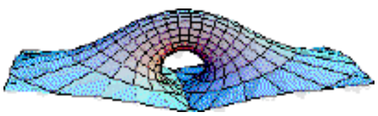
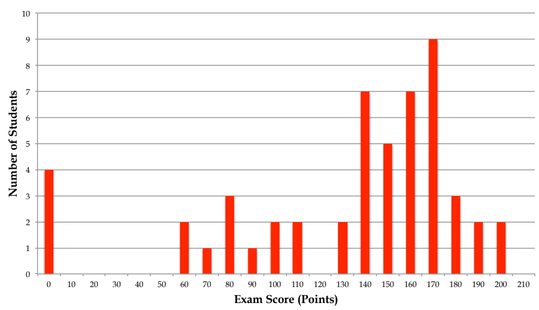
Today in Astronomy 102: black hole evaporation

- ❑ Hawking's "area increase" theorem.
- ❑ Entropy and the area of the event horizon: the thermodynamics of black holes.
- ❑ Quantum-mechanical vacuum fluctuations and the emission of light by black holes: "Hawking radiation."
- ❑ Evaporation of black holes.
- ❑ Exotic matter.

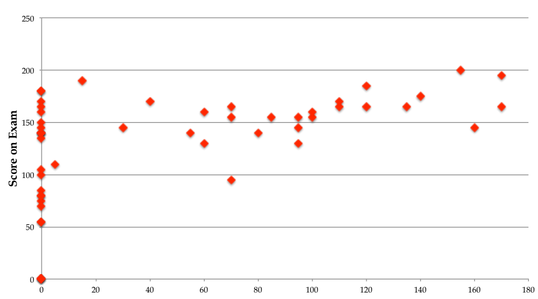
Image: a typical intra-universe wormhole. From Matt Visser, Victoria U. of Wellington



Results Exam # 2



Correlation between practice exam 2 and exam 2.



The horizon area theorem

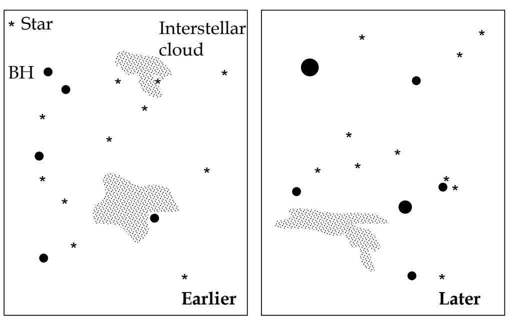
In 1970, Stephen Hawking used general relativity to prove a useful rule called the **horizon area theorem**:

> The total horizon area in a closed system containing black holes never decreases. It can only increase or stay the same.

Increases in total horizon area come from growth of black holes by collapse or accretion of "normal" matter, and by the coalescence of black holes.

Illustration (next page): a closed-off part of the universe. As time goes on, the total area of all the horizons in this closed system increases, owing to the growth of black holes by collapse, accretion and coalescence.

Illustration of Hawking's horizon-area theorem



So why should that seem strange?

The horizon-area theorem is simple (and intuitively obvious), but represents a puzzle at a deeper level when one reflects on the **heat** and **disorder** in the matter that forms or falls into a black hole.

- ❑ Before: the matter is hot, and there are lots of particles sharing the heat among themselves in the form of their random motions. A complete description of the system would thus have different entries for position and velocity for each particle – a vast number of *numbers* required.
- ❑ After: the system can be completely described by only three numbers, its mass, spin and charge. It's orderly!

The problem is, in all other natural processes **matter is never seen to go from a disorderly state to an orderly one all by itself.** This is in fact a law of thermodynamics…

Entropy and the second law of thermodynamics

Compare these these two statements:

The **horizon area theorem**:

- The total horizon area in a closed system never decreases.

The **second law of thermodynamics**:

- The total entropy of a closed system never decreases.

