Today in Astronomy 102: relativity (continued)

- ❑ The Lorentz transformation and the Minkowski absolute interval.
- ❑ The mixing of space and time (the mixture to be referred to henceforth as spacetime) and the relativity of simultaneity: several examples of the use of the absolute interval.
- ❑ Experimental tests of special relativity.

Last new equations before the midterm (we have four, now).

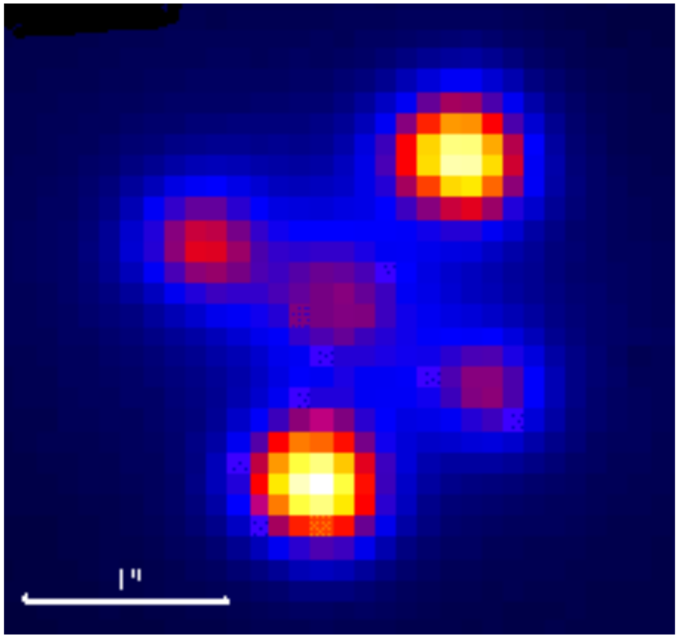


Image: Geraint Lewis and Michael Irwin (1996)

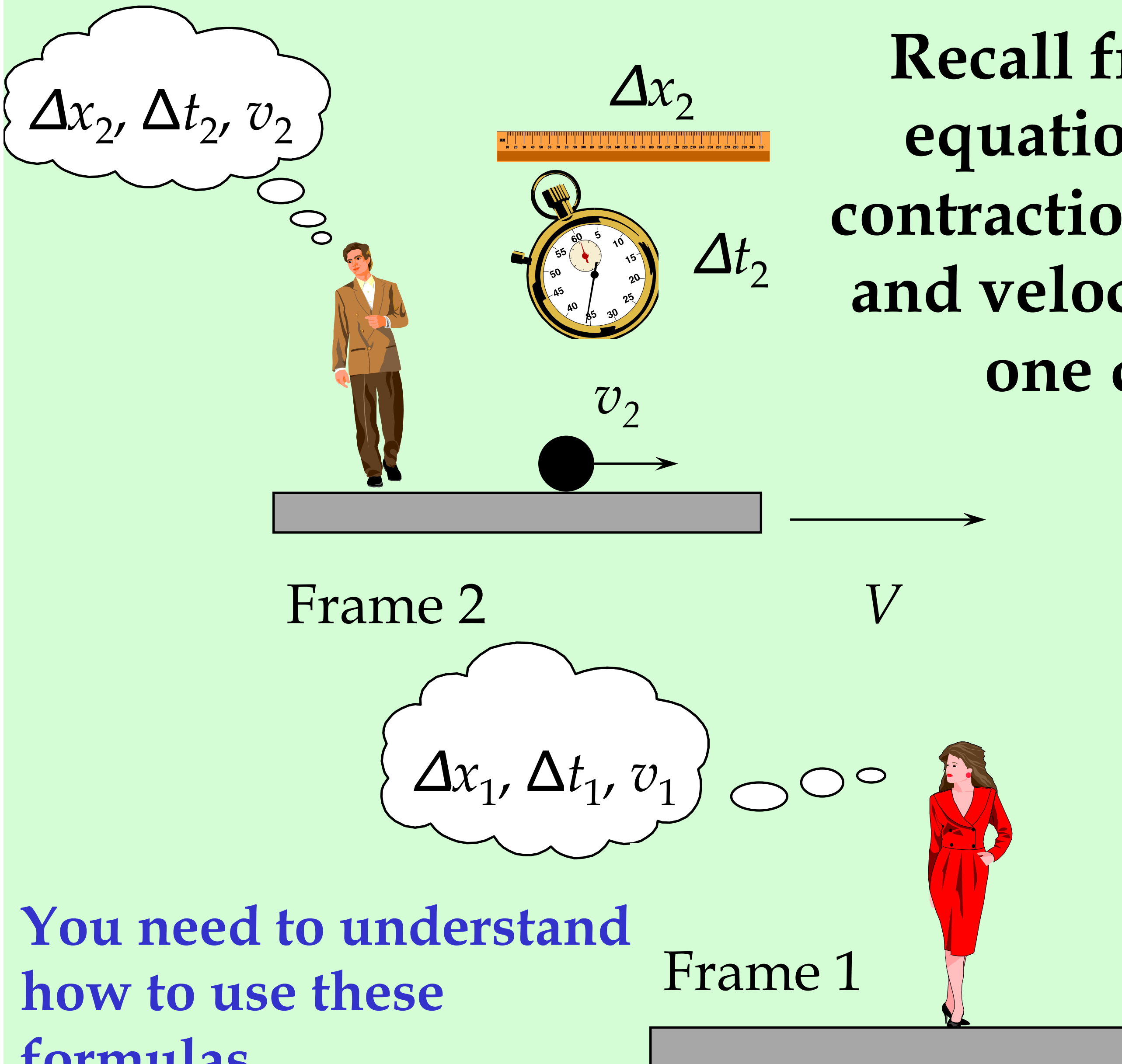
The “Einstein cross”, G2237+0305: a result of gravitational lensing.

Supplies.

