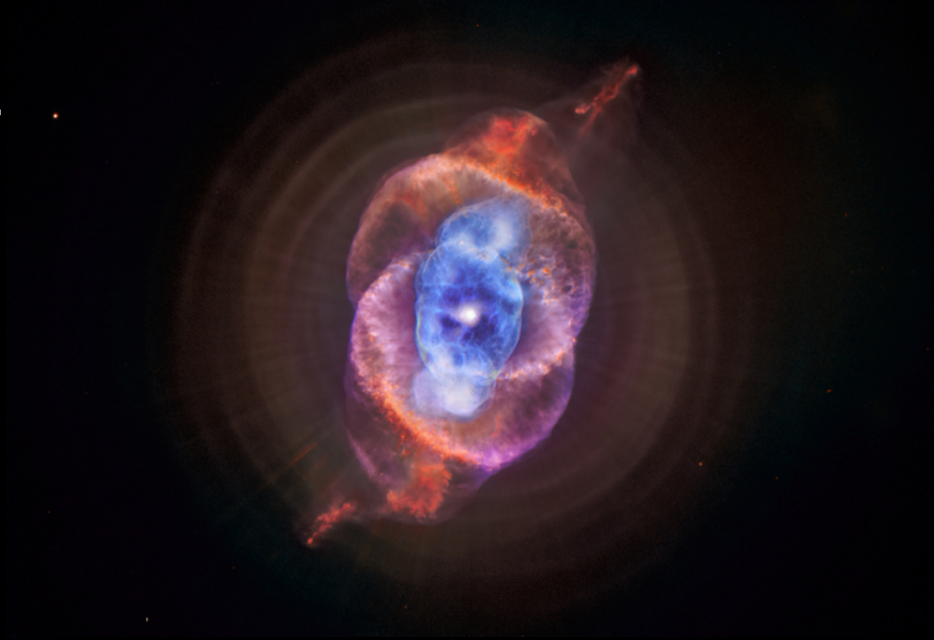
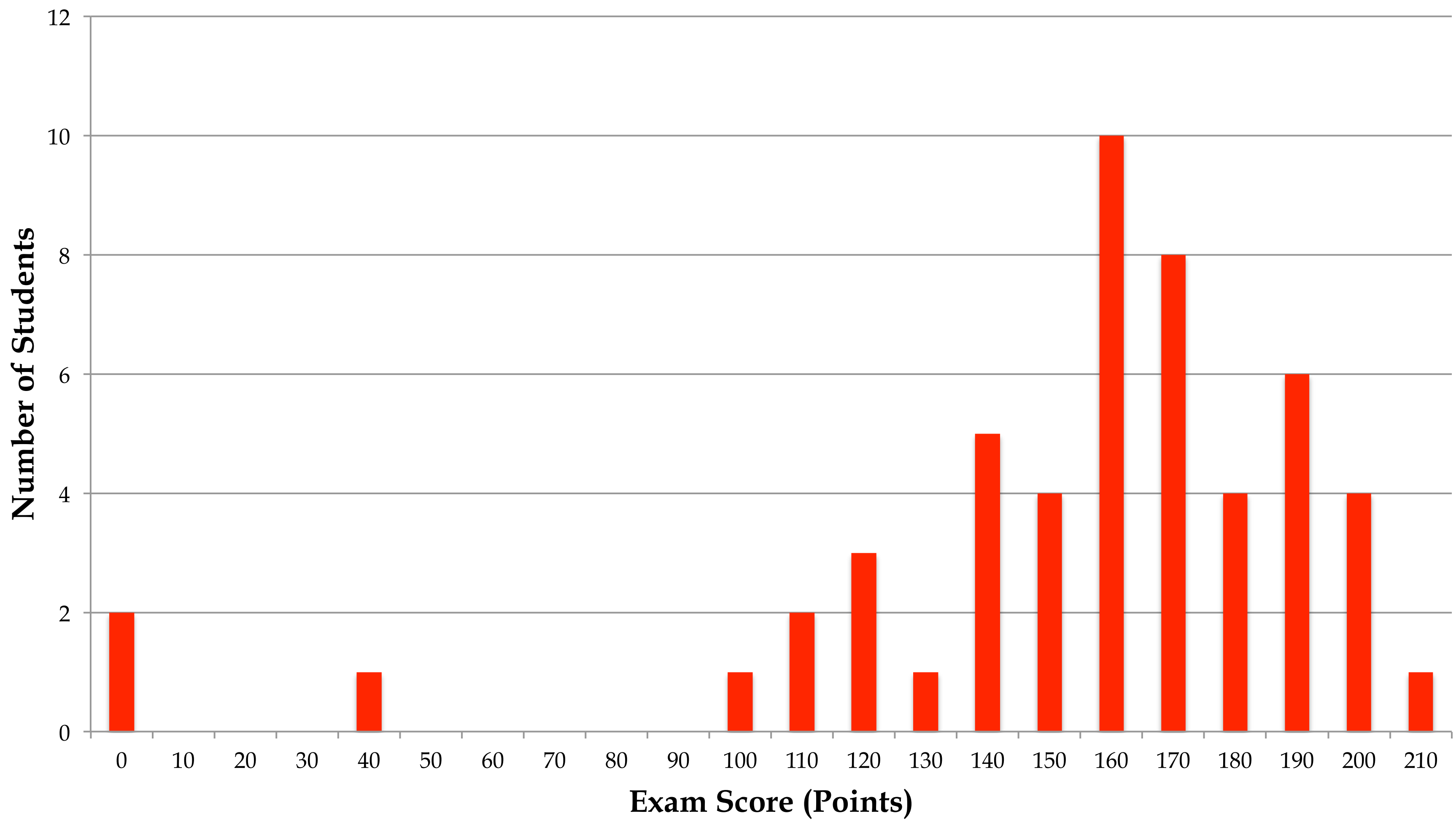
Today in Astronomy 102: black holes and how to prevent them