Entropy = the logarithm of the number of ways all of the atoms and molecules in a system can be rearranged without changing the system's overall appearance. A larger entropy means the system is more disorderly, or more "random."

Do black holes really have entropy as low as they seem to? Does horizon area have anything to do with entropy?

For adepts, scientists and, engineers

Others may ignore this page (i.e. **not on exams or HW**):

Officially, the definition of entropy includes an additional multiplicative factor:

$$S = k \log \Omega$$

where S = entropy,

k = Boltzmann's constant, 1.381×10^{-16} erg K^{-1},

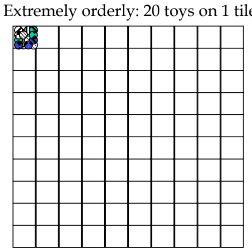
Ω = number of equivalent rearrangements,

and by "log" is meant the natural (base e) logarithm.

- For AST 102: suffice it to say that entropy is some number times the logarithm (any base) of the number of equivalent rearrangements.

Example of entropy: toys in a playroom (Thorne, pg. 424)

Extremely orderly: 20 toys on 1 tile

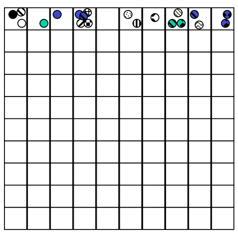


Number of equivalent rearrangements = 1; entropy = 0.

This playroom floor has 100 tiles, on which the kids can arrange 20 different toys. Parents prefer the toys to be kept in an extremely orderly configuration, with all the toys piled on one tile in one corner, as shown. There is only one such arrangement; the entropy of this configuration is thus the logarithm of 1, which is zero.

Entropy in a playroom (continued)

Orderly: 20 toys on 10 tiles



Number of equivalent rearrangements = 10^{20}; "entropy" = 20.

Parents might even accept this somewhat less orderly configuration: 20 different toys on 10 specific tiles. But there are lots of different equivalent arrangements (e.g. swapping the positions of two toys on different tiles produces a different arrangement that's still acceptable): 10^{20} of them, in fact, for an entropy value of $\log(10^{20}) = 20$.

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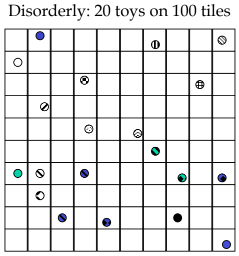
Entropy in a playroom (continued)

Disorderly: 20 toys on 100 tiles



Number of equivalent rearrangements = 10^{40}; "entropy" = 40.

Of course, when the kids are done playing, the floor looks like this: 20 different toys spread randomly over 100 tiles. There are $100^{20} = 10^{40}$ different ways to do that; the entropy is 40.

And kids are like natural physical processes. Through their agency the room will not get less random; the entropy of the room full of toys and kids never decreases by itself.

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Many consider the second law of thermodynamics to be the physical law least likely ever to be broken.

This is because its consequences are so easily tested experimentally that you can see it verified all around you every day. Among the important corollaries to the second law:

- Heat never flows by itself from a lower temperature to a higher temperature.
- Decreasing the temperature of one part of a closed system requires raising the temperature in other parts.
- The mechanical work (*organized* energy) that can be done by a heat engine is always less than the available heat (*disorganized* energy).
- **Irreversible processes**: two sequences energetically allowed, but only one sequence ever happens.

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Aside: irreversible processes and the Arrow Of Time

Greg Louganis, 3 m springboard, 1984 Summer Olympics

The laws of conservation of energy, momentum, and spin are indifferent to the order of time, and if they had the only say, all mechanical processes would be time-reversible.

- For instance, a diver can bound from a springboard, fall 5 meters, and hit the water with kinetic energy about 4×10^{10} erg.
- In the water he is brought to a halt, his kinetic energy converted to heat: this raises the temperature of the water (assumed uniform) by about 4.5×10^{-7} degrees C.

