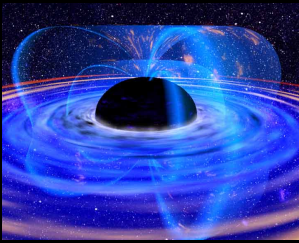


**Today in Astronomy 102: properties of “real” black holes, according to general relativity**

- ❑ Formation of a black hole from collapse of a massive, dead star.
- ❑ Properties of spacetime near black holes.
- ❑ “Black holes have no hair.”
- ❑ “Spacetime is stuck to the black hole.”
- ❑ Spinning black holes.
- ❑ Black holes and gravitational radiation



Artist's rendering of a spinning black hole (Dana Berry).

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**Crashing WebWork**

- During period 1, WebWork did not crash. There appeared to be no problem with 4 students working on the exam at the same time.
- During period 2, WebWork did not crash. There appeared to be no problem with 5 students working on the exam at the same time.
- During period 3, WebWork did not crash but response rate was very slow for those working on the exam. This was with 9 students working on the exam at the same time. Some students were not able to generate an exam.
- I determined that grading the exam takes 13 CPU seconds per student.
- I changed some of the server settings ..... Let's see what happens during period 4.

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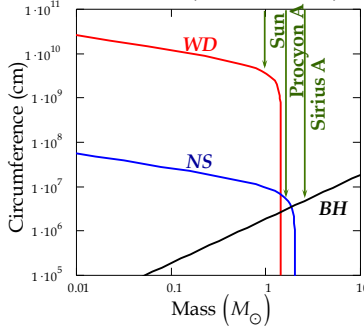
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**Final collapse of burned-out stars:  
white dwarf, neutron star, or black hole?**



If these stars do not eject mass while in their death throes, their fates are as follows: the Sun will become a white dwarf, Procyon A would become a neutron star, and Sirius A would become a black hole.

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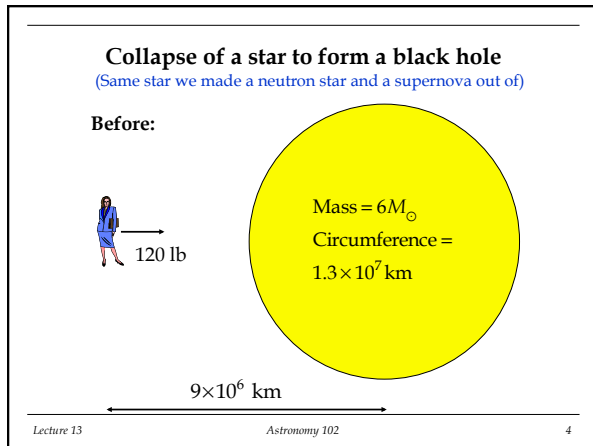
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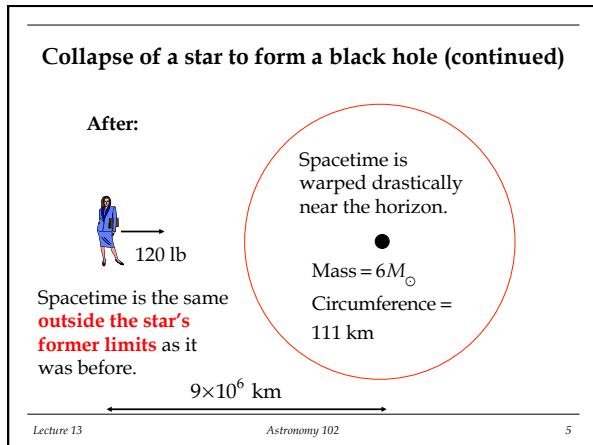
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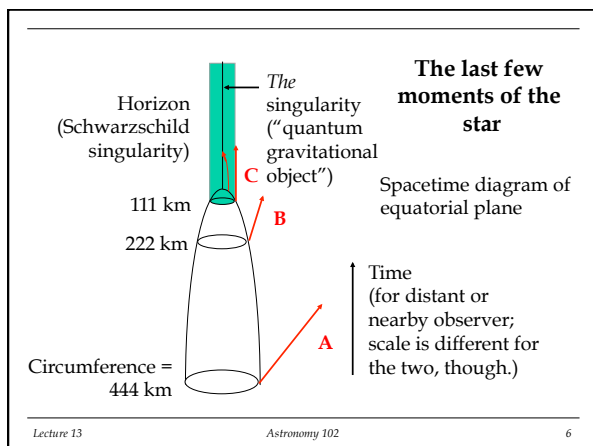
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**Last few moments of the star,  
to an observer on the surface**