Recall from last time: consequences and predictions of Einstein's special theory of relativity

- **Length contraction** (**Lorentz-Fitzgerald contraction**).
- **Time dilation**.
- **Velocities are relative**, except for that of light, and cannot exceed that of light.
- **Spacetime warping**: "distance" in a given reference frame is a mixture of distance and time from other reference frames.
- **Simultaneity is relative.**
- **Mass is relative.**
- **There is no frame of reference in which light can appear to be at rest**.
- **Mass and energy are equivalent.**

Recall from last time: equations for length contraction, time dilation and velocity addition in one dimension.



$$\Delta x_1 = \Delta x_2 \sqrt{1 - \frac{V^2}{c^2}}$$

$$\Delta t_1 = \frac{\Delta t_2}{\sqrt{1 - \frac{V^2}{c^2}}}$$

$$v_1 = \frac{v_2 + V}{1 + \frac{v_2 V}{c^2}}$$

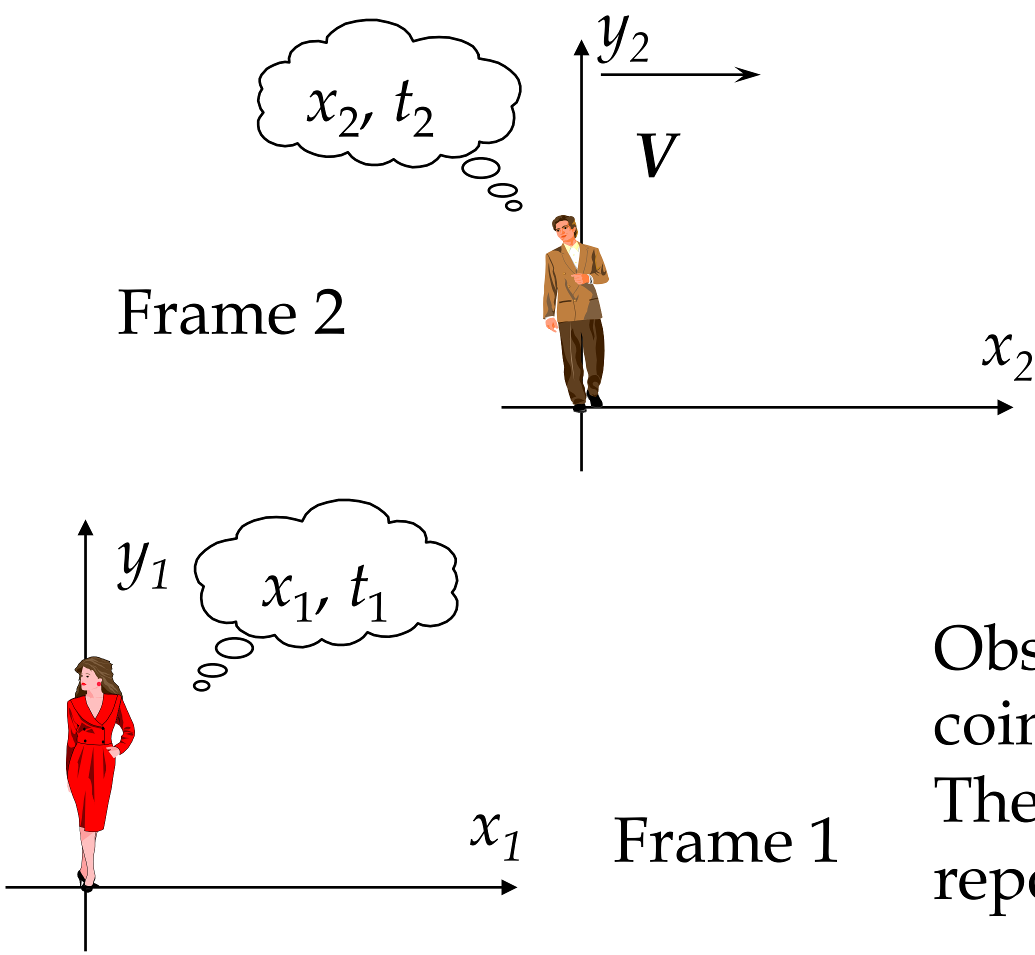
You need to understand how to use these formulas.

PRSs up, please.

Your friend Tim can throw a baseball at 99.9% of the speed of light. You watch the baseball fly past you; because of length contraction it looks like

A. a tiny sphere. B. a thin rod, the baseball's diameter in length.
C. a thin disk, same diameter as the baseball. D. it did before.

Warping (or mixing) of time and space: the Lorentz transformation



"Event"
(say, a firecracker exploding)

Observers' coordinate systems coincide ($x_1 = x_2$) at $t_1 = t_2 = 0$. They both see an event, and report where and when it was.

Warping (or mixing) of time and space: the Lorentz transformation (continued)

The position and time at which the observers see the event are related by the following equations:

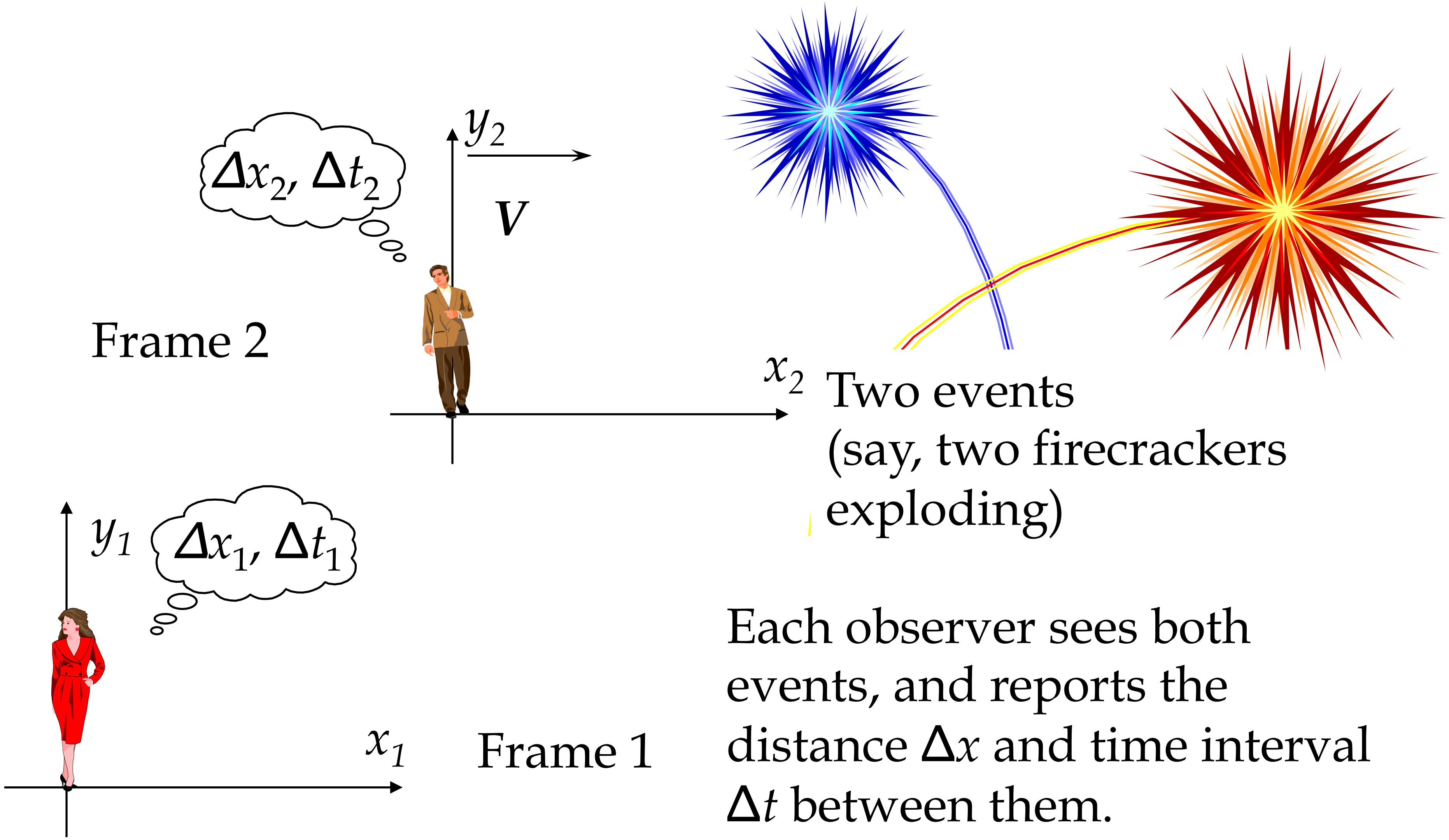
$$x_1 = \frac{x_2 + Vt_2}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad t_1 = \frac{t_2 + \frac{Vx_2}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}}$$

Lorentz transformation

- ❑ Position in one reference frame is a **mixture** of position and time from the other frame.
- ❑ Time in one reference frame is similarly a mixture of position and time from the other frame.
- ❑ The mixture is generally called **spacetime.**

(We won't be using *these* equations on homework or exams.)

Warping (or mixing) of time and space: the Minkowski absolute interval



Two events (say, two firecrackers exploding)

Each observer sees both events, and reports the distance Δx and time interval Δt between them.

Warping (or mixing) of time and space: the Minkowski absolute interval (continued)

The distance and time interval each observer measured for the two events turn out to be related by

$$\Delta x_1^2 - c^2 \Delta t_1^2 = \Delta x_2^2 - c^2 \Delta t_2^2$$

Thus the quantity

$$\text{Absolute interval} = \sqrt{\Delta x_1^2 - c^2 \Delta t_1^2} = \sqrt{\Delta x_2^2 - c^2 \Delta t_2^2}$$
$$= \sqrt{\Delta x^2 - c^2 \Delta t^2}$$

is constant; the same value is obtained with distance Δx and time interval Δt from **any** single frame of reference.

- This can be derived directly from the Lorentz transformation.
- **We will be using the formula for the absolute interval.**

Recall the Nomenclature

We interrupt to remind you that:

- By x_1, we mean the **position** of an object or event along the x axis, measured by the observer in Frame #1 in his or her coordinate system.
- By Δx_1, we mean the **distance** between two objects or events along x, measured by the observer in Frame #1.
- By t_1, we mean the **time** of an object or event, measured by the observer in Frame #1 with his or her clock.
- By Δt_1, we mean the **time interval** between two objects or events, measured by the observer in Frame #1.

Warping (or mixing) of time and space: the Minkowski absolute interval

In other words:

$$\text{Absolute interval} = \sqrt{(\text{distance in Frame 1})^2 - c^2(\text{time interval in Frame 1})^2}$$

$$= \sqrt{(\text{distance in Frame 2})^2 - c^2(\text{time interval in Frame 2})^2}$$

Usually we will have Frame 1 at rest (that makes it "our frame"), and Frame 2 in motion.
Upshot: events that occur simultaneously ($\Delta t = 0$) in one frame may not be simultaneous in another; simultaneity is relative.