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Irreversible processes and the Arrow Of Time

Greg Louganis, 3 m springboard, 1984 Summer Olympics

- So in principle it's possible for the pool to cool down by a tiny amount, transfer that energy to the diver in the form of kinetic energy, and spit him back out of the pool.
- As you know, that never happens: this process is **irreversible**.
- The difference: the diver's kinetic energy is **organized,** the heat in the pool is **disorganized**. That divers never are flung back out of the pool is an expression of the fact that closed systems never become more organized spontaneously.

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Irreversible processes and the Arrow Of Time

Thus a deep connection exists between the second law of thermodynamics and the direction of time, which brings us back to relativity:

> From the standpoint of formal physics there is only one concept which is asymmetric in time, namely entropy. But this makes it reasonable to assume that **the second law of thermodynamics can be used to ascertain the sense of time independently in any frame of reference**; that is, we shall **take the positive direction of time to be that of increasing entropy**.
> (Panofsky and Phillips 1962).

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Mid-lecture break (2 min. 56 sec.).


The Jets of NGC 1097
Image Credit & Copyright: Martin Pugh

Horizon area and entropy (continued)

Black holes form from large collections of atoms and molecules, with extremely large numbers of equivalent rearrangements (large entropy).

What happens to the entropy of this matter when it falls into a black hole? This was a burning issue in 1972.

- **Hawking** (and the rest of the relativists in 1972): The entropy vanishes. Black holes and their horizons are extremely simple objects, with only one possible configuration ("equivalent rearrangement") each: that means zero entropy. Black holes therefore violate the second law of thermodynamics. That doesn't matter very much, because you can't get accreted material back out of the black hole anyway.

Horizon area and entropy (continued)

- **Jacob Bekenstein** (a Princeton graduate student in 1972, and all alone on this side of the argument): The second law of thermodynamics hasn't been violated in any other physical situation; why give up so soon? **The entropy of the ingredients may be preserved, in a form proportional to the horizon area.** If hole has entropy it also must have a temperature, which I find is proportional to the strength of gravity at the horizon.
- **Hawking *et al.* (1972):** But that would mean that the horizon is a **black body** at non-zero temperature that obeys the laws of thermodynamics. Any such body must radiate light - as the hot filament in a lightbulb does, for instance. *Nothing can escape from a black hole horizon*; how can it radiate?

Horizon area and entropy (continued)

- **Hawking *et al.* (1972):** This contradiction implies that black holes cannot have entropy or temperature, and that they must violate the second law of thermodynamics.
- **Bekenstein (1972):** I can't think of any way for light, or anything else, to escape from a black hole; I admit that black holes can't radiate. But there must be something wrong with your viewpoint, because it must be possible for black holes to obey the laws of thermodynamics.

And, actually, Zel'dovich already had thought of a way for horizons to radiate light, a year previously but unknown to these contestants until several years later.

Horizon area and entropy (continued)

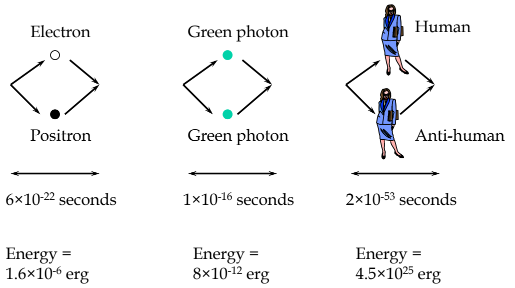
- **Hawking (1974):** Oops.
  There is a way for black holes to emit radiation: it involves quantum-mechanical processes near the black hole's horizon.
  - The emission of light is exactly as one would expect from a black body with temperature that increases as the strength of gravity at the horizon increases.
  - Therefore the black hole has entropy, which increases as the area of the horizon increases.
  - Therefore black holes obey the laws of thermodynamics. **Bekenstein is right after all**.
- The way Hawking found includes the way Zel'dovich had previously found, as a particular case.

