

### Today in Astronomy 102: relativity (continued)

- ❑ The Lorentz transformation and the Minkowski absolute interval.
  - ❑ The mixing of space and time (the mixture to be referred to henceforth as spacetime) and the relativity of simultaneity: several examples of the use of the absolute interval.
  - ❑ Experimental tests of special relativity.
- Last new equations before the midterm (we have four, now).

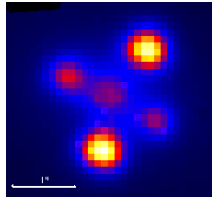


Image: Geraint Lewis and Michael Irwin (1996)

The “Einstein cross”, G2237+0305: a result of gravitational lensing.

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### Supplies.



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### Recall from last time: consequences and predictions of Einstein's special theory of relativity

- ❑ Length contraction (Lorentz-Fitzgerald contraction).
- ❑ Time dilation.
- ❑ Velocities are relative, except for that of light, and cannot exceed that of light.
- ❑ Spacetime warping: “distance” in a given reference frame is a mixture of distance and time from other reference frames.
- ❑ Simultaneity is relative.
- ❑ Mass is relative.
- ❑ There is no frame of reference in which light can appear to be at rest.
- ❑ Mass and energy are equivalent.

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**Recall from last time: equations for length contraction, time dilation and velocity addition in one dimension.**

**You need to understand how to use these formulas.**

$$\Delta x_1 = \Delta x_2 \sqrt{1 - \frac{V^2}{c^2}}$$

$$\Delta t_1 = \frac{\Delta t_2}{\sqrt{1 - \frac{V^2}{c^2}}}$$

$$v_1 = \frac{v_2 + V}{1 + \frac{v_2 V}{c^2}}$$

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**PRs up, please.**

Your friend Tim can throw a baseball at 99.9% of the speed of light. You watch the baseball fly past you; because of length contraction it looks like

A. a tiny sphere.      B. a thin rod, the baseball's diameter in length.  
 C. a thin disk, same diameter as the baseball.      D. it did before.

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**Warping (or mixing) of time and space: the Lorentz transformation**

Observers' coordinate systems coincide ( $x_1 = x_2$ ) at  $t_1 = t_2 = 0$ . They both see an event, and report where and when it was.

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### Warping (or mixing) of time and space: the Lorentz transformation (continued)

The position and time at which the observers see the event are related by the following equations:

$$x_1 = \frac{x_2 + Vt_2}{\sqrt{1 - \frac{V^2}{c^2}}} \quad t_1 = \frac{t_2 + \frac{Vx_2}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \quad \text{Lorentz transformation}$$

- ❑ Position in one reference frame is a **mixture** of position and time from the other frame.
- ❑ Time in one reference frame is similarly a mixture of position and time from the other frame.
- ❑ The mixture is generally called **spacetime**.

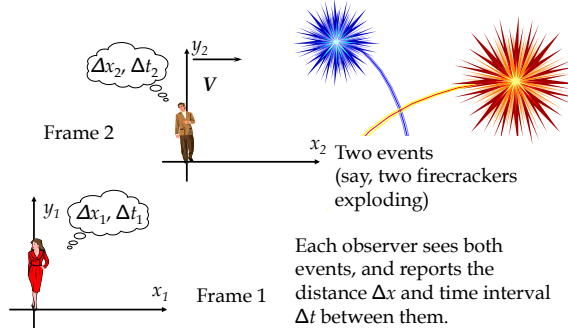
(We won't be using *these* equations on homework or exams.)

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### Warping (or mixing) of time and space: the Minkowski absolute interval



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### Warping (or mixing) of time and space: the Minkowski absolute interval (continued)

The distance and time interval each observer measured for the two events turn out to be related by

$$\Delta x_1^2 - c^2 \Delta t_1^2 = \Delta x_2^2 - c^2 \Delta t_2^2$$

Thus the quantity

$$\text{Absolute interval} = \sqrt{\Delta x_1^2 - c^2 \Delta t_1^2} = \sqrt{\Delta x_2^2 - c^2 \Delta t_2^2} = \sqrt{\Delta x^2 - c^2 \Delta t^2}$$

is constant; the same value is obtained with distance  $\Delta x$  and time interval  $\Delta t$  from **any** single frame of reference.