444 km t = 0		Nothing in particular happens as the star passes through its horizon circumference; the collapse keeps going until the mass is concentrated at a point, which takes very little time.
222 km t = 0.0002 sec		
111 km t = 0.00027 sec		
0 km (!) t = 0.00031 sec		

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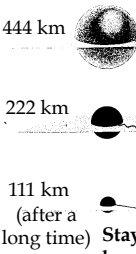
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**Last few moments of the star,  
to a distant observer**

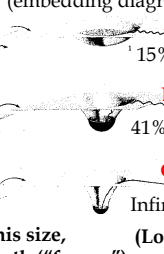
In reality



444 km  
222 km  
111 km  
(after a long time)


**Stays this size, henceforth ("frozen").**

In hyperspace  
(embedding diagram)



**A**  
**B**  
**C**

15% redshift  
41% redshift  
Infinite redshift  
**(Looks black!)**



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**For math adepts**

In case you're wondering where the numbers come from in the calculated results we're about to show: they come from equations that can be obtained fairly easily from the absolute interval that goes with the Schwarzschild metric, which we first saw a few lectures ago:

$$\Delta s = \sqrt{\frac{\Delta r^2}{1 - \frac{4\pi GM}{Cc^2}} + \frac{C^2}{4\pi^2} \Delta \theta^2 + \frac{C^2}{4\pi^2} \sin^2 \theta \Delta \phi^2 - c^2 \left(1 - \frac{4\pi GM}{Cc^2}\right) \Delta t^2}$$

We won't be showing, or making you use, these equations, but we can give you a personal tour of them if you'd like. A  $6 M_{\odot}$  black hole is used throughout unless otherwise indicated.

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### Space and time near the new black hole

**After:**  
Time is warped in strong gravity.

On time.  
↑  
Unchanged

Very slightly  
(factor of 1.000002)  
slower.

Very slow.

$9 \times 10^6 \text{ km}$

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### Gravitational time dilation near the new black hole

Duration of clock ticks (in seconds) a distant observer sees from a clock near a black hole.

If time weren't warped

Orbit circumference, in event horizon circumferences ( $C_s$ ).

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### Space and time near the new black hole (continued)

Physical space

$R = C/2\pi$

**After:**

Space is also strongly warped: for instance, points Y and Z are the same distance apart as points W and X.

Hyperspace

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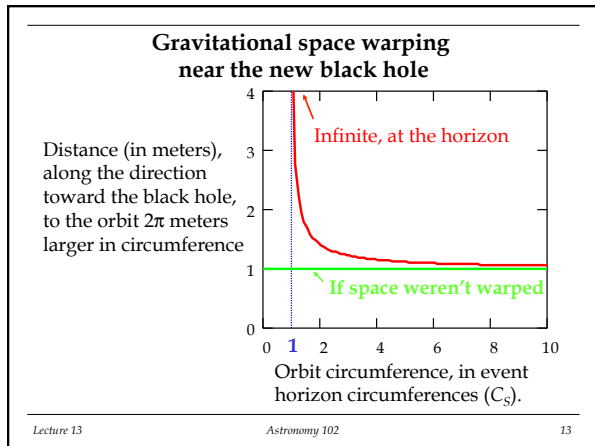
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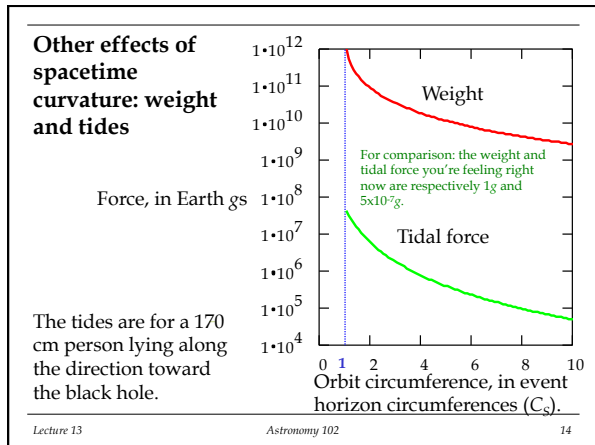
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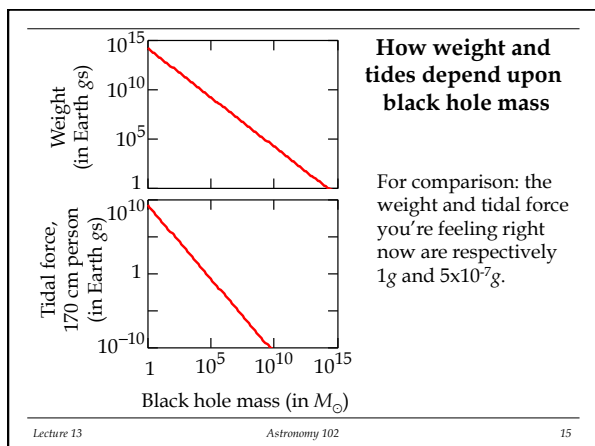
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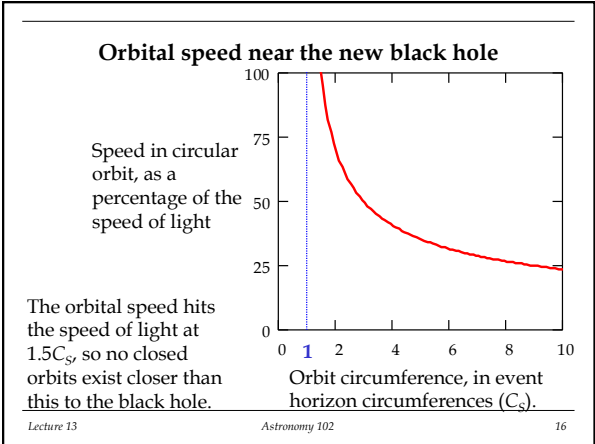
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**Time near a dead star!**