- The Schwarzschild singularity and the sizes of black holes.
- **Degeneracy pressure:** a quantum-mechanical effect that might stop matter from collapsing to form a black hole, when gas pressure or material strength aren't enough.



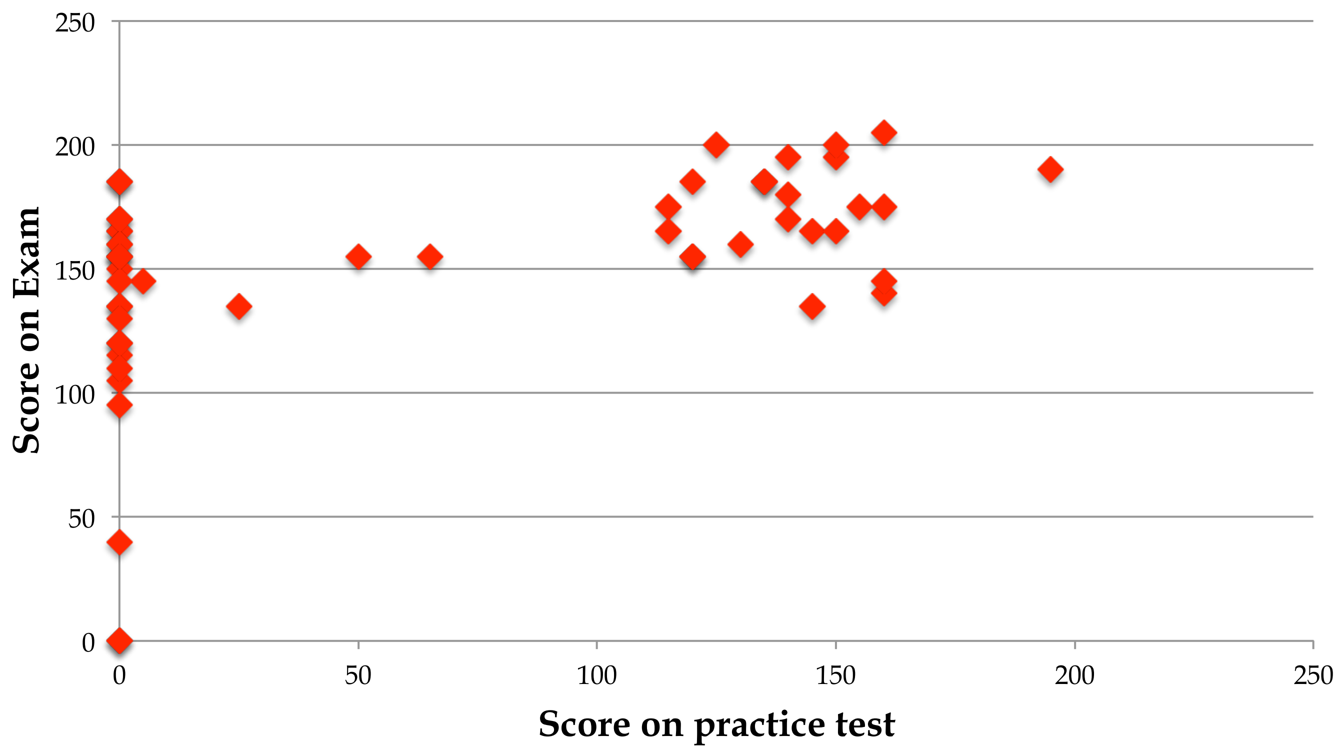
The central star in this planetary nebula, NGC 6543, is well on its way to becoming a white dwarf. *(Hubble Space Telescope and Chandra X-ray Observatory/ NASA, STScI, CfA)*

Results Exam 1



Midterm # 1

- Average = 75%
- Students who took the Practice Midterm did substantially better than those who didn't.