- ❑ This can be derived directly from the Lorentz transformation.
- ❑ **We will be using the formula for the absolute interval.**

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### Recall the Nomenclature

We interrupt to remind you that:

- By  $x_1$ , we mean the **position** of an object or event along the  $x$  axis, measured by the observer in Frame #1 in his or her coordinate system.
- By  $\Delta x_1$ , we mean the **distance** between two objects or events along  $x$ , measured by the observer in Frame #1.
- By  $t_1$ , we mean the **time** of an object or event, measured by the observer in Frame #1 with his or her clock.
- By  $\Delta t_1$ , we mean the **time interval** between two objects or events, measured by the observer in Frame #1.

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### Warping (or mixing) of time and space: the Minkowski absolute interval

In other words:

$$\begin{aligned} \text{Absolute interval} &= \sqrt{(\text{distance in Frame 1})^2 - c^2(\text{time interval in Frame 1})^2} \\ &= \sqrt{(\text{distance in Frame 2})^2 - c^2(\text{time interval in Frame 2})^2} \end{aligned}$$

Usually we will have Frame 1 at rest (that makes it "our frame"), and Frame 2 in motion.

**Upshot:** events that occur simultaneously ( $\Delta t = 0$ ) in one frame may not be simultaneous in another; **simultaneity is relative**.

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### Geometric analogy for the absolute interval: the example of the Mledinans (Thorne pp. 88-90)

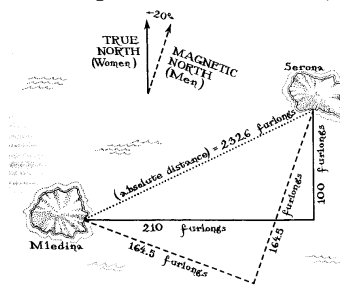


Figure from Thorne, *Black holes and time warps*

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Absolute distance (on a map) covered is the same for men and women, even though they take different paths and have different coordinate systems.

The direction the men call North is a mixture of the women's north and east. The direction the women call North is part north, part west, according to the men.

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**Geometric analogy for the absolute interval: the example of the Mledinans (Thorne pp. 88-90)**

Absolute distance (on the map) is governed by the Pythagorean theorem:

$$\text{Absolute distance} = \sqrt{(\text{distance north})^2 + (\text{distance east})^2}$$

Note the similarity (and the differences) to the Minkowski absolute interval in special relativity :

$$\text{Absolute interval} = \sqrt{(\text{distance})^2 - c^2(\text{time interval})^2}$$

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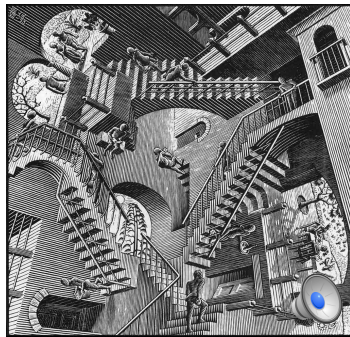
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**Mid-lecture Break (4 minutes 14 seconds).**

- ❑ Homework #2 is due next Monday, February 8, 2016, at 8.30 AM.



Relativity, by M.C. Escher



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**Example of the relativity of simultaneity: the car-and-firecracker experiment (Thorne, p. 74ff.)**

Firecrackers detonated simultaneously, according to driver ("you").



( ♣ ) Figure from Thorne, *Black holes and time warps*



Observer ("I") watches the firecrackers go off, records the time of each explosion.