You stand where the surface of a  $6 M_\odot$  star used to be, but which has collapsed into a black hole. From a great distance, I can see your clock, and it seems to tick

A. at the same rate as mine.  
B. very slightly slower than mine, as would also have been the case before the star collapsed and the black hole formed.  
C. very slightly slower than mine, due to your proximity to the black hole.  
D. much slower than mine, as would also have been the case before the star collapsed and the black hole formed.  
E. much slower than mine, due to your proximity to the black hole.

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**Time near a dead star!**

You stand where the surface of a  $6 M_\odot$  star used to be, but which has collapsed into a black hole. You can see a nearby clock as well as your own, and that clock seems to tick

A. at the same rate as yours.  
B. very slightly slower than yours, as would also have been the case before the star collapsed and the black hole formed.  
C. very slightly slower than yours, due to your proximity to the black hole.  
D. much slower than yours, as would also have been the case before the star collapsed and the black hole formed.  
E. much slower than yours, due to your proximity to the black hole.

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**Mid-lecture break (3 min. 39 sec.)**

- ❑ **Homework # 4** is due on Monday March 21 at 8.30 AM.
- ❑ **Please complete our TA evaluations this week if you have not done so already.**



Artist's rendition of a black hole in a close binary system (M. Weiss, CXO/CfA/NASA)

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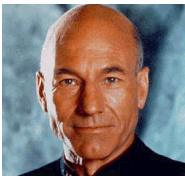
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**"Black holes have no hair"**

**Meaning:** after collapse is over with, the black hole horizon is smooth: nothing protrudes from it; and that almost everything about the star that gave rise to it has lost its identity during the black hole's formation. No "hair" is left to "stick out."

- ❑ Any protrusion, prominence or other departure from spherical smoothness gets turned into **gravitational radiation**; it is radiated away during the collapse.
- ❑ Any magnetic field lines emanating from the star close up and get radiated away (in the form of light) during the collapse.



Visitors to black holes suffer the effects too?  
(CBS Paramount)

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**"Black holes have no hair" (continued)**

- ❑ The identity of the matter that made up the star is lost. Nothing about its previous configuration can be reconstructed.
- ❑ Even the distinction between matter and antimatter is lost: two stars of the same mass, but one made of matter and one made of antimatter, would produce identical black holes.

The black hole has only three quantities in common with the star that collapsed to create it: **mass, spin and electric charge.**

- ❑ Only very tiny black holes can have much electric charge; stars are electrically neutral, with equal numbers of positively- and negatively-charged elementary particles.
- ❑ Spin makes the black hole horizon depart from spherical shape, but it's still smooth.