Geometric analogy for the absolute interval: the example of the Mledinans (Thorne pp. 88-90)

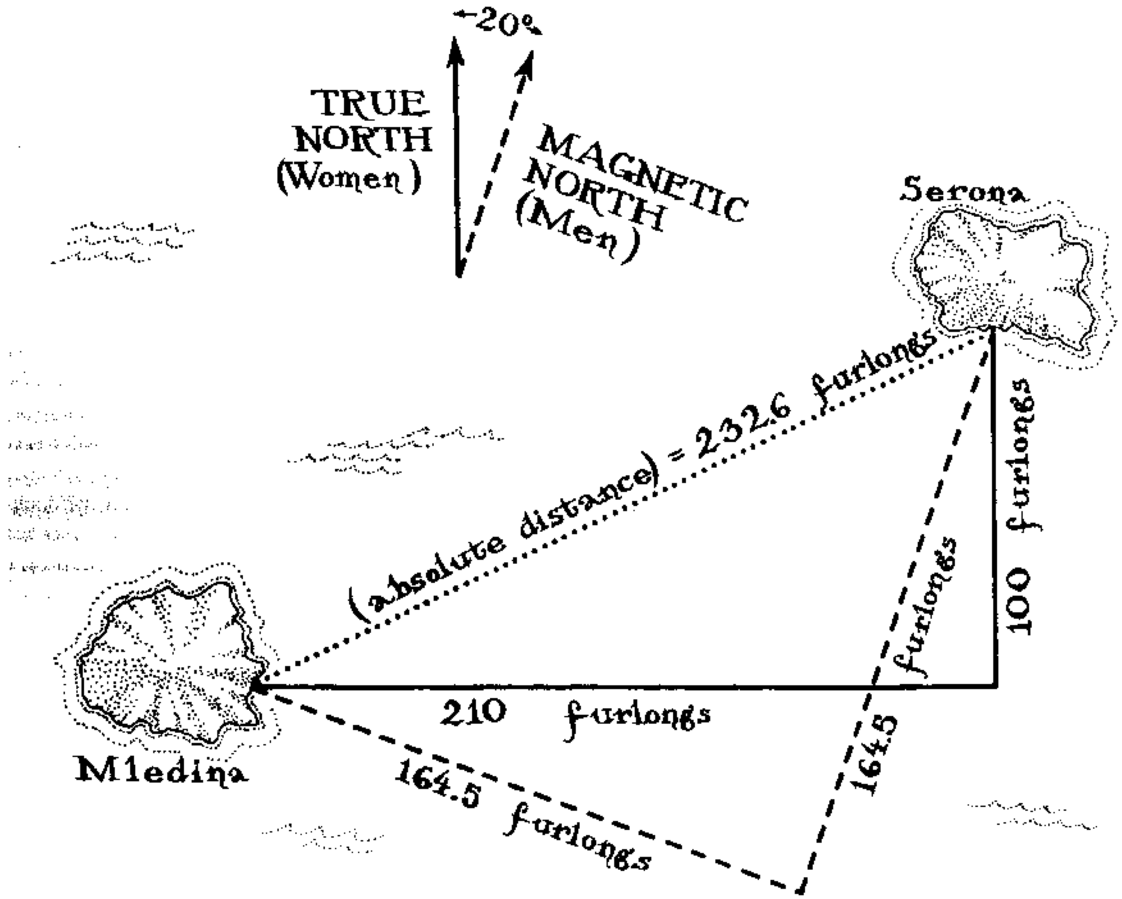


Figure from Thorne, *Black holes and time warps*

Absolute distance (on a map) covered is the same for men and women, even though they take different paths and have different coordinate systems.

The direction the men call North is a mixture of the women's north and east. The direction the women call North is part north, part west, according to the men.

Geometric analogy for the absolute interval: the example of the Mledinans (Thorne pp. 88-90)

Absolute distance (on the map) is governed by the Pythagorean theorem:

$$\text{Absolute distance} = \sqrt{(\text{distance north})^2 + (\text{distance east})^2}$$

Note the similarity (and the differences) to the Minkowski absolute interval in special relativity :

$$\text{Absolute interval} = \sqrt{(\text{distance})^2 - c^2(\text{time interval})^2}$$

Mid-lecture Break (4 minutes 14 seconds).

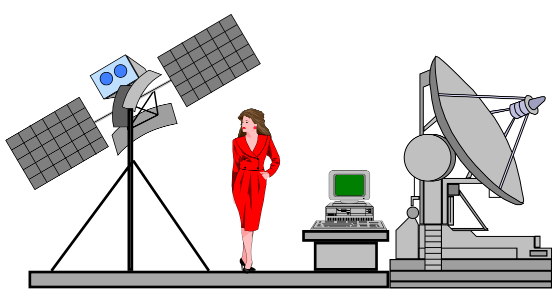
- Homework #2 is due next Monday, February 8, 2016, at 8.30 AM.

Relativity, by M.C. Escher

Example of the relativity of simultaneity: the car-and-firecracker experiment (Thorne, p. 74ff.)

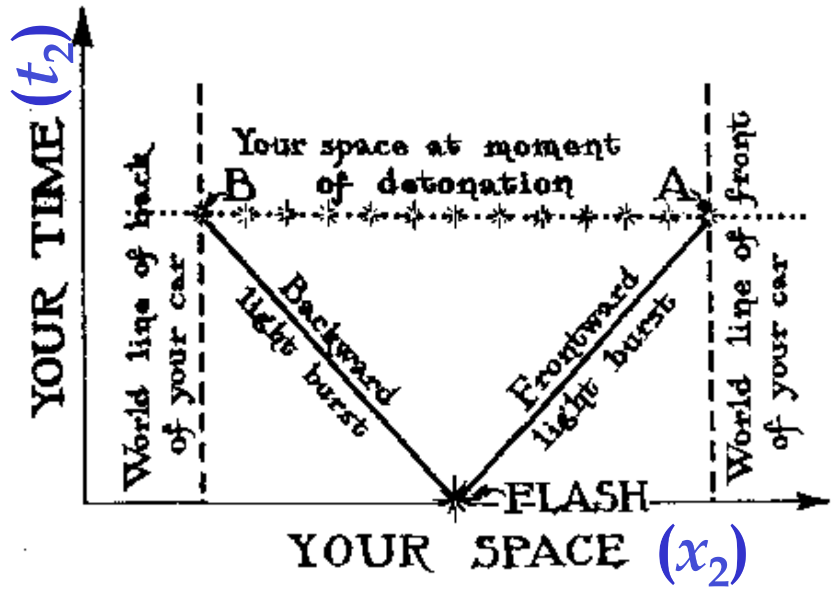
Firecrackers detonated simultaneously, according to driver ("you").