What do you think of electronic exams?

Comparing WeBWorK exams like Exam #1 to ordinary paper exams during class, I like

A. paper exams a LOT better.
B. paper exams better.
C. neither better than the other.
D. WeBWorK exams better.
E. WeBWorK exams a LOT better.

What do you think of electronic exams? (2)

The aspect of the WeBWorK Exam #1 I liked best was

A. Comfortable surroundings of my choice.
B. Flexible hours.
C. Quick grading.
D. Access to all electronic course material during exam.
E. Less stressful overall.

What do you think of electronic exams? (3)

I thought the worst aspect of the WeBWorK Exam #1 was

A. slow WeBWorK computer.
B. too hard to ask questions.
C. lack of incentive from the pressure of in-class exams.
D. the Yankees question
E. other nasty aspects not listed (send me email!).

Back to work.

- The Schwarzschild singularity and the sizes of black holes.
- **Degeneracy pressure:** a quantum-mechanical effect that might stop matter from collapsing to form a black hole, when gas pressure or material strength aren't enough.

The central star in this planetary nebula, NGC 6543, is well on its way to becoming a white dwarf. *(Hubble Space Telescope and Chandra X-ray Observatory/ NASA, STScI, CfA)*

The Schwarzschild singularity

According to Schwarzschild's solution to Einstein's field equation for spherical objects, the gravitational redshift becomes infinite (i.e. time appears to a distant observer to stop) if an object having mass M is confined within a sphere of circumference C_S, given by

$$C_S = \frac{4\pi GM}{c^2}$$

Schwarzschild circumference

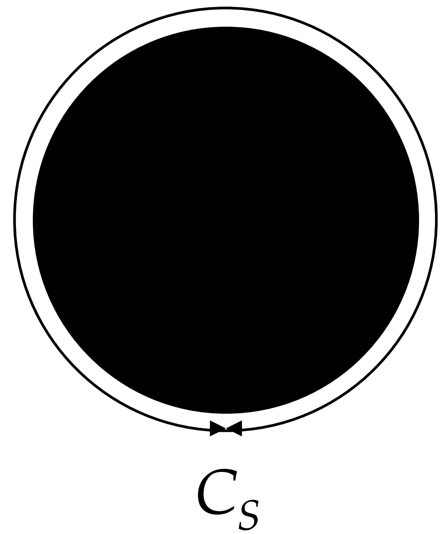
where $G = 6.674215\times10^{-8}$ cm^3/(gm sec^2) is Newton's gravitational constant, and c = 2.99792458×10^{10} cm/sec is, as usual, the speed of light (and π = 3.14159265359....).

You need to understand this formula.

The Schwarzschild singularity (continued)

Any object with mass *M*, and circumference smaller than C_S, would not be able to send light (or anything else) to an outside observer -- that is, it would be a black hole.

The sphere with this critical circumference - the Schwarzschild singularity itself - is what we have been calling the **event horizon**, or simply the **horizon**, of the black hole.



Examples: calculation using the horizon (Schwarzschild) circumference

Example 1: what is the horizon circumference of a $10M_\odot$ black hole?

$$C_S = \frac{4\pi GM}{c^2}$$

$$= \frac{4\times 3.14\times 6.67\times 10^{-8}\ \dfrac{\text{cm}^3}{\text{sec}^2\text{gm}}}{\left(3.00\times 10^{10}\ \dfrac{\text{cm}}{\text{sec}}\right)^2}\times 10M_\odot\times\frac{2.0\times 10^{33}\ \text{gm}}{1M_\odot}$$

$$= 1.86\times 10^{7}\ \text{cm}$$

$$= 1.86\times 10^{7}\ \text{cm}\times\frac{\text{km}}{10^{5}\ \text{cm}} = 186\ \text{km}$$

(Compare to the discussion of the black hole Hades, pg. 29 in Thorne)

Examples: calculation using the horizon (Schwarzschild) circumference, continued

Example 2: what is the horizon circumference of a black hole with the same mass as the Earth (6.0×10^{27} gm)?

$$C_S = \frac{4\pi GM}{c^2}$$

$$= \frac{4\times3.14\times6.67\times10^{-8}\ \frac{\text{cm}^3}{\text{s}^2\,\text{gm}}}{\left(3.00\times10^{10}\ \frac{\text{cm}}{\text{s}}\right)^2}\times6.0\times10^{27}\ \text{gm}$$

$$= 5.6\ \text{cm}$$ **(!!)**

Examples: calculation using the horizon (Schwarzschild) circumference, continued

Example 3: what is the mass of a black hole that has a horizon circumference equal to that of the Earth (4.0×10^9 cm)?