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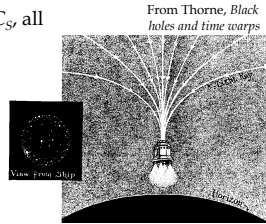
### "Space and time are stuck at black hole horizons"

**Time is stuck at the event horizon.**

- ❑ From the viewpoint of a distant observer, time appears to stop there (infinite gravitational time dilation).

**Space is stuck at the event horizon.**

- ❑ Inside a circle with  $C = 1.5 C_S$ , all **geodesics** (paths of light or freely-falling masses) terminate at the horizon, because the orbital speed is equal to the speed of light at  $C = 1.5 C_S$ ; nothing can be in orbit closer than that.



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### Space and time are stuck at black hole horizons (continued)

- Thus: from near the horizon, the sky appears to be compressed into a small range of angles directly overhead; the range of angles is smaller the closer one is to the horizon, and vanishes at the horizon. (The objects in the sky appear bluer than their natural colors as well, because of the gravitational Doppler shift).
- Thus space itself is stuck to the horizon, since one end of each geodesic is there.

**If the horizon were to move or rotate, the ends of the geodesics would move or rotate with it.** Black holes can drag space and time around.

This is a **very** important effect, since virtually all black holes would be expected to move and/or spin.

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### Time Dilation.

Gravitational time dilation approaches infinity – time is stuck – at a black hole's horizon. This means that, if you are just slightly outside a black-hole horizon,


- everything appears to you to happen in slow motion.
- everything far away from the black hole appears to you to happen in slow motion.
- everything you can see on the inside of the horizon appears to you to happen in slow motion.
- you feel as if you are moving in slow motion.
- you seem to distant observers to be moving in slow motion.

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### Complexity.

Which of these objects is the *least* complex, conceptually, mathematically, and physically?

A. A carbon atom.   B. A single-cell organism.  
C. The Sun.   D. A white dwarf.   E. A black hole.

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### Spinning black holes

Close enough to a rotating black hole, spacetime is dragged around so well that it becomes impossible for a body to hover in such a way that they would appear stationary to a distant observer. This region is called the **ergosphere**.

The ergosphere represents a large fraction of the rotational energy of the black hole.

- ❑ 0-30% of the total energy of the black hole can be present in this rotation, outside the horizon. (The faster it rotates, the higher the percentage.)
- ❑ There is a **maximum rotation rate**, for which an object at the horizon would appear to a distant observer to be moving at the speed of light.

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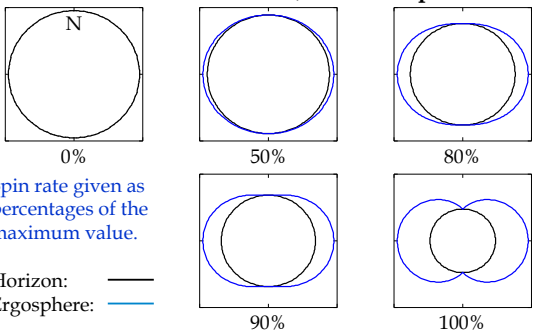
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### Cross sections (through N and S poles) of black holes with same mass, different spins



Spin rate given as percentages of the maximum value.

Horizon: —  
Ergosphere: —

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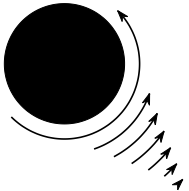
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### Spinning black hole (continued)

The closer to the horizon one looks, the faster space itself seems to rotate (Kerr, 1964). This appears as a “tornado-like swirl” in hyperspace (see Thorne p. 291).



Motion of several bodies trying to hover motionless above the horizon of a spinning BH, as seen by a distant observer above the north pole.

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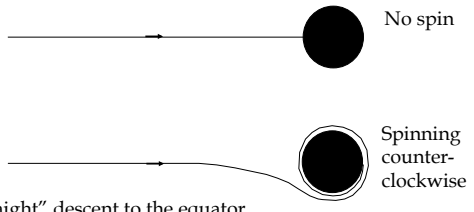
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### Spinning black holes (continued)



No spin

Spinning counter-clockwise

“Straight” descent to the equator of a black hole, as it appears to a distant observer who looks down on the north pole.