How black holes emit light and other particles

- In subatomic matter, we see that energy conservation can be violated, though only temporarily and very briefly. This is expressed in one of Heisenberg's uncertainty principles:
  $$\Delta E \times \Delta t \approx h \approx 10^{-27} \text{ erg-sec}$$
- **Vacuum fluctuations**: the shorter the time interval Δt, the larger the energy ΔE that can be **temporarily** produced. For extremely short time intervals, enough energy can be borrowed from the vacuum (i.e. nothingness) to produce photons, or even massive particles.
- The particles thus made are called **virtual particles**. They vanish again at the end of the time interval Δt.
- Virtual particles are produced as **particle-antiparticle pairs**.

How black holes emit light (continued)

Examples of particle-antiparticle pairs made from the vacuum:



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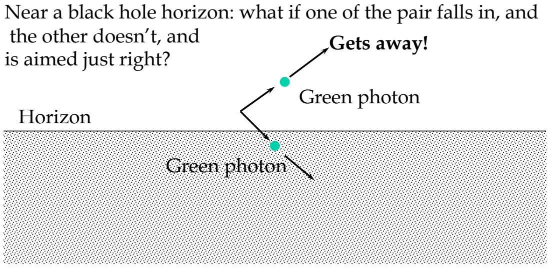
How black holes emit light (continued)

Normally, virtual pairs vanish too quickly to be noticed, or to interact much with anything else.

Near a black hole horizon: what if one of the pair falls in, and the other doesn't, and is aimed just right?



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How black holes emit light (continued)

Details of black-hole emission, nowadays called **Hawking radiation**:

- Virtual pairs, produced by vacuum fluctuations, can be split up by the strong gravity near a horizon. Both of the particles can fall in, but it is possible for one to fall in with the other escaping.
- The escaping particle is seen by a distant observer as emission by the black hole horizon: **black holes emit light** (and other particles), though only in this weird way.
- The energy conservation "debt" involved in the vacuum fluctuation is paid by the black hole itself: the black hole's mass decreases by the energy of the escaping particle, divided by c^2. The emission of light (or any other particle) costs the black hole mass and energy.

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Black hole evaporation

Hawking radiation is emitted more efficiently if the gravity at the horizon is stronger (i.e. its temperature is higher).

- Recall: horizon gravity is stronger for **smaller**-mass black holes.

Thus an isolated black hole will eventually evaporate, as it radiates away all of its mass-energy. The smaller the black hole mass is, the larger the evaporation rate is.

The time it takes to evaporate:

- $10^9\ M_\odot$ black hole: 10^{94} years.
- $2\ M_\odot$ black hole: 10^{67} years.
- 10^8 gram black hole: 1 second (!)

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Black hole evaporation (continued)

It's easy enough to derive the equation for the time it takes a black hole to evaporate *via* Hawking radiation that I usually assign it as a homework problem in Astronomy 142. The answer, for a black hole that starts off with mass M, is

$$t = \frac{10240\pi^2 G^2}{hc^4} M^3$$

$$= \left(8.407\times10^{-26}\ \frac{\text{sec}}{\text{gm}^3}\right) M^3 \quad ,$$

where $h = 6.626\times10^{-27}$ erg sec is Planck's constant, and you already know all the others.

(You need to understand, and learn how to use, this formula.)

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Relativistic Heavy-Ion Collisions. Brookhaven National Laboratory, NY, USA

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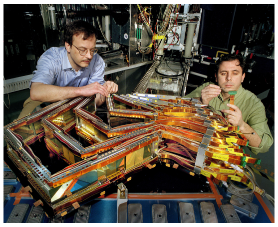
Relativistic Heavy-Ion Collisions. Brookhaven National Laboratory, NY, USA.



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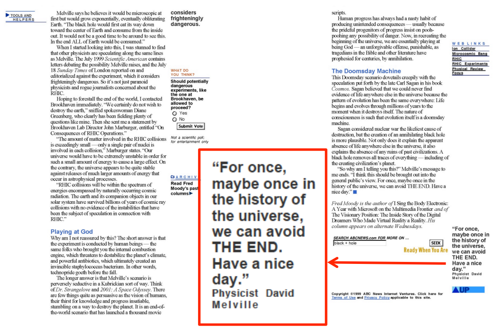
Creating Black Holes on Long Island.