First, rearrange the formula:

$$C_S = \frac{4\pi GM}{c^2}$$

$$\frac{c^2}{4\pi G} C_S = \frac{4\pi GM}{c^2} \frac{c^2}{4\pi G}$$

$$\frac{C_S c^2}{4\pi G} = M$$

Another form in which you need to understand the equation

Examples: calculation using the horizon (Schwarzschild) circumference, continued

Then, put in the numbers:

$$M = \frac{C_S c^2}{4\pi G} = \frac{4\times 10^9\,\mathrm{cm}\times\left(3.00\times 10^{10}\,\dfrac{\mathrm{cm}}{\mathrm{sec}}\right)^2}{4\times 3.14\times 6.67\times 10^{-8}\,\dfrac{\mathrm{cm}^3}{\mathrm{sec}^2\mathrm{gm}}}$$

$$= 4.3\times 10^{36}\,\mathrm{gm}$$

$$= 4.3\times 10^{36}\,\mathrm{gm}\times\frac{1\,M_\odot}{2.0\times 10^{33}\,\mathrm{gm}} = 2.15\times 10^3\,M_\odot$$ **(!!)**

Now, you try:

What is the horizon circumference of a black hole with mass equal to that of the bright star Vega, $M = 2.9 M_\odot$?

A. 53.8140502 km B. 54 km C. 65.1429 km
D. 69.8 km E. 186 km

And try again:

What is the horizon circumference, in centimeters, for a black hole with mass 10^{33} gm?

Reaction to the Schwarzschild singularity (from last time)

- ❑ Einstein showed that a stable object with a singularity cannot exist.
- ❑ From this he concluded (**incorrectly**) that this meant the singularity could not exist in nature.
- ❑ Einstein's calculation was correct, but the correct inference from the result is that **gas pressure cannot support the weight of stars similar in size to the Schwarzschild circumference**.
- ❑ If nothing stronger than gas pressure holds them up, such stars cannot be stable: they will collapse to form black holes – in which case the singularity is real.
  - Stronger than gas pressure: **degeneracy pressure**.

Mid-lecture Break (2 min. 59 s.)

- Homework #3 is now available on WeBWorK; it is due on Monday, February 29, at 8:30 AM.

Einstein and his violin

Degeneracy pressure

This involves a concept from quantum mechanics called the **wave-particle duality**:

- All elementary particles from which matter and energy are made (including light, electrons, protons, neutrons...) have simultaneously the properties of particles and waves.
- Which property they display depends upon the situation they're in.

Degeneracy pressure consists of a powerful resistance to compression that's exhibited by the elementary constituents of matter when these particles are confined to spaces small enough to reveal their wave properties.

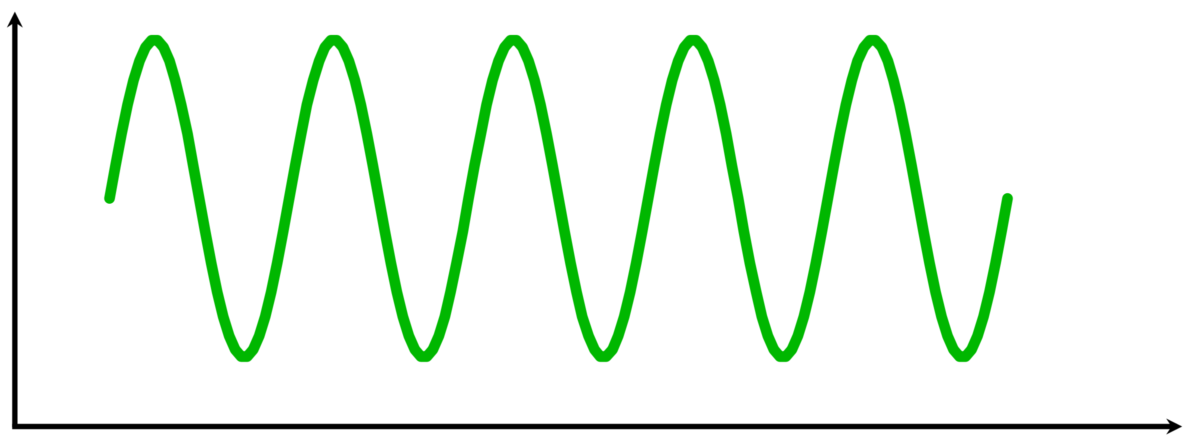
In more detail….