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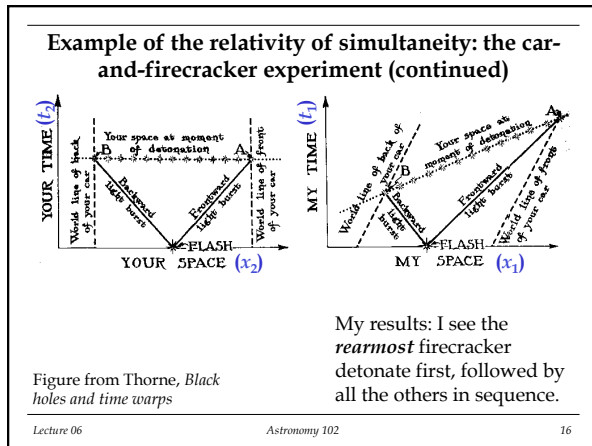
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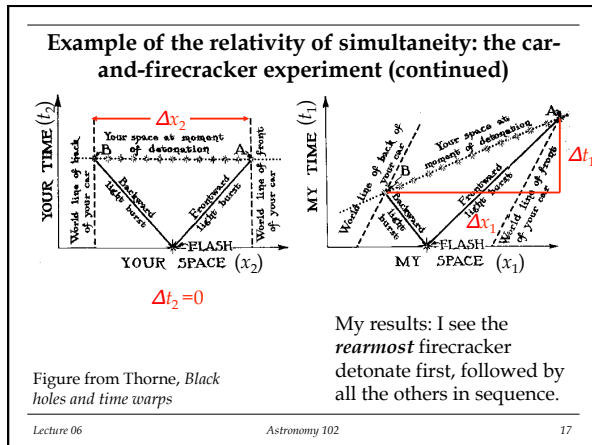
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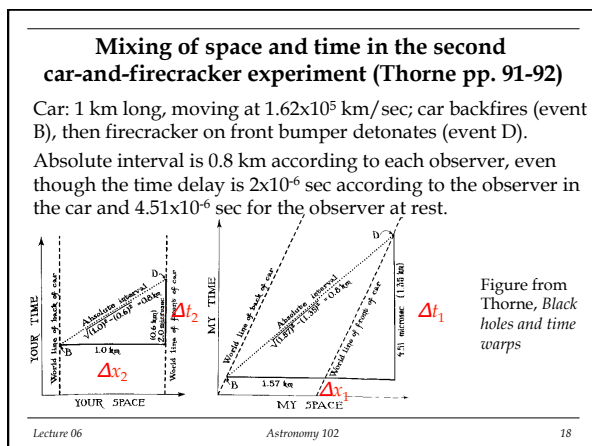
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### Other simple uses of the absolute interval (1)

Suppose you (at rest) were told that events B and D happened simultaneously, viewed from the car, and were also told that they appeared to be separated by 1.19 km along the road (instead of 1 km: longer, because of the motion of the car).

**Are they simultaneous seen from rest?**

The formula for Absolute Interval can be rearranged thus:

$$\begin{aligned}\Delta t_1 &= \frac{1}{c} \sqrt{\Delta x_1^2 - \Delta x_2^2 + c^2 \Delta t_2^2} \\ &= \frac{\sqrt{(1.19 \text{ km})^2 - (1 \text{ km})^2 + (299792 \text{ km/sec})^2 (0 \text{ sec})^2}}{299792 \text{ km/sec}} \\ &= 2.14 \times 10^{-6} \text{ sec.} \quad \text{No.}\end{aligned}$$

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### Other simple uses of the absolute interval (2)

Suppose you (at rest) saw the events B and D happen simultaneously, separated by 0.84 km along the road (shorter than 1 km: this is exactly the setup for Lorentz length contraction). **What was the time interval between events B and D seen in the car?**

Again we rearrange the formula for the absolute interval:

$$\begin{aligned}\Delta t_2 &= \frac{1}{c} \sqrt{\Delta x_2^2 - \Delta x_1^2 + c^2 \Delta t_1^2} \\ &= \frac{\sqrt{(1 \text{ km})^2 - (0.84 \text{ km})^2 + (299792 \text{ km/sec})^2 (0 \text{ sec})^2}}{299792 \text{ km/sec}} \\ &= 1.80 \times 10^{-6} \text{ sec}\end{aligned}$$

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### Other simple uses of the absolute interval (3)

Suppose the events B and D both happen at the same spot on the car (the tailpipe, say), separated by  $2 \times 10^{-6}$  sec as seen from the car, and you see them to be  $2.38 \times 10^{-6}$  sec apart (longer: this is exactly the setup for time dilation). **How far apart along the road do events B and D appear to you to be?**

Again we rearrange the formula for the absolute interval:

$$\begin{aligned}\Delta x_1 &= \sqrt{\Delta x_2^2 - c^2 \Delta t_2^2 + c^2 \Delta t_1^2} \\ &= \sqrt{(0 \text{ km})^2 - (299792 \text{ km/sec})^2 (2 \times 10^{-6} \text{ sec})^2 + (299792 \text{ km/sec})^2 (2.38 \times 10^{-6} \text{ sec})^2} \\ &= 0.387 \text{ km}\end{aligned}$$

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 **Practice problem: recognize what concept to use.**

What kind of problem is this? (What formula should you use?)  
One type of radioactive particle decays in  $2 \times 10^{-6}$  second on the average, if it's at rest. How long does it take if it's moving at 0.995 times  $c$ ?