(a) Figure from Thorne, *Black holes and time warps*



Observer ("I") watches the firecrackers go off, records the time of each explosion.

Example of the relativity of simultaneity: the car-and-firecracker experiment (continued)



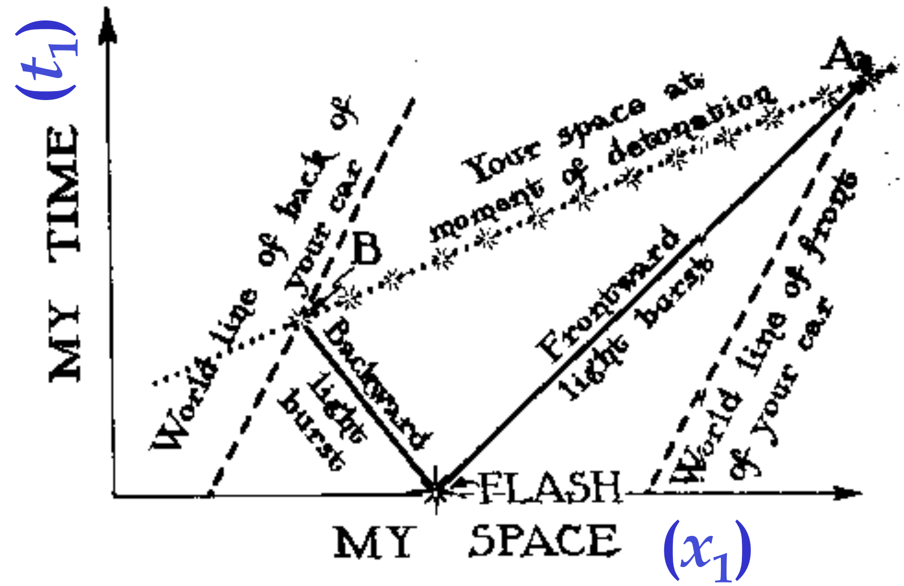
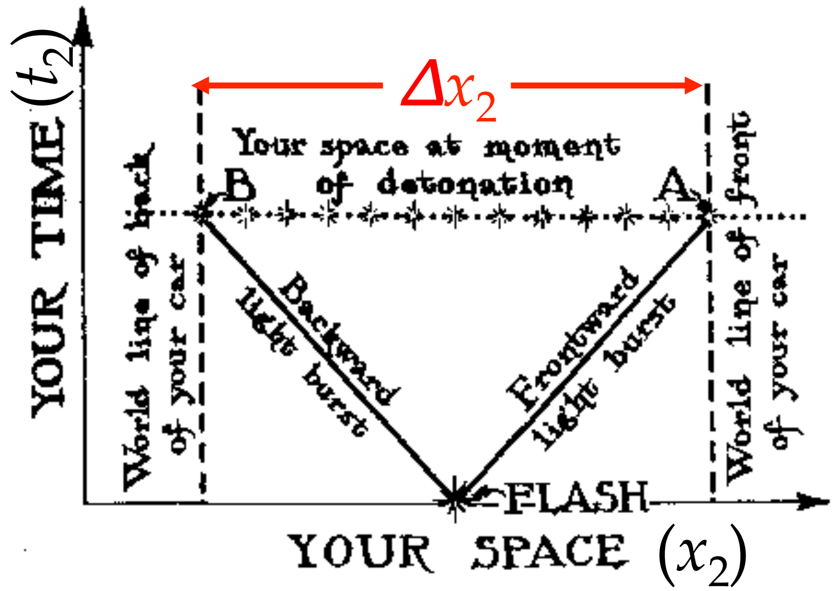
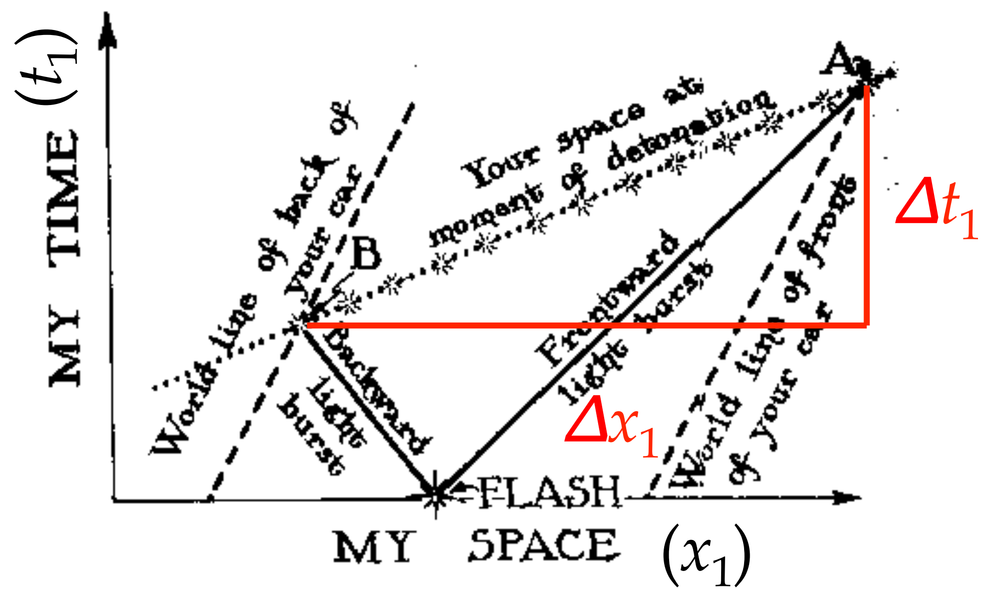


Figure from Thorne, *Black holes and time warps*

My results: I see the ***rearmost*** firecracker detonate first, followed by all the others in sequence.