Particles and waves

- Particles exist only at a point in space.

Waves extend over a region of space.

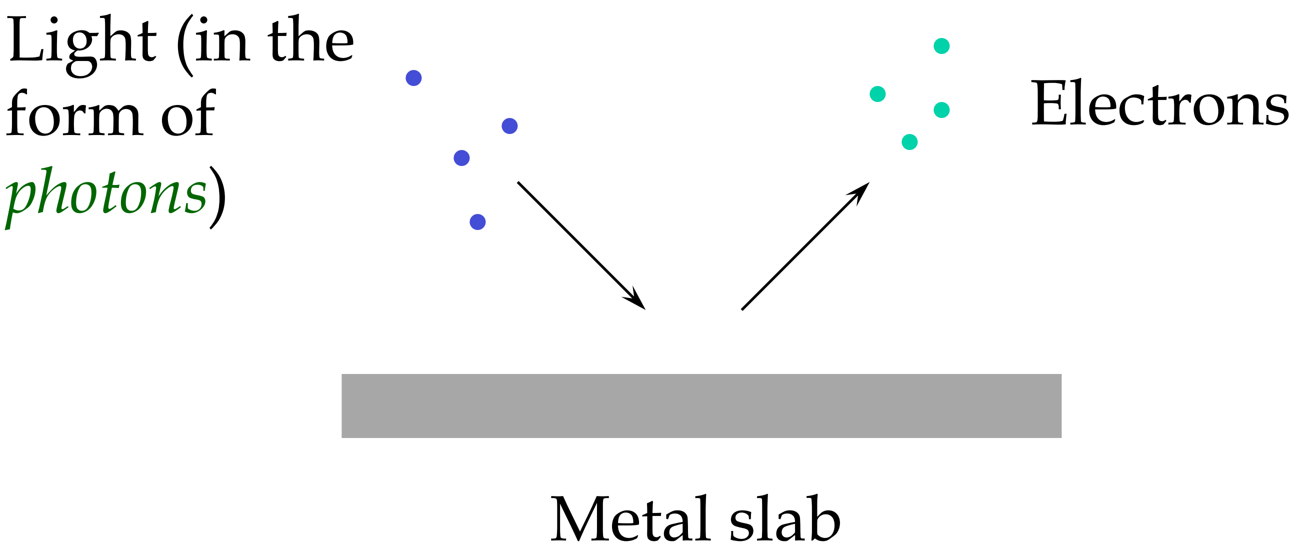
Electric Field, for instance



Location in space

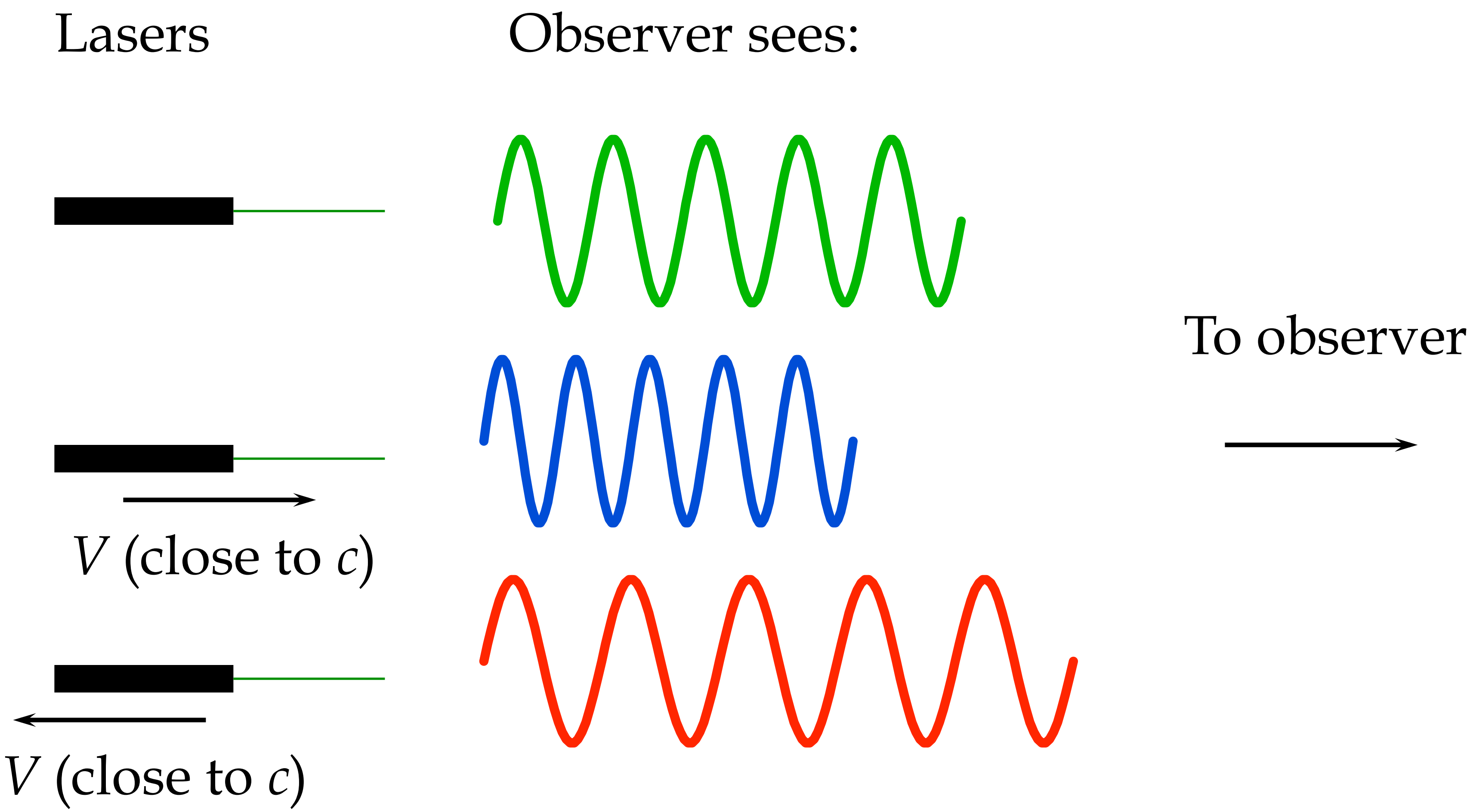
Light can be either a particle or a wave