A. Length contraction

B. Time dilation

C. Velocity addition

D. Absolute interval

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 **Practice problem: recognize what concept to use.**

What kind of problem is this? (What formula should you use?)  
In my car, 1 km long and moving at 99% of the speed of light, I flash my headlights and taillights simultaneously; you see the flashes to be delayed by  $4 \times 10^{-6}$  sec. How far apart along the road are the spots where the flashes appeared to you to occur?

A. Length contraction

B. Time dilation

C. Velocity addition

D. Absolute interval

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 **Practice problem: recognize what concept to use.**

What kind of problem is this? (What formula should you use?)  
I throw a meter stick, so that it moves parallel to its length; it looks to you to be only half a meter long. How fast is it moving, relative to us?

A. Length contraction

B. Time dilation

C. Velocity addition

D. Absolute interval

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### Experimental tests of relativity

Einstein's theories of relativity represent a rebuilding of physics from the ground up.

- ❑ New postulates and assumptions relate the most basic concepts, such as the relativity of space and time and the "absoluteness" of the speed of light.
- ❑ The new theory is logically consistent and mathematically very elegant.
- ❑ The new theory *contains* classical physics, as an approximation valid for speeds much smaller than the speed of light.
- ❑ Still: it would be worthless if it didn't agree with reality (i.e. experiments) better than classical physics.

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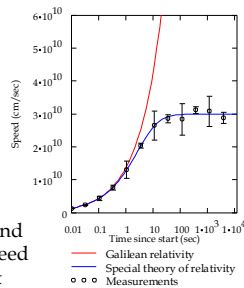
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### Experimental tests of scientific theories

No scientific theory is valid unless:

- ❑ it is mathematically and logical consistent,
- ❑ **and** its predictions can be measured in experiments,
- ❑ **and** the theoretical predictions are in precise agreement with experimental measurements.

**Example:** prediction by the "old" and "new" relativity theories for the speed of a body accelerated with constant power. Both theories are valid for low speeds but only special relativity is valid over the whole range of measurements.



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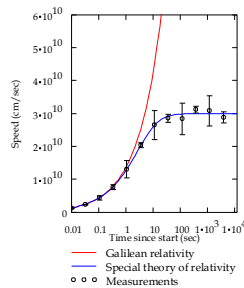
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### Experimental tests of scientific theories (continued)

What constitutes a valid scientific experiment?

- ❑ Measurements are made with accuracy (the size of potential measurement errors) sufficient to test the predictions of prevailing theories,
- ❑ **and** the accuracy can be estimated reliably,
- ❑ **and** the measurements are **reproducible**: the same results are obtained whenever, and by whomever, the experiments are repeated.



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### Experimental tests of relativity (continued)

Some experiments testing special relativity:

- ❑ Michelson or Michelson-Morley type: speed of light always the same in all directions, to **extremely** high accuracy. Repeated many times, and not just in Cleveland.
- ❑ **High-energy accelerators** used in elementary particle physics:
  - radioactive particles are seen to live much longer when moving near light speeds than when at rest (direct observation of time dilation).
  - though accelerated particles get extremely close to the speed of light, none ever exceed it.
- ❑ Nuclear reactors/bombs: mass-energy equivalence ( $E=mc^2$ ).

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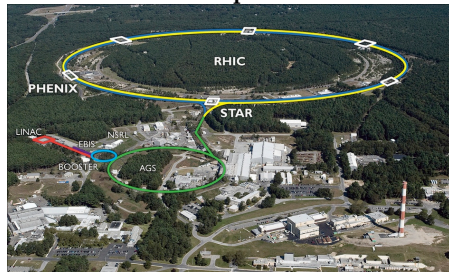
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### Every day, many millions of special-relativity experiments



The Relativistic Heavy Ion Collider at BNL.  
Recreating the early universe!

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### Done! Stars and Dust Across Corona Australis.



Image Credit & Copyright: Ignacio Diaz Bobillo

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