Example of the relativity of simultaneity: the car-and-firecracker experiment (continued)





$\Delta t_2 = 0$

Figure from Thorne, *Black holes and time warps*

My results: I see the ***rearmost*** firecracker detonate first, followed by all the others in sequence.

Mixing of space and time in the second car-and-firecracker experiment (Thorne pp. 91-92)

Car: 1 km long, moving at 1.62x10^5 km/sec; car backfires (event B), then firecracker on front bumper detonates (event D).

Absolute interval is 0.8 km according to each observer, even though the time delay is 2x10^{-6} sec according to the observer in the car and 4.51x10^{-6} sec for the observer at rest.

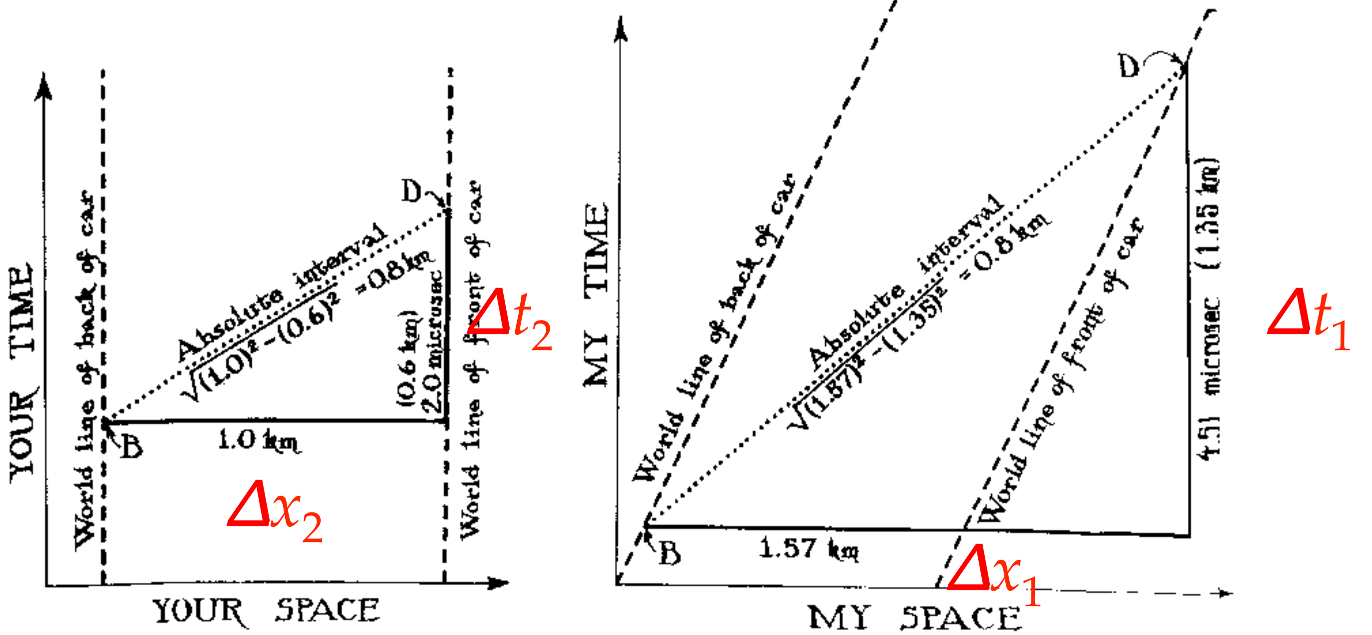


Figure from Thorne, *Black holes and time warps*

Other simple uses of the absolute interval (1)

Suppose you (at rest) were told that events B and D happened simultaneously, viewed from the car, and were also told that they appeared to be separated by 1.19 km along the road (instead of 1 km: longer, because of the motion of the car).

Are they simultaneous seen from rest?

The formula for Absolute Interval can be rearranged thus:

$$\Delta t_1 = \frac{1}{c}\sqrt{\Delta x_1^2 - \Delta x_2^2 + c^2 \Delta t_2^2}$$

$$= \frac{\sqrt{(1.19\ \text{km})^2 - (1\ \text{km})^2 + (299792\,\text{km/sec})^2 (0\ \text{sec})^2}}{299792\,\text{km/sec}}$$

$$= 2.14 \times 10^{-6}\ \text{sec.}$$

No.

Other simple uses of the absolute interval (2)

Suppose you (at rest) saw the events B and D happen simultaneously, separated by 0.84 km along the road (shorter than 1 km: this is exactly the setup for Lorentz length contraction). **What was the time interval between events B and D seen in the car?**

Again we rearrange the formula for the absolute interval:

$$\begin{aligned}\Delta t_2 &= \frac{1}{c}\sqrt{\Delta x_2^2 - \Delta x_1^2 + c^2 \Delta t_1^2} \\ &= \frac{\sqrt{(1\ \mathrm{km})^2 - (0.84\ \mathrm{km})^2 + (299792\ \mathrm{km/sec})^2 (0\ \mathrm{sec})^2}}{299792\ \mathrm{km/sec}} \\ &= 1.80\times 10^{-6}\ \mathrm{sec}\end{aligned}$$

Other simple uses of the absolute interval (3)

Suppose the events B and D both happen at the same spot on the car (the tailpipe, say), separated by $2\text{x}10^{-6}$ sec as seen from the car, and you see them to be $2.38\text{x}10^{-6}$ sec apart (longer: this is exactly the setup for time dilation). **How far apart along the road do events B and D appear to you to be?**

Again we rearrange the formula for the absolute interval:

$$\Delta x_1 = \sqrt{\Delta x_2^2 - c^2 \Delta t_2^2 + c^2 \Delta t_1^2}$$

$$= \sqrt{\begin{array}{l}(0\text{ km})^2 - (299792\text{ km/sec})^2 (2\times 10^{-6}\text{ sec})^2 \\ \quad + (299792\text{ km/sec})^2 (2.38\times 10^{-6}\text{ sec})^2\end{array}}$$

$$= 0.387\text{ km}$$

Practice problem: recognize what concept to use.