Particle example: the **photoelectric effect** -- the 1905 explanation of which, in these terms, won Einstein the 1921 Nobel Prize in physics.



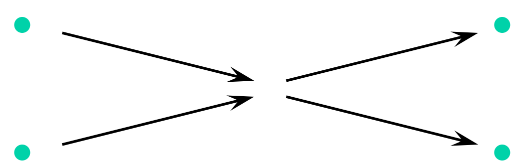
Light can be either a particle or a wave (continued)

Wave example: the **Doppler effect**.

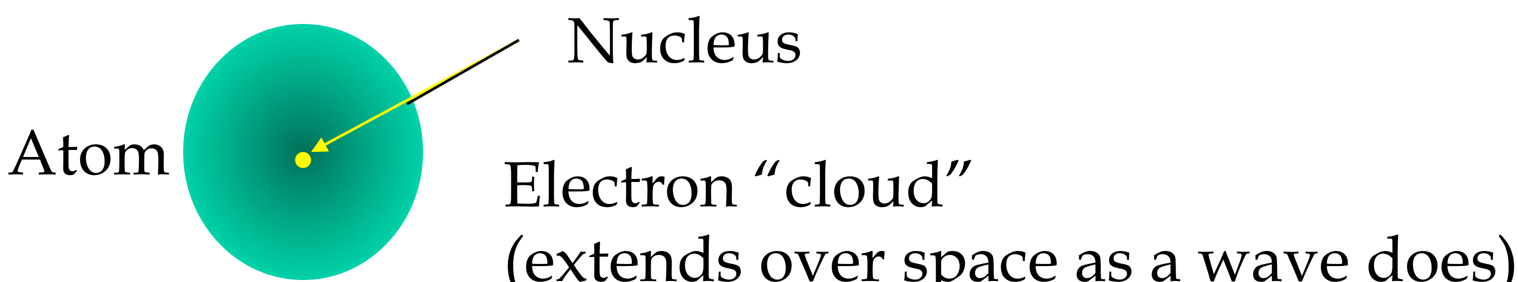


Electrons can be particles or waves

Particle example: collisions between free electrons are "elastic" (they behave like billiard balls).



Wave example: electrons confined to atoms behave like waves.



How to evoke the wave properties of matter

All the elementary constituents of matter have both wave and particle properties.

If a subatomic particle (like an electron, proton or neutron) is **confined to a very small space**, it acts like a **wave** rather than a particle.

How small a space?

- The size of an atom, in the case of electrons (about 10^{-8} cm in diameter).
- A much smaller space for protons and neutrons (about 10^{-11} cm diameter).
- Generally, the more massive a particle is, the smaller the confinement space required to make it exhibit wave properties.