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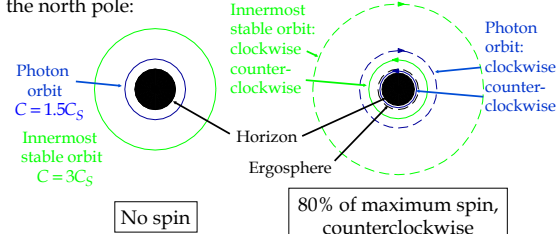
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### There are stable orbits closer to spinning black holes than non-spinning ones.

In the reference frame of a distant observer, anyway, and for orbits in the same direction as the spin. Here are two black holes with the same mass, viewed from a great distance up the north pole:



Photon orbit  $C = 1.5C_S$

Innermost stable orbit  $C = 3C_S$

Horizon

Ergosphere

No spin

80% of maximum spin, counterclockwise

Innermost stable orbit: clockwise counter-clockwise

Photon orbit: clockwise counter-clockwise

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### Black holes and gravitational radiation

Because space is stuck to event horizons, rapid changes in the size or shape of a black hole can generate gravitational radiation. The effect is often likened to ripples of curvature propagating through spacetime, and in turn to the ripples produced by throwing a rock in a pond. Examples:

- ❑ Formation of a horizon by stellar collapse.
- ❑ Nonradial pulsation of a horizon: spindle through sphere to pancake, and back again.



- ❑ Sudden growth of a horizon by the coalescence of two black holes.

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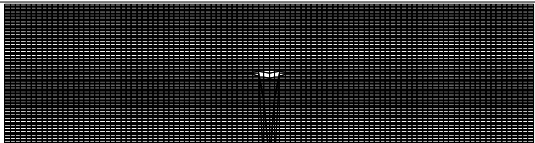
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Form a black hole instantaneously...

**Generation of gravitational radiation by stellar collapse (view from hyperspace)**

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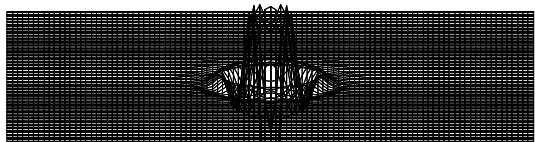
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...and ripples are created in hyperspace....

**Generation of gravitational radiation by stellar collapse (view from hyperspace)**

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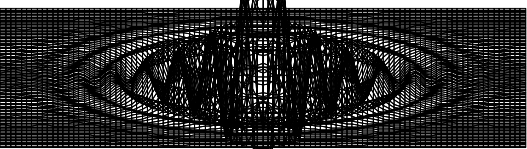
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**Generation of gravitational radiation by stellar collapse (view from hyperspace)**

... that propagate outwards as time (for a distant observer) goes on.

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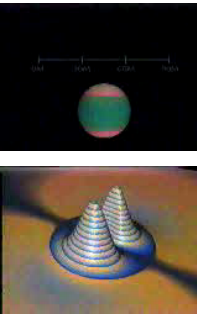
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**Black hole pulsation and gravitational radiation (continued)**

Event horizons are easily “rung” when they are formed, or when the black hole accretes a substantial lump of mass.

**Simulations:** the horizon of a small nonradial pulsation in a horizon (top), and the embedding diagram of the equatorial plane of a distorted black hole, showing emission of gravity waves (bottom). By Ed Seidel *et al.*, NCSA/U. Illinois.

Find simulations at  
<http://www.ncsa.uiuc.edu/Cyberia/NumRel/MoviesEdge.html>



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**Black hole - black hole collision and gravitational radiation**

The most energetic source of gravitational radiation hitherto conceived is the coalescence of two black holes.

**Simulation:** equatorial-plane embedding diagrams for the head-on collision and coalescence of two equal-mass black holes. (By Ed Seidel *et al.*, NCSA/ U. Ill. Urbana-Champaign.)



Case 2 ( $\mu=2.2$ ):  
 The initial distance between the holes is  $L=8.92M$ , where  $M$  is 1/2 the ADM Mass of the system.  
 In this case, the holes do not have a common apparent or event horizon initially.

Find simulation at  
<http://www.ncsa.uiuc.edu/Cyberia/NumRel/MoviesEdge.html>

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**Which would not generate gravity waves?**

A. A stationary black hole spinning at a constant rate.

B. Two black holes orbiting each other.

C. Two neutron stars orbiting each other.

D. The collapse of a star past the neutron-star limit.

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**Done!**

**Venus, Zodiacal Light, and the Galactic Center.**




Image Credit & Copyright:  
Juergen Schmoll  
(Durham University, CfAI)

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