What kind of problem is this? (What formula should you use?)

One type of radioactive particle decays in 2×10^{-6} second on the average, if it's at rest. How long does it take if it's moving at 0.995 times c?

A. Length contraction B. Time dilation C. Velocity addition
D. Absolute interval

Practice problem: recognize what concept to use.

What kind of problem is this? (What formula should you use?)

In my car, 1 km long and moving at 99% of the speed of light, I flash my headlights and taillights simultaneously; you see the flashes to be delayed by 4×10^{-6} sec. How far apart along the road are the spots where the flashes appeared to you to occur?

A. Length contraction B. Time dilation C. Velocity addition
D. Absolute interval

Practice problem: recognize what concept to use.

What kind of problem is this? (What formula should you use?)

I throw a meter stick, so that it moves parallel to its length; it looks to you to be only half a meter long. How fast is it moving, relative to us?

A. Length contraction B. Time dilation C. Velocity addition
D. Absolute interval

Experimental tests of relativity

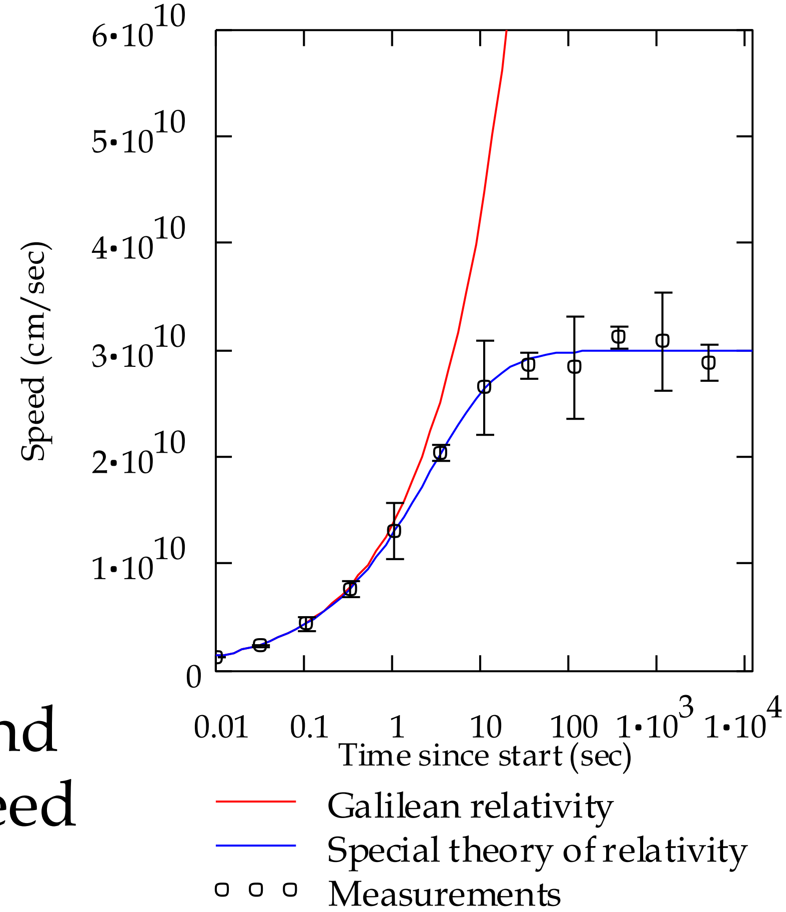
Einstein's theories of relativity represent a rebuilding of physics from the ground up.

- New postulates and assumptions relate the most basic concepts, such as the relativity of space and time and the "absoluteness" of the speed of light.
- The new theory is logically consistent and mathematically very elegant.
- The new theory *contains* classical physics, as an approximation valid for speeds much smaller than the speed of light.
- Still: it would be worthless if it didn't agree with reality (i.e. experiments) better than classical physics.

Experimental tests of scientific theories

No scientific theory is valid unless:

- it is mathematically and logically consistent,
- **and** its predictions can be measured in experiments,
- ***and*** the theoretical predictions are in precise agreement with experimental measurements.