Elementary particle masses

In a reference frame in which the particle is at rest,

$$m = 9.1094 \times 10^{-28} \text{ gm} \quad \text{(electron)}$$

$$m = 1.6726 \times 10^{-24} \text{ gm} \quad \text{(proton)}$$

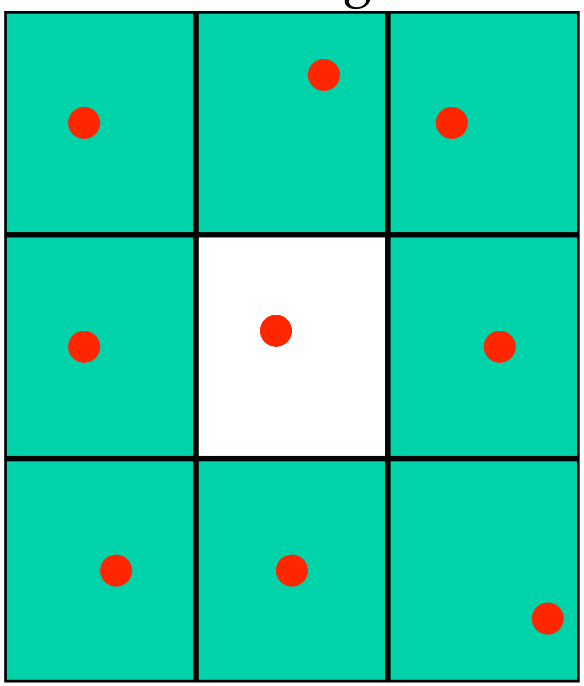
$$m = 1.6750 \times 10^{-24} \text{ gm} \quad \text{(neutron)}$$

- To reveal their wave properties, electrons need to be confined to atomic dimensions (about 10^{-8} cm); thus neutrons and protons to a space a factor of about 1836 smaller (in round numbers, about 10^{-11} cm), that number being the ratio of these particles' masses to that of the electron.
- Photons -- particles of light -- have rest mass 0. (This goes with them having no rest frame; they always appear to travel at the speed of light.)

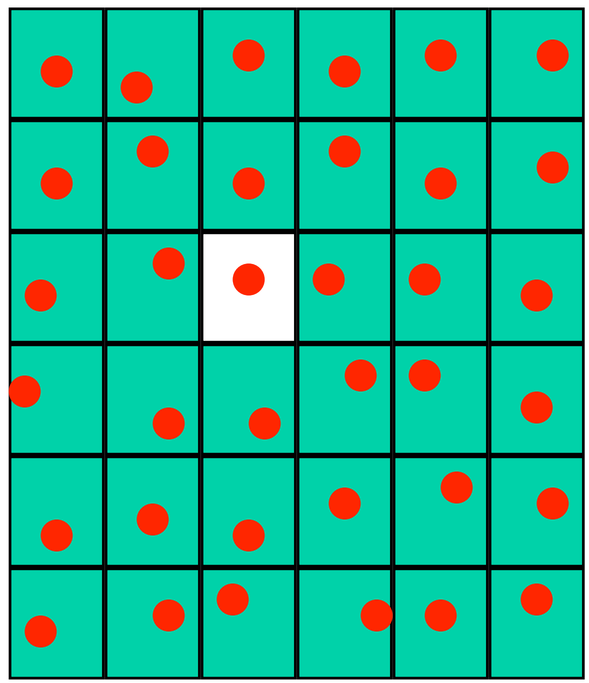
Confinement of elementary particles

Particles like electrons, protons and neutrons can be confined to a small space by being surrounded by other particles of the same type, very nearby.

9 electrons sharing a two-dimensional region.



36 electrons sharing the same area. Each is confined to a smaller space.



(They don't even wander into each other's cell! Why not?)