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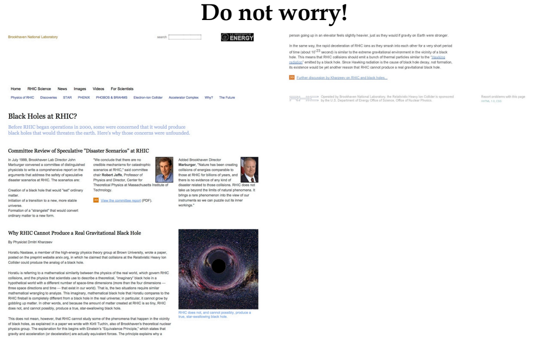
Creating Black Holes on Long Island. Do not worry!



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How long is that?

The Large Hadron Collider (LHC) in Switzerland **might, conceivably,** be able to produce tiny black holes, with mass 2.50×10^{-20} gm. How long will they last before evaporating?

A. 1.31×10^{9} sec B. 1.31 sec C. 1.31×10^{-24} sec
D. 1.31×10^{-54} sec E. 1.31×10^{-84} sec

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Hawking radiation is really tiny and usually insignificant compared to other sources of light commonly found near black holes.

That's why it takes massive black holes so long to evaporate. Consider how it works out for a 2 $M_\odot$ black hole.

Luminosity, if accreting at its maximum (Eddington) rate:

$$L = 2.5\times10^{38} \text{ erg/sec} = 6.6\times10^{4} L_\odot$$

Luminosity due to Hawking radiation:

$$L = 2.3\times10^{-22} \text{ erg/sec} = 5.9\times10^{-56} L_\odot$$

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Exotic matter

Part of this story should strike you as weird: Why is it that the black hole can *consume* a particle, and wind up *decreasing* in mass and energy?

- Because in the strongly warped spacetime near the horizon, virtual particles made from vacuum fluctuations turn out to have negative energy density.
  - **Energy density** = energy per unit volume.
- These particles indeed have positive mass -- look at the one that escaped! -- but their mass is distributed very strangely over spacetime. (Quantum-mechanically speaking, particles have nonzero volume; this is an aspect of the wave-particle duality.)
- Matter with negative energy density is generally called **exotic matter.**

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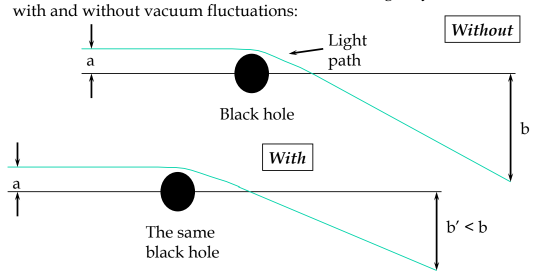
Exotic matter (continued)

Theoretical details of exotic matter, according to the present partial marriage of general relativity and quantum mechanics (incompletely known; only studied intensively since 1985):

- Quantum mechanical vacuum fluctuations in flat spacetime - far from any strong gravitational field - always have zero net energy density; they can never be exotic.
- However, in warped spacetime, vacuum fluctuations are in general exotic: their net energy density is negative, according to a distant observer measuring the energy density by observation of the deflection of light by the ensemble of fluctuations. The stronger the curvature, the more negative the energy density looks.

Exotic matter (continued)

Results of **theoretical calculations** of deflection of light by a black hole, with and without vacuum fluctuations:



Light is deflected less when vacuum fluctuations are included in the calculations: thus these fluctuations are "anti-gravity" (i.e. exotic).

Done!
In the Shadow of Saturn.

Image Credit: Cassini Imaging Team, SSI, JPL, ESA, NASA