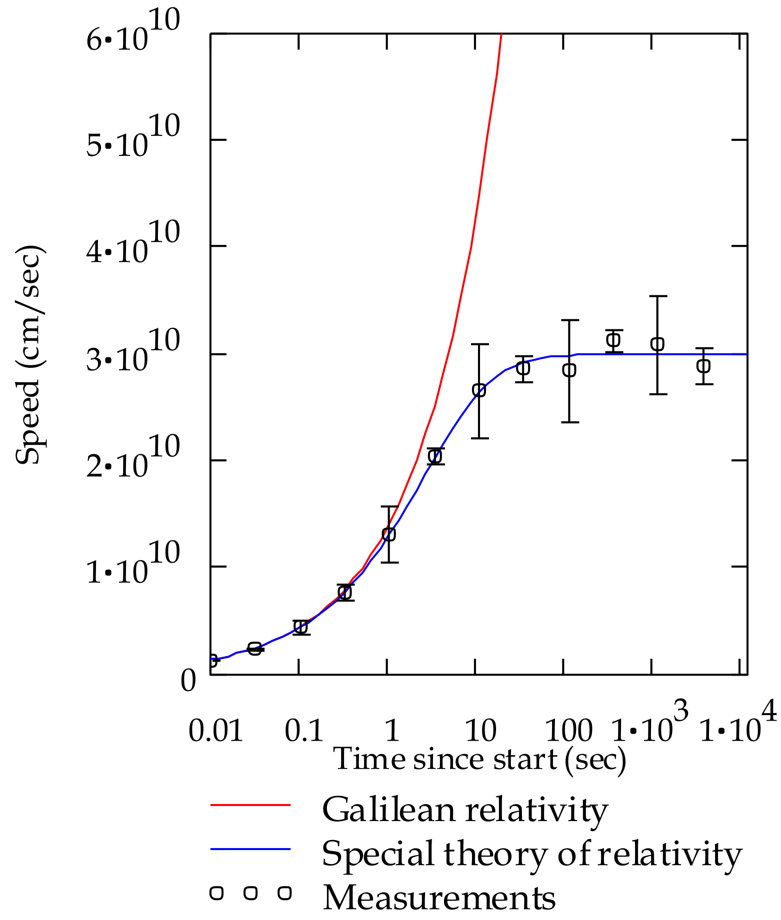


Example: prediction by the "**old**" and "**new**" relativity theories for the speed of a body accelerated with constant power. Both theories are valid for low speeds but only special relativity is valid over the whole range of measurements.

Experimental tests of scientific theories (continued)

What constitutes a valid scientific experiment?

- ❑ Measurements are made with accuracy (the size of potential measurement errors) sufficient to test the predictions of prevailing theories,
- ❑ **and** the accuracy can be estimated reliably,
- ❑ ***and*** the measurements are **reproducible**: the same results are obtained whenever, and by whomever, the experiments are repeated.

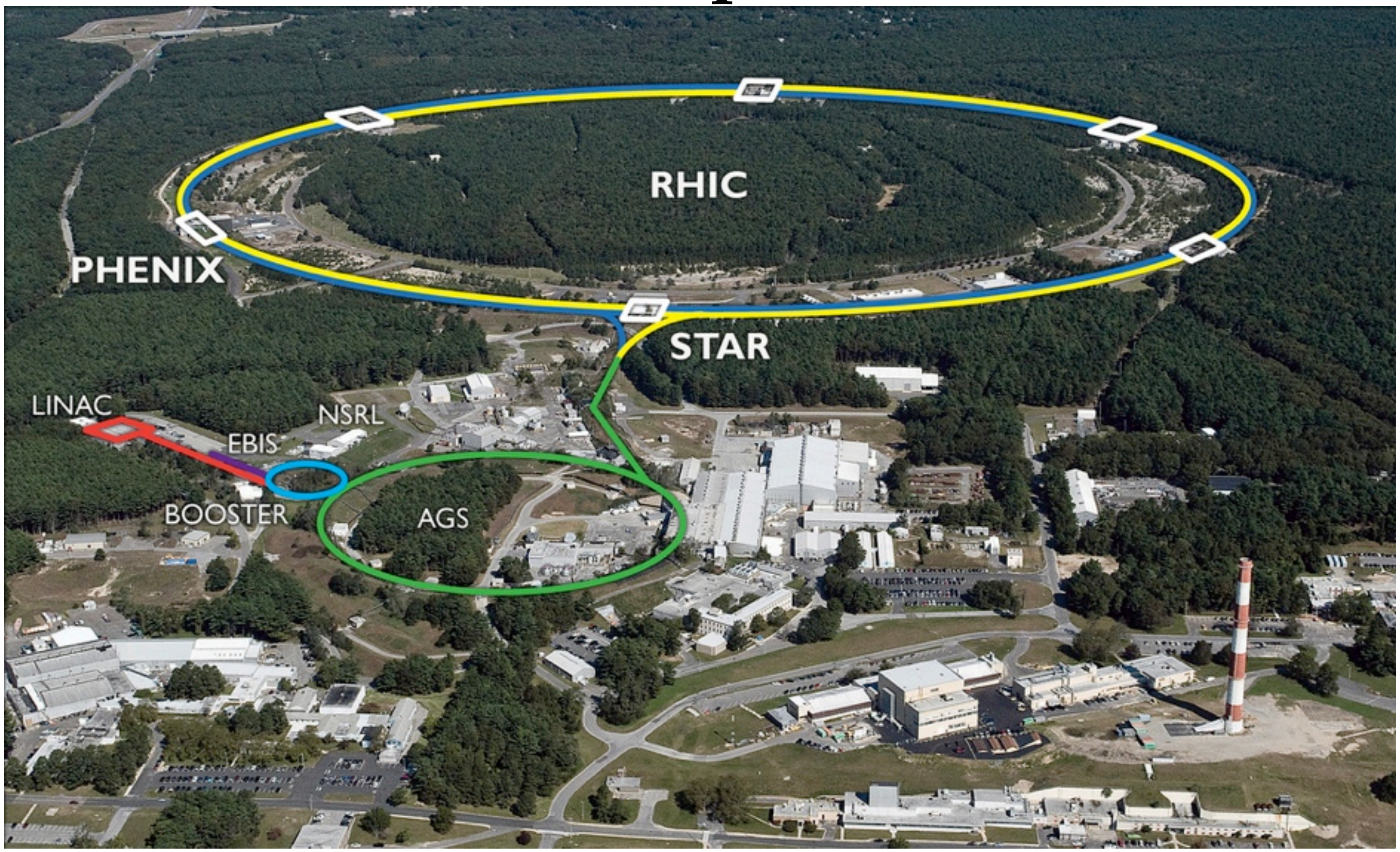


Experimental tests of relativity (continued)

Some experiments testing special relativity:

- ❑ Michelson or Michelson-Morley type: speed of light always the same in all directions, to **extremely** high accuracy. Repeated many times, and not just in Cleveland.
- ❑ **High-energy accelerators** used in elementary particle physics:
  - radioactive particles are seen to live much longer when moving near light speeds than when at rest (direct observation of time dilation).
  - though accelerated particles get extremely close to the speed of light, none ever exceed it.
- ❑ Nuclear reactors/bombs: mass-energy equivalence ($E=mc^2$).

Every day, many millions of special-relativity experiments



The Relativistic Heavy Ion Collider at BNL.

Recreating the early universe!

Done!
Stars and Dust Across Corona Australis.