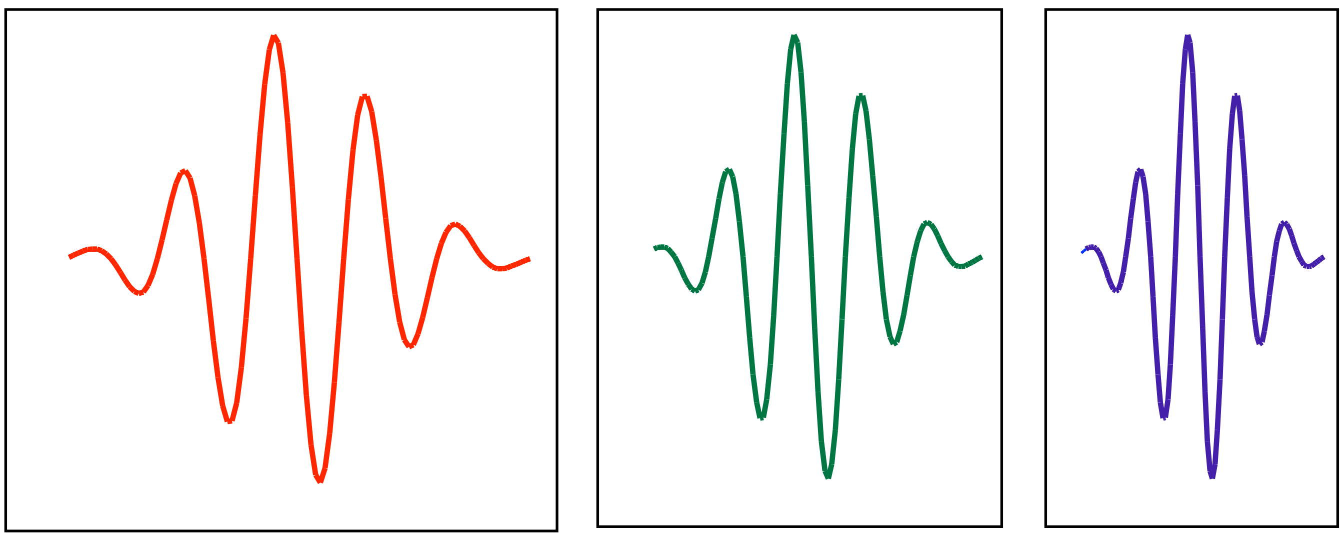
Confinement of elementary particles (continued)

This confinement has to do not only with the electric repulsion they may experience; there is an additional **quantum-mechanical repulsion** of electrons by each other, which sets in at very small distances, such that wave properties are displayed.

- If the separation is small enough that this quantum repulsion is bigger than the electric repulsion, the electrons are said to be **degenerate**.
- Note for those who have taken physics or chemistry before: you may know this quantum repulsion as the **Pauli exclusion principle**.
- Protons can confine each other in a similar fashion; so can neutrons. Because electrons are less massive, though, they become degenerate with less confinement (a space roughly 1800 times larger, as we have seen).
- Photons do not do this; the Pauli principle does not apply to light.

Implications of confinement arising from the wave properties of elementary particles

If one confines an electron wave to a smaller space, its wavelength is made shorter.



Just as is the case for light, a shorter wavelength means a larger energy for each confined electron.

Implications of confinement arising from the wave properties of elementary particles (cont'd)

With this increase in energy, each electron exerts itself harder on the walls of its "cell;" this is the same as an increase in pressure. So:

- squeeze a lot of matter from a very small space into an even smaller space…
- electrons are more tightly confined…
- thus the electrons have more energy and exert more pressure against their confinement.

This extra pressure from the increase in wave energy under very tight confinement is **degeneracy pressure**, first described by British physicist Ralph Fowler in 1926.

Sir Ralph Fowler (AIP).

Implications of confinement arising from the wave properties of elementary particles (cont'd)

- Another, equivalent, way to view the wave-particle duality-induced extra resistance to compression is to invoke the **Heisenberg uncertainty principle**:

  The more precisely the position of an elementary particle is determined along some dimension, the less precisely its momentum (mass times velocity) along that same direction is determined.

- In other words: confining a bunch of elementary particles each to a very small distance (thus determining each position precisely) leads to a very large variation in their momenta and speeds.
- Confine to smaller space => increase speed of particles on average => increase the force they exert on their "cell walls" (degeneracy pressure).

Let’s check quickly…

Deduce, from what you’ve just heard, which of these statements is **false**:

A. Degeneracy pressure can hold ordinary objects together.

B. A degenerate object made entirely of neutrons would be smaller than an object of the same mass made entirely of electrons.

C. The more tightly confined electrons are, the larger is their degeneracy pressure.

D. If I add mass to a degenerate object, it should get smaller in diameter, not bigger.

E. The more tightly confined electrons are, the larger is their momentum likely to be.

Electron degeneracy pressure and the prevention of black holes

Questions:

- ❑ Most stable stars are stable because their weight is held up by gas pressure. Do stars exist that are held up by electron degeneracy pressure, rather than gas pressure?
  - Yes: **white dwarfs.**
- ❑ How are such stars made?
  - From normal stars at the end of life, when they have run out of fuel, can't generate pressure, and collapse under their own weight.
- ❑ Can electron degeneracy pressure balance gravity for all compact stars, preventing them from collapsing so far that they acquire horizons and become black holes?
  - **Not entirely**, as we'll see next time.

Done!

Milky Way over the Pinnacles in Australia
Image Credit: Michael Goh