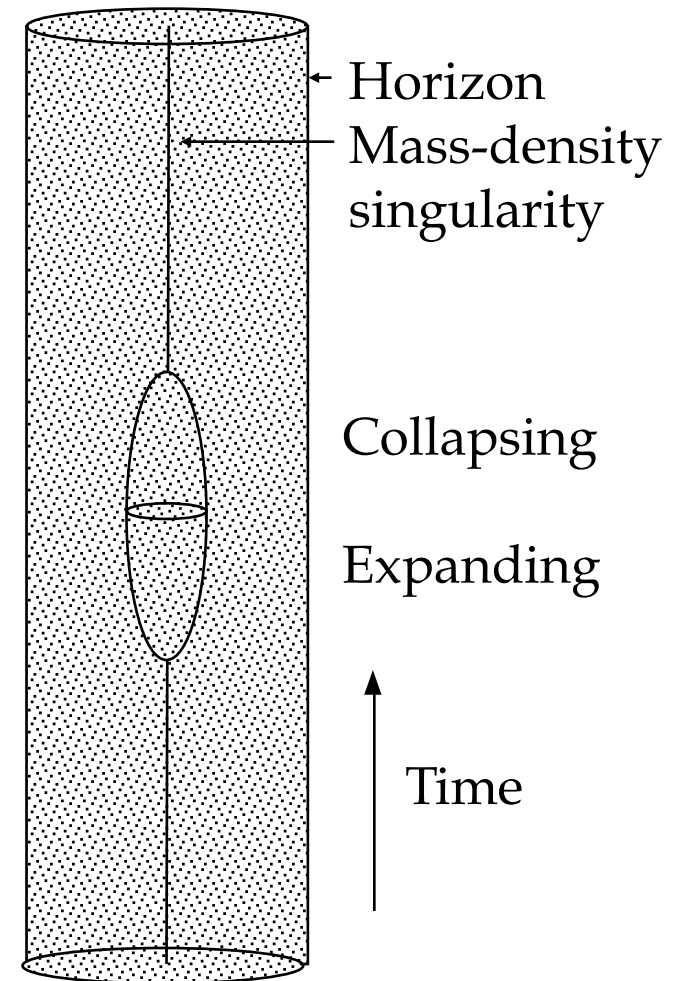
## Expanding and collapsing singularities

We don't know enough about quantum gravity to understand the properties of this “foam” in much detail, but:

- ❑ An infinite variety of “foam” configurations are possible; a particle falling into a mass-density singularity has a certain nonzero probability of finding each possible configuration.
- ❑ The next infalling particle would most likely find it (or cause it to be in) in a different configuration.
- ❑ Since time doesn't exist in the foam, there is no natural tendency for this “time origin” to connect in any predetermined way to spacetime outside the mass-density singularity.

# Expanding and collapsing singularities (continued)

- ❑ Some of the foam configurations might in fact connect better to surrounding spacetime in which **expansion** takes place, as in the Big Bang (as we'll see), rather than the contraction characteristic of black-hole formation.
  - Another way to look at this is that **in expanding mode, time flows out of the mass-density singularity** (like in the real Big Bang), rather than *in* (like in black hole formation).
- ❑ Thus it seems as though the mass-density singularity **might** switch back and forth between collapsing and expanding modes as it interacts with other masses and energies in the black hole's interior.



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## Expanding and collapsing singularities (continued)

### Implications:

- ❑ As it switches states, the mass-density singularity pushes and pulls the spacetime within the black hole's horizon. (Remember, spacetime *ends* at this singularity.)
  - If it really switches back *and* forth, it can create something resembling the “mixmaster” configuration of a black-hole interior (see Thorne, page 475).
- ❑ “Baby universes” may form inside massive black holes. (This is the basis of many a science-fiction story...)
- ❑ Black holes with their mass-density singularities in expanding configuration provide a useful paradigm for the formation of **wormhole**: a connection through hyperspace of two regions in spacetime that contain such singularities.



## Switching to expanding mode

Which of these could most easily cause a mass-density singularity to switch spontaneously from black-hole type to big-bang type?

- A. Matter falling in from outside the horizon.
- B. Light and/or gravity waves falling in from outside the horizon.
- C. Vacuum fluctuations very near the singularity, and their associated “Hawking radiation.”
- D. Disconnection in flow of time at singularity.

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## Can one see a mass-density singularity directly, and report the results to others?

**Penrose (1969): No.** In a survey of analytical-mathematical solutions of the Einstein field equation for various collapsing objects, a horizon was always produced. I propose, but cannot yet prove, the converse of my horizon-singularity theorem, the **cosmic censorship conjecture** :

Any solution to the Einstein field equation that involves the formation of a mass-density singularity also involves the formation of a horizon.

**Teukolsky and Shapiro (1991): Maybe.** In a survey of numerical, computer solutions to the Einstein field equation for very lopsided collapsing star clusters, some **naked singularities** were produced, lacking horizons for a time. Whether they can exist in nature remains to be seen.

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## Can one see a mass-density singularity directly, and report the results to others? (cont'd)

**Choptuik (1997): In a manner of speaking.** A numerical solution to the field equations for a collapsing spherical body, under some admittedly artificial initial conditions that probably would never be found in nature, produced a mass-density singularity before it produced a horizon.

Mostly on the strength of the Choptuik result, and amid much fanfare and press coverage at Caltech, **Stephen Hawking (1997)** conceded the bet he had made with **Kip Thorne** and **John Preskill**, as presented on page 481 of Thorne's book. It cost him £100 and two T-shirts.

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## The Hawking-Preskill-Thorne bet

Whereas Stephen W. Hawking firmly believes that naked singularities are an anathema and should be prohibited by the laws of classical physics,

And whereas John Preskill and Kip Thorne regard naked singularities as quantum gravitational objects that might exist unclothed by horizons, for all to see,

Therefore Hawking offers, and Preskill/Thorne accept, a wager with odds of 100 pounds sterling to 50 pounds sterling, that when any form of classical matter or field that is incapable of becoming singular in flat spacetime is coupled to general relativity via the classical Einstein equations, the result can never be a naked singularity.

The loser will reward the winner with clothing to cover the winner's nakedness. The clothing is to be embroidered with a suitable concessionary message.



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# Done!

## Horsehead and Orion Nebulas.



Image Credit & Copyright: Roberto Colombari & Federico Pelliccia